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Some properties of q-regularity and q-normality in supra topological spaces

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Abstract

The objective of this article is, first, to introduce and investigate the notions of supra q-regular and supra q-normal by using the concepts of q-regularity and q-normality. Second, to introduce some types of supra q-regular and supra q-normal continuous mappings and investigate some properties and point out their main features of them. Finally, we introduce and investigate supra q-regular (resp; supra q-normal) homeomorphism and supra q-regular (resp; supra q-normal) irresolute map also supra q-regular (resp; supra q-normal) contra continuity map and investigate some properties of these mappings with several examples are presented.

Keywords: supra q-regular, supra q-normal, supra irresolute map, supra contra continuity map.
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1 Introduction

Mashhour et al. [7] introduced the supra topological spaces and studied s-continuous maps and $s*$-continuous maps. In 2010, Sayed and Noiri [8] introduced on supra b-open sets and supra b-continuity. After that in 2016 appeared the concept of supra-R-open sets [5]. Lately, Abo-elhamayel and Al-shami [1] introduced and studied supra open (supra closed, supra homeomorphism) maps in supra ordered topological spaces.

Finally, Latif [6] was introduced the concept of supra-R-compactness and supra-R-connectedness. In the present work, we established the concepts of q-regularity and the q-normality and we study the relationship among these types with the help of examples and investigate the equivalent conditions for each concept. Finally, we introduce some types of continuous mappings and homeomorphism continuous mappings with investigate some properties of these mappings with various examples given.

2 Preliminaries

Throughout this paper (X, τ) and (Y, σ) are represent two different topological space denote the closure, the interior and the complement of $H \subseteq X$ by $cl(H)$, $int(H)$ and H^c respectively.

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We recall the following definitions which are using in our work.

Definition 2.1. [2] A $\tau \subseteq X$ is said to be supra topology on X if

(i) $X, \emptyset \in \tau$.

(ii) If $H_i \in \tau, \forall i \in J$, then $\bigcup_{i\in J} H_i \in \tau$.

Then (X, τ) is called a supra topological space. The element of τ are called supra open sets in (X, τ) (for short, $sup\tau$)and the complement of supra open set is called supra closed sets and it is denoted by $sup\tau^c$.

Definition 2.2. [2] The supra closure and supra interior of a set H is defined by:

$$\text{sup}cl(H) = \cap\{N : N \text{ is sup } \tau^c \text{ and } H \subseteq N\}$$

and

$$\text{sup}int(H) = \cup\{N : N \text{ is sup } \tau \text{ and } H \supseteq N\}.$$

Definition 2.3. [3] Let (X, τ) be a supra topological space. A subset H of X is called a supra pre-open set (for short, suppo-set), if $H \subseteq cl(int(H))$. The complement of suppo-set is a supra pre-closed set (for short, suppc-set).

Definition 2.4. [8] Let (X, τ) be a topological space and let G be closed set such that $k \notin G$, where k is a point in X. If there exist disjoint open sets W and Q such that $k \in W$ and $G \subseteq Q$, then (X, τ) is called a regular space (for short, r-space).

Definition 2.5. [8] A topological space (X, τ) is called a normal space (for short, n-space), if for every $sup\tau^c$-sets J, P such that $J \cap P = \varnothing$ there exist two disjoint $sup\tau$-sets W and Q such that $J \subseteq W$ and $P \subseteq Q$.

Definition 2.6. [2] Let (X, τ) be a supra topological space. A subset H of X is called a supra β-open set (for short, supβo-set), if $H \subseteq cl(int(H))$. The complement of supβo-set is a supra β-closed set (for short, supβc-set).

Definition 2.7. [6] Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called supra continuous function if $f^{-1}(V)$ is a supra closed in (X, τ), for every supra closed set V of (Y, σ).

Definition 2.8. [6] Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called a supra irresolute if $f^{-1}(V)$ is a supra closed in (X, τ), for every supra closed set $V$of (Y, σ).

