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A risk averse robust portfolio optimization under severe uncertainties by using IGDT approach: Iran Stock Exchange

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Abstract

Portfolio optimization in finance and economy is more than a mathematical model for improving performance under uncertainty constraints. Practically all organizations seek to create value by selecting the best portfolios that consume the least resources and obtaining high expected portfolio returns and controlling risk. In the context of the portfolio selection problem, severe uncertainties would significantly affect the technical and financial aspects. This paper presents a bi-level information gap decision theory (IGDT) risk averse decision-making tool for robust portfolio optimization problems to help organizations or investors for managing their portfolios and finding the best transactions with severe uncertainty variables (price and return) to process the forecast data generated by the learning prediction method in order to construct the optimal stock portfolios that a target profit is guaranteed. The heuristic solution approach is constructed and the augmented ε-constraint method is used to solve the proposed bi-level IGDT robust optimization problem. The effectiveness and efficiency of the proposed model are evaluated on the Iranian Stock Market. The results show the efficiency and effectiveness of the proposed model for selecting the best stocks. The Mont Carlo simulation method is applied for the validation of results.

Keywords: robust portfolio optimization, severe uncertainties, information gap decision theory, bi-level model, Mont Carlo simulationk risk averse
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1 Introduction

Portfolio optimization means that how to selecting the number of items for allocating to assets in different market. The first mathematical model for portfolio selection was presented by Markowitz [33, 34] that is evaluating the mean and variance of investments. A basic and important assumption of Markowitz mathematical model is that the investor knows the exact expected return but in reality, the true expected is not occurred and a lot of factors can change the results [30]. For this reason, there are not enough historical data about distribution of return and it is not easy to forecast the investment return accurately [40].

The classical portfolio not considering the estimation error and performing not strongly in uncertain conditions. Therefore, it is needed to construct a portfolio optimization model considering data uncertainty with statistical methods

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and experts' experience to forecast return of investment. For this reason, factor models are used for solving and considering the unexpected return to evaluating the performance of portfolio managers, to assess return risk, to predict returns, and to construct portfolios [37, 12]. There are some various fund-separation theorems considering the uncertainty such as Asset pricing models Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) [42]. Financial time series forecasting method for managing risk of stock price risk analysis is one of the most difficult problems for researchers and plays important role in trading strategies to identifying opportunities to buy and sell stock [48].

Uncertainties from various contributing source for predicting and forecasting the future return based on historical data is the biggest and most important challenge for any investment. Sources of uncertainty may be divided into two types: aleatory and severe [38, 27].

Aleatory uncertainty is not simplified phenomena that exhibiting natural variation such as different conditions of random event. Severe or epistemic uncertainty results from a less knowledge about the subjective data of system or parameters and approximations in the behavior models that caused reduce obtaining information about the system.

Severe uncertainty is a type of model parameters can be defined with reference to a stochastic quantity whose distribution type or distribution parameters are not precisely known [2], or with reference to a deterministic quantity whose value is not precisely known [22].

Uncertainty in some cases is appeared with distribution function of a random variable that is available showed as intervals given by experts. As a result, the stochastic optimization methods with a risk measurement are employed the uncertain variables are considered by the suitable probability distribution functions. In portfolio optimization stochastic programming is used for select the best choice for buy and sell [25, 49]. In this paper because severe uncertainty the stochastic optimization is applied.

One of the best methods for controlling the uncertainty of stock market is prediction methods and data sources commonly used methods were modeling the relationship between the historical behavior and future movement of the price, and using historical market samples to predict the future trend or value of the price [51]. For financial time traditional statistical methods such as linear regression, auto-regression and moving average (ARMA), and GARCH (Generalized Auto Regressive Conditional Heteroskedasticity) were much used for predict the future.

Robustness methods are another important method for controlling uncertainty as defined the ability of a system to be insensitive to small departures from the assumptions depending that the system operating correctly in the presence of uncertain environmental conditions [21, 23, 35, 17, 50].

The essential segments and components in robust optimization methods are: 1. ensuring objective robustness, 2. ensuring feasibility robustness, 3. estimating mean and measure of variation (e.g., variance) of the performance function, and 4. multiobjective optimization [15].

A detailed description of four elements of robust optimization method can be found in Conditional value at risk (CVaR) is used as risk measurement tool formulated in the target function or constraints of optimization model to achieving profit volatility. The optimality of stochastic optimization methods or robust optimization and the results for dealing with risk by using a measurement index (CVaR) depending on the precision of estimated number of scenarios considered within the optimization procedure the absence of sufficient historical data leads to inaccurate fitted PDFs and consequently wrong results [44]. In one hand risk aversion approach is applied for non-expected utility theories and for uncertainty situation [36]. The risk aversion is a method for decreasing decisions for an increasing risk [45].

In another hand, in related with increasing the number of scenarios for complaining the uncertainty and computational complexity of the optimization problem significantly grows. In due to these reason for risk aversion and increasing the number of scenarios the information gap decision theory (IGDT) was presented for the first time by Ben [2] for maximizing the interval solution for achieving to best results. There are a few research in this field. Majidi et al [32] present review research for applying IGDT method in different aspects. Optimizing the problem for power energy hub with using the IGDT is presented by Jordehi et al [26]. Applying IGDT method for electric autonomous hybrid refueling station is a risk-constrained design presented by Sriyakul and Jermsittiparsert [47]. One research is used for portfolio optimization with IGDT that just optimize basic portfolio model. Table 1 summarizes the portfolio optimization with severe or epistemic uncertainty and with the methods which applied for considering the uncertainty.

Based on the literature review there is no risk averse study considering severe return and severe price uncertainty with IGDT. For this reason, this paper presents a novel non-deterministic and non- probabilistic risk averse method for portfolio selection based on information gap decision theory (IGDT) which has several advantages and requires forecasted values as well as lower and upper bounds of uncertain variables that are easier to obtain from historical data. One of another advantage is risk management and profit maximization are simultaneously performed.

Table 1: Literature review for the portfolio optimization with severe

Authors	Year	Risk averse	Uncertain variables	Type Uncertainty	Method	Case Study
Asadujjaman, M., Zaman,	2019	No	Return	Sever	Moment bounding approach	Chines Market
Berleant et al.	2008	No	Return	Sever	Stochastic dominance	*
Cheong et al.	2007	No	Return	Sever	Stochastic dominance	energy markets
Beck et al,	2012		Return	Sever	Robust Fuzzy optimization	*
Current reserach	-	yes	Return and Price	Sever	**IGDT**	Iran Stock Market

The main novel contributions is proposing a novel bi-level model considering severe uncertainty with using utilizing IGDT as a new non-deterministic and non-probabilistic method for portfolio selection for the first time that has no assumption on the probabilistic estimation of the uncertain variables and it can be using forecasted variables with severe uncertainties. For solving the problem an efficient heuristic solution model is presented to defining any kind of uncertainty measurement index to handle the risk of uncertain variables and guaranteeing a target profit different from the stochastic optimization methods.

The remainder of proposed paper is organized as follows: In Section 2, a deterministic mathematical formulation of portfolio selection including the objective function and constraints is described. Robust portfolio optimization recast into its robust counterpart using IGDT based method and the solution procedure is presented in Sections 3 and 4 respectively. Heuristic solution approach is given in Section 5. Numerical example is shown in Section 6. Validation result is illustrated in section 7. Finally conclusion of some main findings are shown in section 8.

2 Deterministic model

A flowchart of description framework for this novel paper is illustrated in figure 1. An investor in stock market for portfolio selection is responsible for economic. The proposed model in this paper following the optimization processes and decisions. In a time series period the model receives the data from the related of uncertain variables. The forecasting information including macroeconomic variables such as return and price using predication methods extreme learning machine method. Then short-term operational decisions such as the quantity of selling and buying stock price to helping investors in their portfolio management are made by optimal results, including transactions (purchase/sale/hold) with the selecting best portfolios.

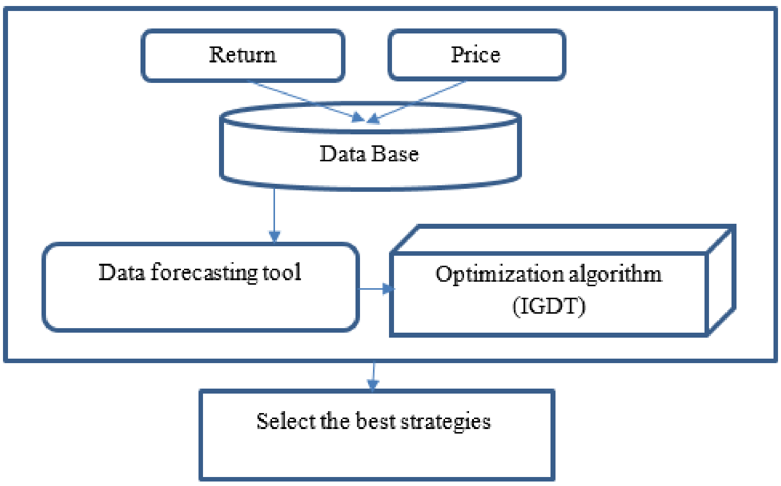


Figure 1: Graphical description of the proposed model

Table 2: Variables and parameters and indices of paper

Indices		p_{0i}	The first price for assets i
t, t'	Indices of optimization period	$\widetilde{\lambda_{t,i}^{p}}$	Forecasted price of assets i in period t
i	Indices for asset i	$\widetilde{\xi_{t,i}^{r}}$	Forecasted expected return of assets i in period t
n	Number of assets	$\overline{\Delta_{r_i}}, \underline{\Delta^{r_i}}$	Minimum and maximum quantity l for return asset i
Parameters		$\overline{\Delta_{p_i}}, \underline{\Delta^{p_i}}$	Minimum and maximum quantity for price asset i
r_i	The expected value of return for asset i	$\text{Profit}^{\text{exp}}$	Expected profit
σ_{ij}	The covariance of the return between assets i and j	DV_1	Target profit level 1
V_i	The maximum allowable portfolio risk	DV_2	Target profit level 2
		ε	Small number
LB_i	The vectors of lower bound of decision variables for assets i	*Variables*	
UB_i	The vectors of upper bound of decision variables for assets i	$\propto i$	Value of uncertainty horizon of return value of asset i
$\check{F}$	Robustness function in IGDT method	βi	Value of uncertainty horizon of return value of price asset i
r_i	Expected value of return for asset i,	λ_{pi}	Uncertainty horizon of return
σ_{ij}	The covariance of the return for assets i and j	ξ_{ri}	Uncertainty horizon of price
p_i	Price of assets i	x_i	Fraction of capital invested in asset i
w_i	Weighting coefficients for assets i		
ϑ_i	Weighting coefficients for covariance i		
$\underline{\sigma_{ij}}\overline{\sigma_{ij}}$	Minimum and maximum for covariance		

In reality only a small number of data points may be available for the input variables and a lot of lack for missing the exact values therefore information about random input variables may only be specified as intervals by gathering data from expert opinions. Based on the first mathematical model for portfolio optimization developed by Markowitz [33, 34], this paper by considering the severe uncertainty proposes a new methodology for robustness-based portfolio has to take into account for these input data uncertainty (severe uncertainty), causing uncertainty regarding the expected value and covariance of the returns and prices. For a given level of severe uncertainty the investor may choose the portfolio with the highest expected return. Table 1 shows the variables and indices used in this paper.

The basic classical formulation for maximizing the expected return for an upper limit on the variance can be written as follows:

$$\max f(x_i) = w \sum_{i=1}^{n} r_i x_i \tag{2.1}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq V \tag{2.2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{2.3}$$

where x_i is the fraction of capital invested in asset i, x_j is the fraction of capital invested in asset j, r_i is the expected value of return for asset i, σ_{ij} is the covariance of the return between assets i and j, and V is the maximum portfolio risk. There are different risk-return measures for portfolio optimization which can be maximized considering mean or median and risk.

Some studies are based on different approaches: 1. minimizing variation of variance [8, 11, 14], 2. minimizing lower semi-variance is another approach for solving the basic portfolio model [6], 3. mean absolute deviation [46, 28].In robust knowledge sample median is used instead of sample mean because it is not affected by the outlier which are : Value-at-risk [18], conditional value-at-risk [41, 24, 20], partitioned value-at-risk [20, 7], asymmetry- robust value-at-risk [13], worst-case value-at-risk [24, 31], worst-case polyhedral value-at-risk [53], worst-case quadratic valueat-risk [53], sharp ratio [39].

In the rest of the paper for the proposed model robust optimization model is applied.

3 Robust portfolio optimization

In robustness portfolio optimization all input parameters are estimated using expected values so that the resulting solution is less sensitive to the differences of the input random variables.

The proposed robustness-based portfolio optimization problem under sever uncertainty for each period t can be formulated as follows:

$$\max\ f(x_i) = w_i \sum_{i=1}^{n} r_i x_i - \nu_i \sum_{i,j=1}^{n} (x_i x_j \sigma_{ij}) \quad \forall\ t \tag{3.4}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq V_i \quad \forall\ t \tag{3.5}$$

$$\sum_{i=1}^{n} x_i = 1 \quad \forall\ t \tag{3.6}$$

$$LB_i \leq x_i \leq UB_i \quad \forall\ t \tag{3.7}$$

$$r_i = H_i(\frac{D_1}{p_{0i}} + \frac{p_{ti} - p_{0i}}{p_{0i}}) \quad \forall\ t \tag{3.8}$$

$$\underline{\Delta^{r_i}} \leq r_i \leq \overline{\Delta_{r_i}} \quad \forall\ t \tag{3.9}$$

$$\underline{\Delta^{p_i}} \leq p_i \leq \overline{\Delta_{p_i}} \quad \forall\ t$$

$$\underline{\sigma_{ij}} \leq \sigma_{ij} \leq \overline{\sigma_{ij}} \quad \forall\ t \tag{3.10}$$

$$\left[\underline{\sigma_{ij}}\overline{\sigma_{ij}}\right] = [\left(r_{ij} \times \sigma_i \times \sigma_j\right) (\overline{r_{ij}} \times \overline{\sigma_i} \times \overline{\sigma_j})] \quad \forall\ t \tag{3.11}$$

where $w_i \geq 0$ and $\nu_i \geq 0$ are the weighting coefficients that represent the relative importance of the target. In some cases, the investors are preferred by others to invest a specific amount of capital in particular assets, so the fractions of capital invested in different assets have lower and upper bounds LB and UB are the vectors of lower and upper bounds of decision variables x_i.