3 q-Regularity and q-Normality in Supra Topological Spaces

In this section, we introduce the notion of q-Regularity and q-Normality in supra topological spaces and investigate some properties of them.

Definition 3.1. Let (X, τ) be a topological space. A subset H of X is called q-closed set (for short, qc-set), if $pcl(H) \subseteq clint(U)$, whenever $H \subseteq U$, U is an open set. The complement of the qc-set is q-open set (for short, qo-set).

Definition 3.2. Let (X, τ) be a supra topological space. A subset H of X is called a supra q-closed set (for short, supqc-set) if $pcl(h) \subseteq clint(U)$, whenever $H \subseteq U$, U is a supβo- set. The complement of supqc-set is supra q-open set (for short, supqo-set).

Definition 3.3. The supra q-closure and supra q-interior of a set H is defined by:

$$\text{supra}cl(H) = \cap\{N : N \text{ is a supqc-set and } H \subseteq N\}$$

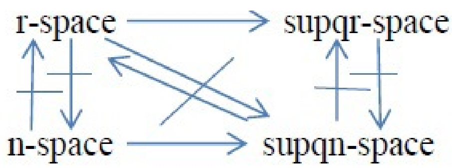
and

$$\text{supra}int(H) = \cup\{N : N \text{ is a supqo-set and } H \supseteq N\}.$$

Definition 3.4. Let (X, τ) be a supra topological space and let G be a supra of q-closed set such that $k \notin G$, where k is point in X. If there exist disjoint supqo-sets W and Q such that $k \in W$ and $G \subseteq Q$ then (X, τ) is called a supra q-regular space (for short, supqr-space).

Definition 3.5. A supra topological space (X, τ) is called supra q-normal space (for short, supqn-space), if for every supqc-sets J, P such that $J \cap P = \varnothing$ there exist two disjoint supqo-set W and Q such that $J \subseteq W$ and $P \subseteq Q$.

Proposition 3.6. Let (X, τ) be a topological space. Then the following impactions are satisfied:



Proof . r-space $\longrightarrow$ supqr-space.
Let (X, τ) be an r-space and let $k \in X$, G is a closed set such that $k \notin G$. Since every closed set is a $sup\tau^c$ set. Thus J, P are q-closed sets. Since $(X, \ \tau\ )$ is an r-space, there exist disjoint open-sets W and Q such that $k \in W$ and $G \subseteq Q$, hence W and Q are supqo. Therefore (X, τ) is supqr-space.
n-space $\longrightarrow$ supqn-space.
Let $(X, \ \tau\ )$be a n-space and let J, P are closed sets such that $J \cap P = \varnothing$. Since every closed set is q-closed set, J, P are q-closed set, since $(X, \tau\ )$ is a n-space. Then there exist disjoint open-sets W and Q such there exist two disjoint open sets W and Q such that $J \subseteq W$ and $P \subseteq Q$, hence W and Q are supqo. Therefore (X, τ) is a supqn-space. □

The converse of the above Proposition 3.6 are not true as shown by the following examples.

Example 3.7. Let $X = \{a, b, \ c\}$ and $\tau = \{X, \varnothing, \{b\}, \{a, b\}, \{b, c\}\}$. Then $(X, \tau\ )$ is a supqr-space because for every q-closed set G that is not contain k points, there exist two disjoint W and Q that are supqo such that $k \in W$ *and* $G \subseteq Q$. But $(X, \ \tau\ )$ is not r-space, because $a \notin \{b, c\}$ and there exist two open-sets $\{a, b\}, \{b, c\}$ such that $a \in \{a, b\}$ and $\{b, c\} \subseteq \{b, c\}$ but $\{a, b\} \cap \{b, c\} = \{a\} \neq \varnothing$.