4 Proposed IGDT method

The information gap decision theory (IGDT) method maximizes the uncertainty horizons and finds a solution that guarantees a certain expectation for the objective. The IGDT method helps investors to maximize the robustness of its decisions, against the uncertain variables. The uncertainty model in this method does not have any assumptions on the probability distributions therefore it is suitable in the situations with high level of uncertainty or missing of sufficient historical data [4]. In this paper for portfolio decisions macroeconomic variables return and price forecasts for every day with machine learning techniques then IGDT-based decisions guarantee a specified target profit.

The IGDT method is essentially based on the gap between actual and forecasted values of uncertain variables. We model uncertain price and return with IGDT method as a risk aversion method as given in (4.12) and (4.13), respectively:

$$\lambda_{pi}\left(\propto, \widetilde{\lambda_{t,i}^{p}}\right) = \left\{ \lambda_{t,i}^{p} \middle| - \propto i \leq \frac{\lambda_{t,i}^{p} - \widetilde{\lambda_{t,i}^{p}}}{\widetilde{\lambda_{t,i}^{p}}} \leq \propto i \right\} \tag{4.12}$$

$$\xi_{ri}\left(\beta, \widetilde{\xi_{t,i}^{r}}\right) = \left\{ \xi_{t,i}^{r} \middle| -\beta i \leq \frac{\xi_{t,i}^{r} - \widetilde{\xi_{t,i}^{r}}}{\widetilde{\xi_{t,i}^{r}}} \leq \beta i \right\} \tag{4.13}$$

The objective of the IGDT method is to maximize the robustness that a target profit is achieved. In this way, the uncertainty modelling and optimization are accomplished together:

$$\widehat{\Gamma}\left(DV_1, DV_2, Profit^{\exp}\right) = \max_{DV_1}\left\{\left(\alpha, \beta\right)\middle|\max_{DV_2}\text{Profit}(\lambda_{pi}, \xi_{ri})\right\}$$
$$\geq Profit^{\exp} \times (1-\sigma i) \qquad \forall\, t \tag{4.14}$$

The IGDT method should handle two kinds of variables: decision variables of portfolio selection as indicated in x_i and uncertainty horizon of macroeconomic variables $(\alpha i, \beta i)$ as indicated in:

$$DV_1 = \{x_i\} \tag{4.15}$$
$$DV_2 = \{\alpha i, \beta i, \lambda_{pi}, \xi_{ri}\} \tag{4.16}$$

The proposed IGDT-based formulation in is a bi-level problem which explained in the next section.

4.1 Bi-level IGDT based problem

The risk aversion bi-level IGDT based problem can be formulated as follows:

$$\text{Profit} = \max_{DV_1}\ (x_i) = w_i \sum_{i=1}^{n} r_{it} x_i - \nu_i \sum_{i,j=1}^{n} (x_i x_j \sigma_{ij}) \quad \forall \in t \tag{4.17}$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \leq V_i \quad \forall \in t \tag{4.18}$$
$$\sum_{i=1}^{n} x_i = 1 \quad \forall \in t \tag{4.19}$$
$$LB_i \leq x_i \leq UB_i \quad \forall \in t \tag{4.20}$$
$$r_i = H_i\left(\frac{D_i}{pi_0} + \frac{f(\lambda_{pi}, \xi_{ri}) - p_{i0}}{p_{i0}}\right) \tag{4.21}$$
$$\underline{\Delta^{r_i}} \leq r_i \leq \overline{\Delta_{r_i}} \quad \forall\, t \tag{4.22}$$
$$\underline{\Delta^{p_i}} \leq p_i \leq \overline{\Delta_{p_i}} \quad \forall\, t \tag{4.23}$$
$$\underline{\sigma_{ij}} \leq \sigma_{ij} \leq \overline{\sigma_{ij}} \quad \forall\, t \tag{4.24}$$
$$\left[\underline{\sigma_{ij}}\overline{\sigma_{ij}}\right] = \left[(r_{ij} \times \sigma_i \times \sigma_j)\,(\overline{r_{ij}} \times \overline{\sigma_i} \times \overline{\sigma_j})\right] \quad \forall\, t \tag{4.25}$$

Level 2:

$$\text{profit} = \max_{DV_2}\ (\alpha i.\beta i) \tag{4.26}$$
$$\text{Profit}\ \ (\lambda_{pi}, \xi_{ri}) \geq Profit^{\exp} \times (1-\sigma i) \quad \forall \in t \tag{4.27}$$
$$-\propto i \leq \frac{\lambda^p_{t,i} - \widetilde{\lambda^p_{t,i}}}{\widetilde{\lambda^p_{t,i}}} \leq \propto i \quad \forall \in t \tag{4.28}$$
$$-\beta i \leq \frac{\xi^r_{t,i} - \widetilde{\xi^r_{t,i}}}{\widetilde{\xi^r_{t,i}}} \leq \beta i \quad \forall \in t \tag{4.29}$$

Since covariance is a concave function with regarding to both variance and correlation coefficient which are estimated by the methods described above. In this model we have:

$$[\overline{\sigma_{ij}}, \underline{\sigma_{ij}}] = [\underline{r_{ij}} \times \underline{\sigma_i} \times \underline{\sigma_j}\ , \overline{r_{ij}} \times \overline{\sigma_i} \times \overline{\sigma_j}] \tag{4.30}$$

In reality it is impractical to calculate the correlation coefficients among the asset returns that are described by interval data [52]. In this paper we assumed interval data of correlations among the input variables are unknown and therefore can range from −1 to 0 or 0 to +1.

First level in this bilevel model determines the short term operational decision to maximize the robustness while guaranteeing that the target profit is achieved and the second level determines the worst case of horizons of return and price respectively. In this paper a heuristic method is applied for solving the problem.

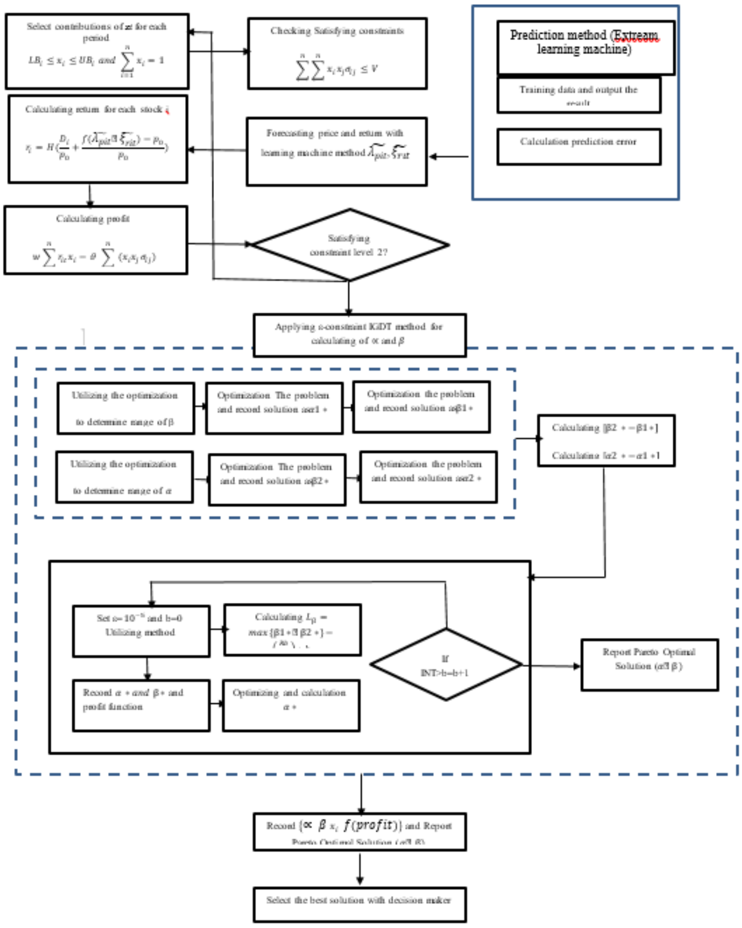


Figure 2: Heuristic solution 2

5 Solution approach

In this paper we construct for the first time a novel bilevel model with information gap decision theory for robust optimization portfolio problem. The bilevel programming problem is as an optimization problem which has another optimization problem in its constraints and includes two level such as leader and follower level. Leader in the upper level is a robust optimization model and the follower in the lower level is affected hierarchically by the leader's decisions [29]. One of the difficult approach in solving bilevel problems is that maybe a solution is optimal for the lower level but it is not feasible for the overall problem therefore finding optimal distances for α and β is difficult. In this paper we study the impact of using presented heuristic method in the lower-level problem that how near-optimal solutions on the lower level can affect the upper-level target function values. This study considers a heuristic method which can solve the problem and we can arrive to solution. All steps for this heuristic model is illustrated in figure 2.

5.1 Augmented ε-constraint method

The augmented ε-constraint method is a optimization method that the uncertainty horizon of one of the macroeconomic variables is maximized and the uncertainty horizon of another macroeconomics is divided into equal intervals through grid points [16]. Therefore, problems should be optimized to obtain Pareto optimal solutions. The augmented ε-constraint of macroeconomic variables (return and price) is indicated in:

$$\max \quad (\alpha + \varepsilon \times (\frac{S_\beta}{R_\beta}) \tag{5.31}$$

$$\beta - S_\beta = l_\beta \geq 0 \tag{5.32}$$

$$l_\beta = \beta^{\max} - \left(\frac{\beta^{\max} - \beta^{\min}}{iNT}\right) \times b \qquad b = 0, 2m \ldots int \tag{5.33}$$

where, ε is a small number typically between 10^{-3} and 10^{-6}, S_β are slack variables and l_β are the maximum and minimum uncertainty horizons.

The Pareto optimal solutions are obtained by applying different value of b solving the single Pareto set is achieved, but the computational time is increased. As a result, a trade-off between density of Pareto set and computational time is needed.

5.2 Forecasting method

In time series forecasting methods Extreme learning machine (ELM) is a powerful training algorithm based on statistical approaches for single hidden layer feed forward neural network (SLFN) that converges much faster than traditional models. Another forecasting model like ARIMA models can only be applied to stationary time series which properties not depend on the time and they fail to capture seasonality [24].
The notations $(zRi, z'Pi')$ and $(oRi', o'Pi')$ show the input and output vectors. F and M represent the weight from input to hidden layer and the bias of hidden layer and β shows the output weight. The formulations for forecasting return and price are as follow:

$$\sum_{i'} G_{i'} g(F_{i'} zRi' + T) = \boldsymbol{oRi'} \tag{5.34}$$

$$\sum_{i'} G_{i'} g(M_{i'} z'Pi' + T') = \boldsymbol{o'Pi'} \tag{5.35}$$

6 Numerical results

In this study, stocks of 10 companies of Tehran Stock Exchange are selected as portfolio. Because of reducing the correlations between stocks these companies are chosen from among different industries with random techniques. Time series historical data for uncertainty variable price are collected from archive of Finance information processing of Iran (FIPIRAN) between 02/01/2010 to 13/07/2022. And returns time series data consists of 1016 observations which are divided to 472 out sample observations for evaluating estimated risk measures. Table 2 demonstrates names of these companies and some descriptive statistics of them. Also, figures of daily price and returns for 4 companies as examples are shown in Figure 2.

Table 3: Companies of iran stock exchange

i	**Company name**	**Mean**	**Std. dev.**	**Skewness**	**Kurtosis**	**Jarque-Bera**
1	Darosazi JAber (IT)	0.00030	0.05000	25.91	859.05	57880990.24
2	Traktorsazi Iran (TI)	-0.00080	0.02230	-1.270	13.940	9912.42
3	Nosazi & Sakhteman (NS)	-0.00130	0.02818	0.736	9.6900	3687.15
4	Iran Transfor (DJ)	-0.00029	0.02807	8.640	200.11	3079971.24
5	Siman Sepahan(SS)	0.00013	0.02220	1.060	13.260	8622.78
6	Pertol Abadan (PD)	-0.00010	0.03990	18.090	544.03	23129982.54
7	Mes Shahid Bahonar (MSB)	-0.00039	0.02588	-0.990	13.870	9619.58
8	Tooka Fulad (TF)	-0.00110	0.02816	2.290	31.100	63785.63
9	Sarmayegozari Alborz (SM)	-0.00047	0.02403	2.520	28.980	55128.94
10	Pars Khodro (PK)	0.00035	0.04033	13.34	374.36	10905084.69