Example 3.8. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \varnothing, \{a\}, \{a, b, d\}, \{a, c\}\}$. Then (X, τ) is a supnr-space because for all J, P that are two disjoint q-closed set there exist two disjoint W and Q that are supqo such that $k \in W$ and $G \subseteq Q$. But $(X, \ \tau\ )$ is not n-space, because there exist two open-sets $\{a\}, \{a, c\}$ such that $\{b\} \subseteq \{a, b, d\}$ and $\{c\} \subseteq \{a, c\}$, but $\{a, b, d\} \cap \{a, c\} = \{a\} \neq \varnothing$.

Example 3.9. Recall Example 3.7, we see that (X, τ) is supqr-space, but (X, τ) is not supqn-space, because there exist two supqo-sets $\{a, b\}, \{b, c\}$ s.t $\{a\} \subseteq \{a, b\}$ and $\{b, c\} \subseteq \{b, c\}$ but $\{a, b\} \cap \{b, c\} = \{b\} \neq \varnothing$. Also (X, τ) is not n-space, because there exist two open-sets $\{a, b\}, \{b, c\}$ such that $\{b\} \subseteq \{a, b\}$ and $\{b, c\} \subseteq \{b, c\}$ but $\{a, b\} \cap \{b, c\} = \{a\} \neq \varnothing$.

Example 3.10. Recall Example 3.8, we see that (X, τ) is supqn-space. But (X, τ) is supqr-space, because there not exist disjoint supqo-sets contain supqc-set. Also (X, τ) is not r-space, because there exist two open-sets $\{a, b\}, \{b, c\}$ such that $a \in \{a, b, d\}$ and $\{b, d\} \subseteq \{b, d\}$ but $\{a, b, d\} \cap \{b, d\} = \{b\} \neq \varnothing$.

4 Some Types of Supra q-Regular and Supra q-Normal Continuous Mappings

In this section we introduce some types of supra q (resp; supra q-regular supra q-normal) continuous mappings and investigate some properties of them.

Definition 4.1. A map $f : (X, \ \tau) \longrightarrow (Y, \sigma)$ is called supq (resp; supqr, supqn)-continuous function if the inverse image of every supqo-set in (Y, σ) is supqo (resp; supqro, supqno)-set in (X, τ).

Definition 4.2. A map $f : (X, \ \tau\ ) \ \longrightarrow \ (Y, \ \sigma)$ is called strongly supq (resp; supqr, supqn)-continuous function if the inverse image of every supqc (resp; supqrc, supqnc)-set in $(Y, \ \sigma)$ is supc (resp; suprc, supnc)-set in $(X, \ \tau)$.

Definition 4.3. A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called perfectly supq (resp; supqr, supqn)--continuous function if the inverse image of every supqc (resp; supqrc, supqnc)-set in (Y, σ) is both supqo (resp; supqro, supqno)-set and supqc (resp; supqrc, supqnc)- in (X, τ).

Proposition 4.4. Every perfectly supq (resp, supqr, supqn)-continuous function is strongly supq (resp, supqr, supqn)-continuous function.

Proof . Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a perfectly q-continuous function. Let H be supc (resp; suprc, supnc) - set in (Y,σ). Since f is perfectly supq (resp; supqr, supqn)-continuous function then $f^{-1}(H)$ is both supqo (resp; supqro, supqno) set and supqc (resp; supqrc, supqnc) set in (X,τ). Therefore f is strongly supra q-(resp; supqr, supqn)-continuous function. □

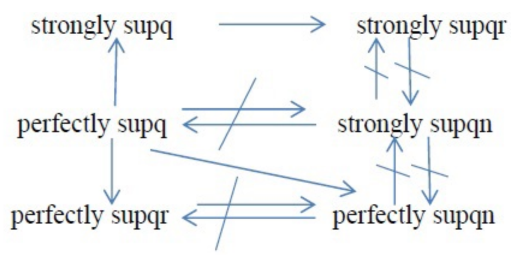
The converse of the above proposition is not be true. The following example show is not true.