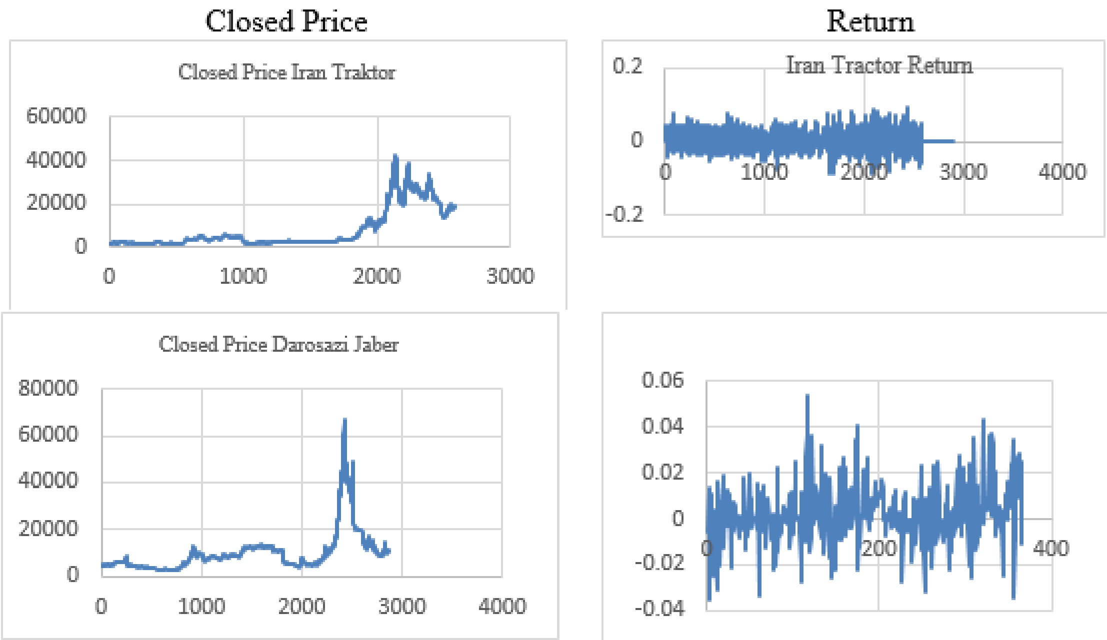
The coefficient of each stock is gathered in Table 4.

Table 4: Weight coefficient for each stock

W1	W2	W3	W4	W5	W6	W7	W8	W9	W10
10000	11000	10000	12000	10000	11000	12000	11000	12000	10000

The data time series of companies of Tehran Stock Exchange for priced and returns are illustrated in Figure 3.

All steps for numerical example are calculated as follow:



Step1: Construct all of $n > 0$ contributions of x_i which $\sum_{i=1} x_i = 1$ and $0 \leq x_i \leq 1$ for $t = 1$.With OR solution techniques we have 887 feasible solutions for each $tj(j = 1 - 365)$. All steps by one and one are implied for each tj. We start for $tj = 1$ and fix all steps for all of contributions $(tj = 1 \ldots 365)$. Because of large numbers of outputs table 4is illustrated 10 of 887 contribution results.

Table 5: Ten results of contributions

Contribution		X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	X1	0.1	0.9	0	0	0	0	0	0	0	0
2	X2	0.2	0	0.2	0.2	0.1	0.1	0	0	0	0
3	X3	0.2	0.2	0	0.2	0.1	0.1	0	0	0	0
4	X4	0.2	0.2	0.2	0	0.1	0.1	0.2	0	0	0
5	X5	0.2	0.2	0.2	0.2	0	0.1	0.1	0	0	0
6	X6	0	0	0	0.1	0.1	0	0.2	0.2	0.2	0.2
7	X7	0	0	0	0.2	0.1	0.1	0	0.2	0.2	0.2
8	X8	0	0	0	0.1	0.1	0.2	0.2	0	0.2	0.2
9	X9	0	0	0	0.1	0.1	0.2	0.2	0.2	0	0.2
10	X10	0	0	0	0.1	0.1	0.2	0.2	0.2	0.2	0

Step 2: For all days $(tj = 1, \ldots, 365)$ stocks price and return are predicted with time series learning prediction method which illustrated in figure 4.

Step 3: Based on the forecasted price prediction in step 2 the return of each stock is calculated for each contribution. The results of calculation of forecasted returns for each stock in $(x1 = 0.1$ and $X2 = 0.9)$ for $t = 1$ are shown in figure 5.

For all stocks $(i = 1, \ldots, 10)$ the price and return are predicted for all times $tj = 1, \ldots, 365$.

Step 4: The target profit is calculated for level 1. Because of a large number of outputs figure 6 shows the feasible profit for one contribution in all tj. For this reason, first fix $tj = 1$ and all contributions are considered for calculating. For each tj based on the profit the diagrams are illustrated. The target profit for all tj and for $x1 = 0.1, x2 = 0.9$ and for all contributions is illustrated in figure 6.

The rest of steps is illustrated just for $t = 1$ and all contributions. The target profit for all contributions for $t = 1$ is shown in figure 7.

Step 5: Applying IGDT method for achieving the α and β. The results of all steps for IGDT method based on ε