Example 4.5. Let $X=Y=\{a,\ b,\ c,d\}$ and $\tau=\{X,\varnothing,\{a\},\{b\},\{b,c,d\},\{a,b\}\},\sigma\ =\{Y,\varnothing,\{a,b\}\}$. A function $f\ :(X,\tau)\longrightarrow(Y,\sigma)$ defined by $f(a)=c,\ f(b)=c,f(c)=a,f(d)=b$. Thus f is strongly supq-continuous but not perfectly supq-continuous, since $\{b,c\}$ is supqc in Y, but $f^{-1}(\{a,b\})=\{c\}$ is not supqo-set in X.

Example 4.6. Let $X=Y=\{a,b,c,d\}$ and $\tau=\{X,\varnothing,\{a\},\{b,c,d\}\}$, $\sigma=\ \{Y,\varnothing,\ \{a,b\}\}$. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ defined by $f\,(a)=d,f\,(b)=c,f\,(c)=b,f\,(d)=a$. Thus f is strongly supqr-continuous but not perfectly supqr-continuous, since $\{b,c\}$ is supqrc in Y, but $f^{-1}(\{b,c,d\})=\{a,b,c\}$ is not supqro-set in X.

Example 4.7. Let $X=Y=\{a,b,c\}$ and $\tau\ =\{X,\varnothing,\{a\},\{a,b\}\}$, $\sigma\ =\{Y,\varnothing,\{a,b\}\}$. A function $f\ :(X,\ \tau)\longrightarrow(Y,\sigma)$ defined by $f\,(a)=c,f\,(b)=b,f\,(c)=a$. Thus f is strongly supqn -continuous but not perfectly supqn-continuous, since $\{b,c\}$ is supqc in Y , but $f^{-1}(\{a,c\})=\{a,c\}$ is not supqno-set in X .

Proposition 4.8. Let (X,τ) be a topological space. Then the following impaction are satisfied:



Example 4.9. Recall Example 4.6, we see that (X,τ) is strongly supqr-continuous function. But (X,τ) is not strongly supq-continuous function, also (X,τ) is not strongly supqn continuous function.

Example 4.10. Recall Example 4.7, we see that (X,τ) is strongly supqn-continuous function. But (X,τ) is not perfectly supq-continuous function, also (X,τ) is not perfectly supqr-continuous function.

Example 4.11. Let $X=Y=\{a,b,c,d,e\}$ and $\tau\ =\{X,\ \varnothing,\{a\},\{a,b,c\},\{b,c\}\}$, $\sigma=\{Y,\varnothing,\{a\},\{a,b\}\}$. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ defined by $f\,(a)=d,f\,(b)=e,f\,(c)=a,f\,(d)=b,f\,(e)=a$. Thus f is perfectly supqr -continuous but not perfectly supqn-continuous, since $\{d,a\}$ is supqnc in Y , but $f^{-1}(\{d,a\})=\{a,e\}$ is not supqno-set in X. Also, (X,τ) is not perfectly supq-continuous function.

Example 4.12. Let $X=Y=\{a,b,c\}$ and $\tau=\{X,\varnothing,\ \{c\},\{a,b\}\}$, $\sigma=\{Y,\varnothing,\{a,b\}\}$. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ defined by $f\,(a)=b,\ \ f\,(b)=a,\ \ f\,(c)=a$. Thus f is perfectly supqn-continuous but not perfectly supqr-continuous, since $\{b,c\}$ is supqrc in Y, but $f^{-1}(\{a\})=\{b,c\}$ is not supqro-set in X . Also, $(X,\tau\ )$ is not perfectly supq-continuous function.

Proposition 4.13. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be strongly supq (resp, supqr, supqn)-continuous and $g:\ (Y,\sigma)\longrightarrow(Z,\delta)$ be strongly supq (resp, supqr, supqn)-continuous then their composition $g\circ f:(X,\tau)\longrightarrow(Z,\delta)$ is a strongly supq (resp, supqr, supqn)- continuous function.

Proof . Let H be supq (resp, supqr, supqn)-set in (Z,δ). Since g is strongly supq (resp, supqr, supqn)-continuous, $g^{-1}(H)$ is supqc (resp, supqrc, supqnc)-in (Y,σ). Since every supra closed set is supqc (resp, supqrc, supqnc)-set, $g^{-1}\,(H)$ is supqc (resp, supqrc, supqnc)-set in (Y,σ). Since f is strongly supq (resp, supqr, supqn)-continuous, $f^{-1}(g^{-1}(H))$ is supqc (resp, supqrc, supqnc)-set in (X,τ), thus $(g\circ f)(H)$ is supqc (resp, supqrc, supqnc)-set in (X,τ). Therefore $g\circ f$ is strongly supq (resp, supqr, supqn)-continuous function. □

5 Supra q-Regular Homeomorphism and Supra q-Normal Homeomorphism Continuous Mappings

In this section, we introduce supra q (resp; supra q-regular supra q-normal) homeomorphism and supra q (resp; supra q-regular supra q-normal) irresolute map also supra q (resp; supra q-regular supra q-normal) contra continuity map normal and investigate some properties of them.

Definition 5.1. A bijection $f : (X, \tau\ ) \longrightarrow (Y, \sigma)$ is called supq (resp; supqr, supqn)-homeomorphism if f is both supq (resp; supqr, supqn)-continuous function and f^{-1} is supq (resp, supqr, supqn)-continuous function.

Proposition 5.2. Let $f : (X,\ \tau) \longrightarrow (Y, \sigma)$ be a bijective supq (resp; supqr, supqn)-continuous map. Then the following are equivalent

1) f is a supq (resp; supqr, supqn)-open map
2) f is a supq (resp; supqr, supqn)--homeomorphism
3) f is a supq (resp; supqr, supqn)-closed map.

Proof . 1) $\Longrightarrow$ 2) If f is a bijective supq (resp; supqr, supqn)-continuous function, suppose (1) holds. Let H be supqc (resp; supqrc, supqnc)-set in (X, τ) then H^c is supqo (resp; supqro, supqno) in $(X, \tau\ )$. Since f is supq (resp; supqr, supqn)-open map, $f(H^c)$ is H^c-open in (Y, σ). Hence $f(H)$ is supqc (resp; supqrc, supqnc)-set in (Y, σ). Thus f^{-1} is supq (resp; supqr, supqn)-continuous. Therefore, f is a supq (resp; supqr, supqn)-homeomorphism.
2) $\Longrightarrow$ 3)
Suppose f is a supq (resp; supqr, supqn)-homeomorphism and f is bijective supq (resp; supqr, supqn)-continuous function then from the definition 5.1, we get $f^{-1}$is supq (resp; supqr, supqn)-continuous. Therefore, f is supq (resp; supqr, supqn)-map.
3) $\Longrightarrow$ 1)
Suppose f is supq (resp; supqr, supqn)-map. Let H be supqo (resp; supqro, supqno)-set in (X, τ) then $H^c$is supqc (resp; supqrc, supqnc)-in(X, τ). Since f is supq (resp; supqr, supqn)-map, $f(H^c)$ is supq (resp; supqr, supqn)-set in (Y, σ). Thus $f(H)$ is supqo (resp; supqro, supqno)-set in (Y, σ). Therefore f is a supq (resp; supqr, supqn)-map. □

Remark 5.3. The composition of two supq (resp; supqr, supqn)-homeomorphism are not be a supq (resp; supqr, supqn)-homeomorphism. Since composition of two supq (resp; supqr, supqn)-continuous function are not supq (resp; supqr, supqn)-continuous and composition of two supq (resp; supqr, supqn)-map are not supq (resp; supqr, supqn)-closed map. The following example show this case.