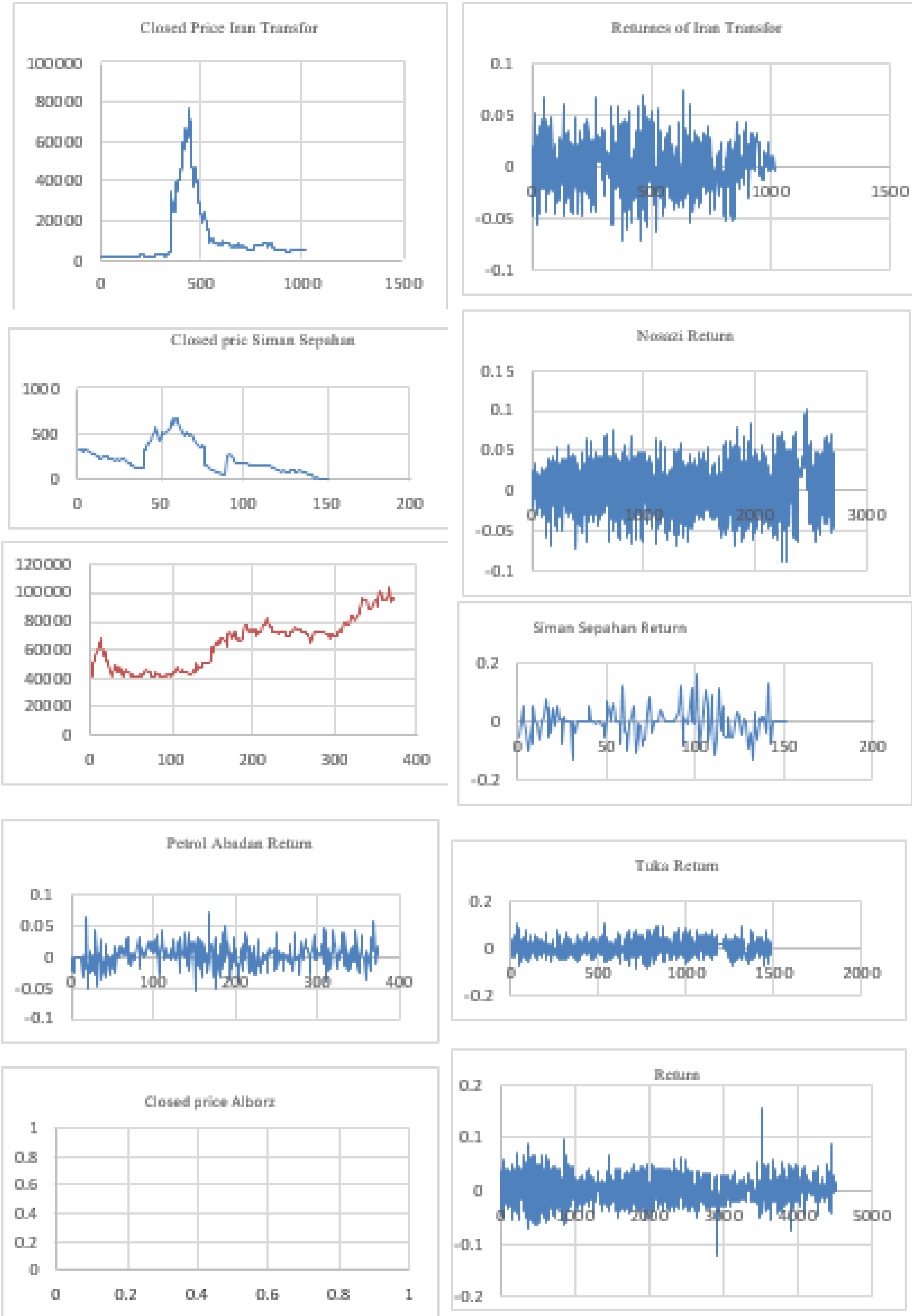
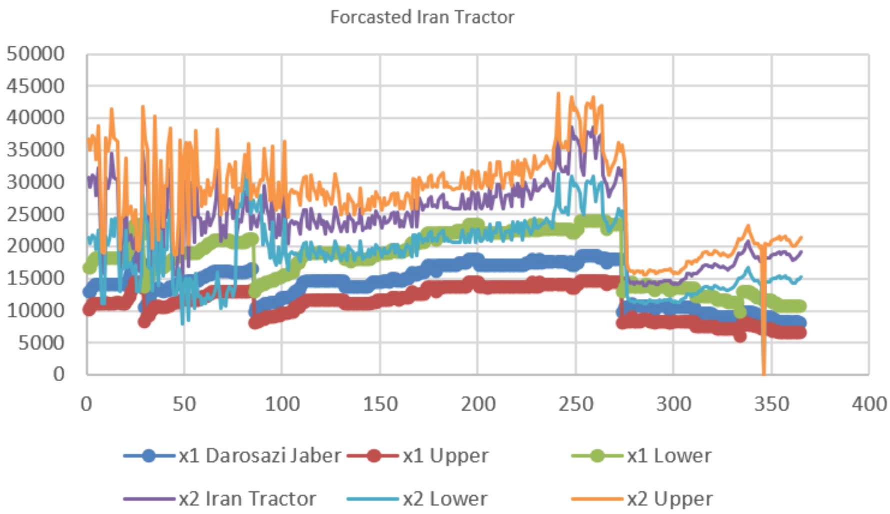
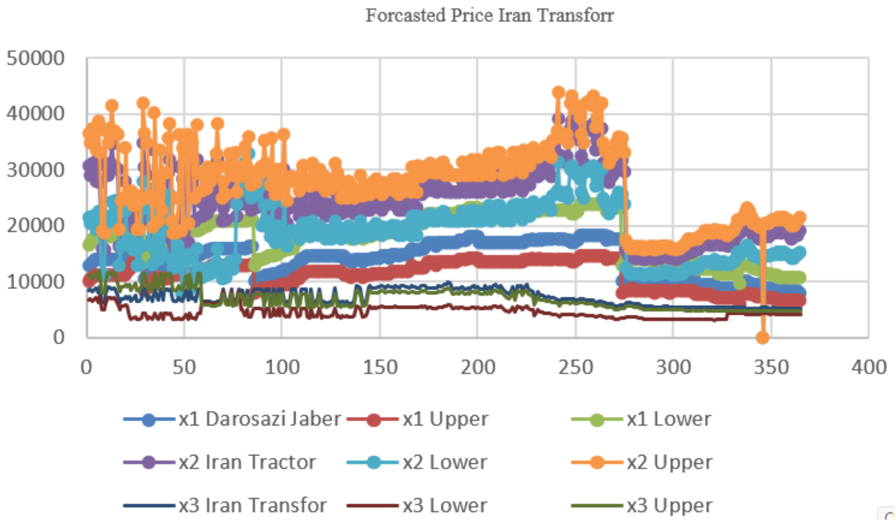


Figure 3: The data time series of companies of tehran stock exchange

constraint. The results for calculating of α and β for $t = 1$ is showed in figure 8. For this reason, the maximum profit is selected for each α and β.

Step 6: Applying the step 2-5 for $t = 1$ for all contributions. Because of large number the results are not shown and