Example 5.4. Let $X = Y = Z = \{a, b, c, d\}$ and $\tau = \{X,\ \varnothing, \{a\}, \{b, c, d\}\}, \sigma = \{Y, \varnothing, \{b\}\}, \delta = \{Z, \varnothing, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = c,\ f(b) = b, f(c) = a, f(d) = d$. and $g : (Y, \sigma) \longrightarrow (Z, \delta)$ be the function defined by $g(a) = c,\ g(b) = b,\ g(c) = d, g(d) = a$. Thus f and g is supq-closed map, but the composition is not supq-closed map, since $g \circ f(\{a, d\}) = \{c, d\}$ is not supq-closed map in Z. Therefore $g \circ f$ is not supq-homeomorphism.

Example 5.5. Let $X = Y = Z = \{a, b, c\}$ and $\tau = \{X,\ \varnothing,\ \{a\}, \{b, c\}\}, \sigma = \{Y, \varnothing,\ \{b\}\}$. $\delta = \{Z, \varnothing, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b,\ f(b) = c,\ f(c) = a$. and $g : (Y, \sigma) \longrightarrow (Z, \delta)$ be the function defined by $g(a) = b, g(b) = a, g(c) = c$. Thus f and g is supqr-closed map, but the composition is not supqr-closed map, since $gof\ (\{a, c\}) = \{a, b\}$ is not supqr-closed map in Z. Therefore $g \circ f$ is not supqr-homeomorphism.

Example 5.6. Let $X = Y = Z = \{a, b\}$ and $\tau = \{X, \varnothing, \{a\}, \{b\}\}, \sigma = \{Y, \varnothing, \{a\}\}$. $\delta = \{\ Z,\ \varnothing\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b, f(b) = a$. and $g : (Y, \sigma) \longrightarrow (Z, \delta)$ be the function defined by $g(a) = b,\ g(b) = b$,. Thus f and g is supqn-closed map, but the composition is not supqn-closed map, since $g \circ f(\{a\}) = \{b\}$ is not supqn-closed map in Z. Therefore $g \circ f$ is not supqn-homeomorphism.

Definition 5.7. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called supq (resp; supqr, supqn)- irresolute map if $f^{-1}(H)$ is supqc (resp; supqrc, supqnc)-set in (X, τ) for every supqc (resp; supqrc, supqnc)-set H of (Y, σ).

Proposition 5.8. Every supq (resp; supqr, supqn)-homeomorphism is supqc (resp; supqrc, supqnc)-continuous.

Proof . It is obvious. □

The converse of the above theorem need not be true. The following example show this case.

Example 5.9. Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \varnothing, \{a\}, \{b, c\}\}, \sigma = \{Y, \ \varnothing, \ \{a, b\}, \{b, c\}, \{b\}\}$. Let $f : (X, \tau\ ) \longrightarrow (Y,\ \sigma)$ be the identity function. Hence, f is supqr-continuous but not supqr-homeomorphism, since f^{-1} is not supqrc-continuous.

Example 5.10. Recall Example 4.7, we see that f is supqn-continuous but not supqn-Homeomorphism, since f^{-1} is not supqnc-continuous.

Example 5.11. Recall Example 4.5, we see that f is supq-continuous but not supq-Homeomorphism, since f^{-1} is not supqnc-continuous.

Proposition 5.12. Every supq (resp; supqr, supqn)-homeomorphism is supqc (resp; supqrc, supqnc)-irresolute.

Proof . It is clear. □

The converse of the above theorem need not be true. It is shown by the following example.

Proposition 5.13. Every supq (resp; supqr, supqn)-irresolute is strongly supqc (resp; supqrc, supqnc)-closed map.

Proof . Since $f : (X, \tau) \longrightarrow (Y, \sigma)$ is supqc (resp; supqrc, supqnc)-irresolute, f^{-1} is supra is supqc (resp; supqrc, supqnc)-irresolute. Therefore, f is strongly is supqc (resp; supqrc, supqnc)-closed map. □

The converse of the above theorem need not be true. It is shown by the following example.