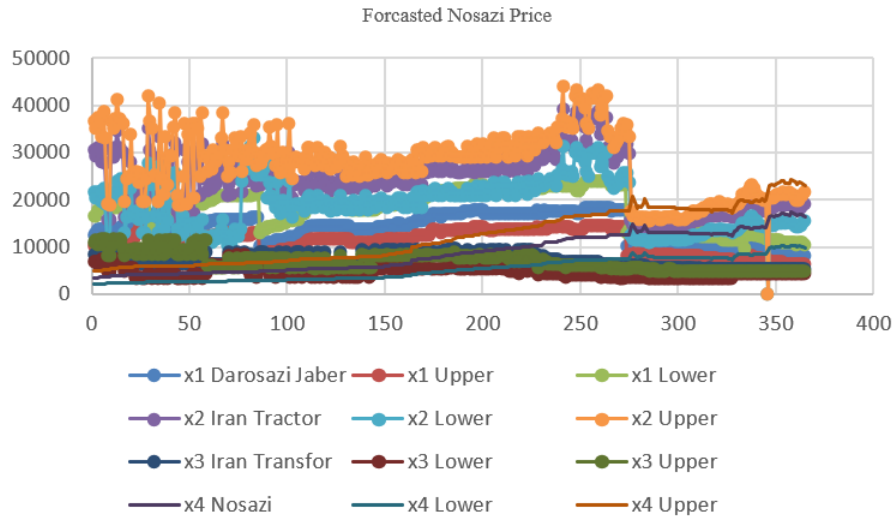


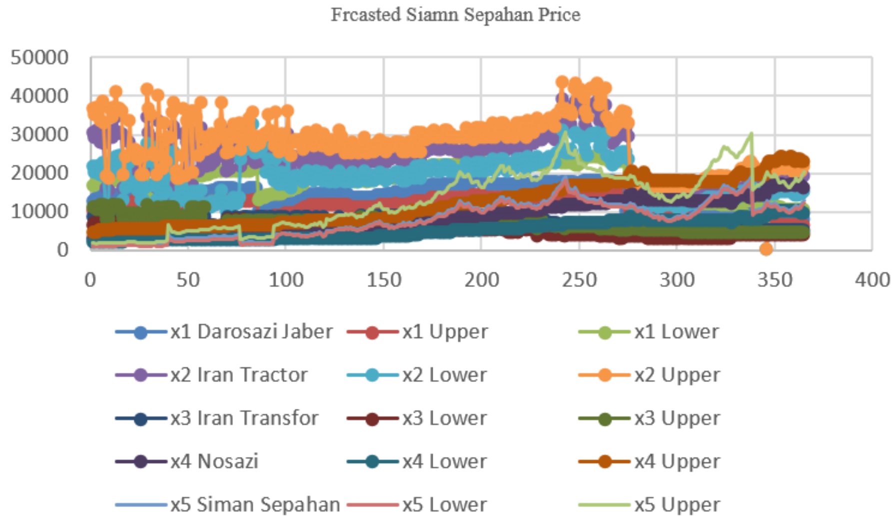


just illustrated in figure 9.

Step 7: Construct and select best solution for $t = 1(Xj, \alpha\beta)$. The best target profit with feasible β is shown in figure 10.

Step 8: Do all steps 1-7 for Step $tj = tj + 1$.





Step 9: Construct the feasible solution for all times.

Step8: Select the best optimized output.

The final results for the problem are gathered in table 6. The final result is shown in figure 11.

Table 6: Best solution for problem for each tj

t	Number of Optimal Contribution	Profit	α	β
1	40	509	0.2	1.67
2	548	803	0.2	1.67
3	692	968	0.2	1.67
4	676	633	0.2	1.67
5	158	1335	0.2	1.67
6	570	958	0.2	1.67
7	440	447	0.2	1.67
8	778	1442	0.2	1.67
9	353	831	0.2	1.67
10	633	388	0.2	1.67
11	539	1135	0.2	1.67

12	739	474	0.2	1.67
13	296	845	0.2	1.67
14	528	1593	0.2	1.67
15	556	225.0964687	0.2	1.67
16	781	230.3608104	0.2	1.67
17	463	265.1725357	0.2	1.67
18	554	248.3408415	0.2	1.67
19	778	252.9248582	0.2	1.67
20	613	253.9686586	0.2	1.67
21	583	248.4711594	0.2	1.67
22	604	249.3344403	0.2	1.67
23	256	230	0.2	1.67
24	218	251.7788131	0.2	1.67
25	782	254.0146658	0.2	1.67
26	718	249.6849714	0.2	1.67
27	560	238.8330064	0.2	1.67
28	354	299.1076595	0.2	1.67
29	114	937	0.2	1.67
30	471	851	0.2	1.67
31	678	888	0.2	1.67
32	81	697	0.2	1.67
33	749	786	0.2	1.67
34	707	415	0.2	1.67
35	254	1396	0.2	1.67
36	159	1242	0.2	1.67
37	148	863	0.2	1.67
38	450	464	0.2	1.67
39	15	659	0.2	1.67
40	359	666	0.2	1.67
41	528	977	0.2	1.67
42	22	375	0.2	1.67
43	497	1117	0.2	1.67
44	81	525	0.2	1.67
45	255	1313	0.2	1.67
46	711	868	0.2	1.67
47	588	83.3241774	0.2	1.67
48	257	119.1654191	0.2	1.67
49	175	277.4043853	0.2	1.67
50	691	-203.660962	0.2	1.67
51	158	11.31593277	0.2	1.67
52	345	63.59766879	0.2	1.67
53	389	-298.0722477	0.2	1.67
54	300	157.2893187	0.2	1.67
55	547	134.7599355	0.2	1.67
56	183	112.6330015	0.2	1.67
57	528	269.519282	0.2	1.67
58	576	237.6827702	0.2	1.67
59	193	99.91136966	0.2	1.67
60	148	-113.8605031	0.2	1.67
61	56	416	0.2	1.67
62	367	1244	0.2	1.67
63	585	1087	0.2	1.67
64	20	345	0.2	1.67
65	155	873	0.2	1.67

66	24	547	0.2	1.67
67	704	901	0.2	1.67
68	239	799	0.2	1.67
69	17	689	0.2	1.67
70	164	878	0.2	1.67
71	139	997	0.2	1.67
72	650	1506	0.2	1.67
73	188	643	0.2	1.67
74	619	993	0.2	1.67
75	500	507	0.2	1.67
76	749	705	0.2	1.67
77	741	552	0.2	1.67
78	798	1430	0.2	1.67
79	471	1288	0.2	1.67
80	127	616	0.2	1.67
81	237	1489	0.2	1.67
82	254	282.5203066	0.2	1.67
83	72	287.3050279	0.2	1.67
84	323	219.7224503	0.2	1.67
85	427	206.6254349	0.2	1.67
86	706	3.228078947	0.2	1.67
87	298	-57.03186009	0.2	1.67
88	582	149.217908	0.2	1.67
89	104	222.4699016	0.2	1.67
90	227	80.8407366	0.2	1.67
91	797	-42.15821698	0.2	1.67
92	102	183.1890889	0.2	1.67
93	513	104.8909549	0.2	1.67
94	323	181.9545776	0.2	1.67
95	177	-70.29534948	0.2	1.67
96	211	300.4756868	0.2	1.67
97	365	-38.19938516	0.2	1.67
98	659	-189.2642334	0.2	1.67
99	357	-26.45090661	0.2	1.67
100	121	37.01044417	0.2	1.67
101	261	234.2974022	0.2	1.67
102	533	492.9254606	0.2	1.67
103	696	217.6547294	0.2	1.67
104	195	-145.9553066	0.2	1.67
105	796	342.0434473	0.2	1.67
106	497	334.9958848	0.2	1.67
107	100	-142.5248766	0.2	1.67
108	438	7.647226163	0.2	1.67
109	293	528.0351693	0.2	1.67
110	681	-247.8567102	0.2	1.67
111	478	201.7017708	0.2	1.67
112	637	277.4457746	0.2	1.67
113	68	287.8898702	0.2	1.67
114	308	298.9877154	0.2	1.67
115	113	272.0264517	0.2	1.67
116	773	298.1893339	0.2	1.67
117	732	276.3685861	0.2	1.67
118	122	-5.520608875	0.2	1.67
119	4	153.7252235	0.2	1.67