Example 5.14. Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \varnothing, \{a, b\}, \{b,\ c\}, \{b\}\}$, $\sigma = \{Y,\ \varnothing,\ \{a\}, \{b, c\}\}$. Let $f : (X, \tau\ ) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b,\ f(b) = c,\ f(c) = a$. Thus f is strongly supq-closed but not f is not supq-irresolute, since $f^{-1}$is not supq-irresolute because f is not supq-irresolute.

Example 5.15. Let $X = Y = \{a, b, c, d\}$ and $\tau = \{X,\ \varnothing, \{a, b\}, \{b,\ c\}\}$, $\sigma = \{Y,\ \varnothing, \{a\}, \{b, c\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b,\ f(b) = c,\ f(c) = a, f(d) = d$. Thus f is strongly supqr-closed but not supq- irresolute, because f is not supq-irresolute.

Example 5.16. Let $X = Y = \{a, b, c\}$ and $\tau = \{X,\ \varnothing, \{c\}, \{b,\ c\}\}$, $\sigma = \{Y, \varnothing, \{a\}\}$. Let $f : (X, \tau\ ) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = a,\ f(b) = a,\ f(c) = a$. Thus f is strongly supqn-closed but not supqn-irresolute, because f is not supqn-irresolute.

Definition 5.17. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called supq (resp; supqr, supqn)-contra continuity if $f^{-1}(H\ )$ is supqc (resp; supqrc, supqnc)-set in (X, τ) for every supqpc (resp; supqprc, supqnpc)-set H in (Y, σ), where supqpc (resp; supqprc, supqnpc) are supra q pre closed set (resp; supra q-regular pre closed set, supra q-normal pre closed set).

Proposition 5.18. Every supq (resp; supqr, supqn)-irresolute is supqc (resp; supqrc, supqnc)-contra continuity map.

Proof . Since $f : (X, \tau\ ) \longrightarrow (Y, \sigma)$ is supq (resp; supqr, supqn)-irresolute, f^{-1} is supra is supqc (resp; supqrc, supqnc)-irresolute. Since every supra closed set is supra pre closed implies f^{-1} is supqc (resp; supqrc, supqnc)-supqpc (resp; supqprc, supqnpc)-set. Therefore, f is supq (resp; supqr, supqn)-contra continuity map. □

The converse of the above theorem need not be true. It is shown by the following example.

Example 5.19. Let $X = Y = \{a, b, c\}$ and $\tau = \{X,\ \varnothing, \{a, b\}, \{b, c\}, \{b\}, \{a\}\}$, $\sigma = \{Y, \varnothing, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b, f(b) = c,\ f(c) = a$. Thus f is supq-contra continuity but not f is not supq-irresolute, since f^{-1} is not supq-irresolute because f is not supq-irresolute.

Example 5.20. Let $X = Y = \{a, b, c, d, e\}$ and $\tau\ = \{X, \varnothing, \{a, b\}, \{c, d, e\}\}, \sigma = \{Y, \varnothing, \{a\}, \{b, c\}, \{d, e\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = b, f(b) = e,\ f(c) = a, f(d) = c, f(e) = a$. Thus f is supqr-contra continuity but not supq-irresolute, because f is not supq-irresolute.

Example 5.21. Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \varnothing, \{c\}, \{b, c\}\}$, $\sigma\ = \{Y, \varnothing, \{c\}\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the function defined by $f(a) = c, f(b) = a,\ f(c) = b$. Thus f is strongly supqn-closed but not supqn-irresolute, because f is not supqn-irresolute.

6 Conclusion

We get the new ideas such supq (resp; supqr, supqn)- and concentrated on certain relations them, later that we presented supq (resp; supqr, supqn)-consistent mappings and emphatically supq (resp; supqr, supqn)-persistent mappings and impeccably supq (resp; supqr, supqn)-ceaseless capacity explore a few properties of them. At long last, we presented supq (resp; supqr, supqn)-homeomorphism mappings and supq (resp; supqr, supqn)-indecisive mappings and supq (resp; supqr, supqn)-contra coherence guide and we examined a portion of their connections.

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