120	617	283.9685065	0.2	1.67
121	180	126.7929822	0.2	1.67
122	527	80.5009828	0.2	1.67
123	411	291.8488915	0.2	1.67
124	520	294.7318412	0.2	1.67
125	536	299.6188975	0.2	1.67
126	227	296.4201087	0.2	1.67
127	84	302.006702	0.2	1.67
128	410	298.5982356	0.2	1.67
129	85	290.4183757	0.2	1.67
130	278	290.4560574	0.2	1.67
131	755	293.2389711	0.2	1.67
132	765	9.263166667	0.2	1.67
133	431	1.982094698	0.2	1.67
134	606	-24.30404213	0.2	1.67
135	46	-19.20692562	0.2	1.67
136	425	7.107974673	0.2	1.67
137	611	260.0013185	0.2	1.67
138	235	283.89262	0.2	1.67
139	277	52.4640072	0.2	1.67
140	596	267.2097221	0.2	1.67
141	665	215.4870953	0.2	1.67
142	254	184.1268173	0.2	1.67
143	541	-41.77733178	0.2	1.67
144	102	264.3599132	0.2	1.67
145	369	43.43537711	0.2	1.67
146	578	15.00110146	0.2	1.67
147	669	-229.5552882	0.2	1.67
148	546	-5.68380303	0.2	1.67
149	557	-109.9435404	0.2	1.67
150	630	66.40883085	0.2	1.67
151	352	-34.79065431	0.2	1.67
152	500	176.62674	0.2	1.67
153	105	217.2155476	0.2	1.67
154	643	130.9171439	0.2	1.67
155	340	190.1887876	0.2	1.67
156	478	65.07295899	0.2	1.67
157	228	105.3097108	0.2	1.67
158	486	183.0113279	0.2	1.67
159	395	236.3932262	0.2	1.67
160	50	155.3722817	0.2	1.67
161	692	289.9293193	0.2	1.67
162	121	286.8280864	0.2	1.67
163	258	294.8143282	0.2	1.67
164	740	307.7690689	0.2	1.67
165	595	286.4357963	0.2	1.67
166	720	138.4721303	0.2	1.67
167	347	-90.70381185	0.2	1.67
168	520	-7.959571666	0.2	1.67
169	7	-9.387531936	0.2	1.67
170	243	-96.74521072	0.2	1.67
171	528	201.5157982	0.2	1.67
172	200	289.6798992	0.2	1.67
173	495	272.1550507	0.2	1.67

174	652	286.4358519	0.2	1.67
175	686	311.141985	0.2	1.67
176	436	180.0547187	0.2	1.67
177	337	-49.505532	0.2	1.67
178	46	-86.50535444	0.2	1.67
179	449	-3.724557503	0.2	1.67
180	317	37.69467112	0.2	1.67
181	48	208.9501648	0.2	1.67
182	167	176.3349683	0.2	1.67
183	776	282.9965	0.2	1.67
184	160	-19.49410505	0.2	1.67
185	120	-51.83406105	0.2	1.67
186	544	-320.6515999	0.2	1.67
187	191	253.6622419	0.2	1.67
188	423	280.1359852	0.2	1.67
189	78	282.6943867	0.2	1.67
190	187	309.3329807	0.2	1.67
191	580	-7.073589158	0.2	1.67
192	195	6.827290881	0.2	1.67
193	651	128.4505507	0.2	1.67
194	786	240.7621144	0.2	1.67
195	666	10.91286222	0.2	1.67
196	782	-213.5652661	0.2	1.67
197	680	51.38571167	0.2	1.67
198	690	-37.37134807	0.2	1.67
199	586	-164.0017345	0.2	1.67
200	16	0.791903463	0.2	1.67
201	23	174.0274365	0.2	1.67
202	266	-4.185368761	0.2	1.67
203	550	-128.267428	0.2	1.67
204	536	-4.354319672	0.2	1.67
205	431	-80.57061761	0.2	1.67
206	563	-109.114535	0.2	1.67
207	342	237.1089682	0.2	1.67
208	363	0.876801864	0.2	1.67
209	297	-162.1399554	0.2	1.67
210	610	293.1026265	0.2	1.67
211	28	20.47286458	0.2	1.67
212	354	335.8524941	0.2	1.67
213	122	191.748311	0.2	1.67
214	268	290.7859733	0.2	1.67
215	269	218.5718439	0.2	1.67
216	557	127.3324487	0.2	1.67
217	145	209.0604817	0.2	1.67
218	631	310.8944912	0.2	1.67
219	201	289.0779489	0.2	1.67
220	277	314.4727994	0.2	1.67
221	92	-0.419893443	0.2	1.67
222	594	20.27515958	0.2	1.67
223	577	318.3079917	0.2	1.67
224	658	201.9896724	0.2	1.67
225	239	272.915334	0.2	1.67
226	685	298.8984557	0.2	1.67
227	500	292.8397712	0.2	1.67

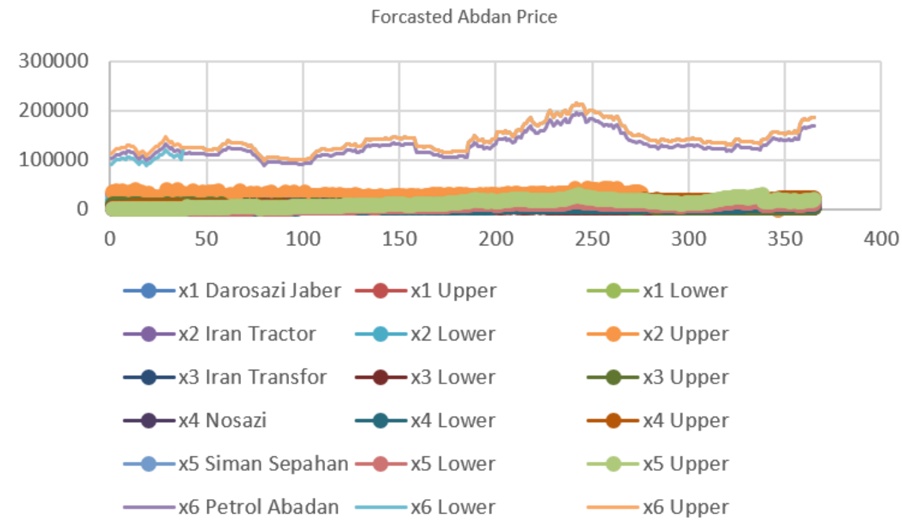
228	64	-132.041518	0.2	1.67
229	559	-54.28811538	0.2	1.67
230	672	33.35566308	0.2	1.67
231	246	297.9284895	0.2	1.67
232	303	310.2544961	0.2	1.67
233	183	304.703981	0.2	1.67
234	731	271.2236267	0.2	1.67
235	190	-42.90879843	0.2	1.67
236	484	201.182344	0.2	1.67
237	765	-52.54681723	0.2	1.67
238	789	-79.91126674	0.2	1.67
239	211	62.39977869	0.2	1.67
240	137	-97.09531335	0.2	1.67
241	657	-96.98804322	0.2	1.67
242	452	6.168516461	0.2	1.67
243	429	10.4711564	0.2	1.67
244	137	-62.33073926	0.2	1.67
245	729	306.0873887	0.2	1.67
246	283	-45.03506432	0.2	1.67
247	441	65.41375551	0.2	1.67
248	764	-140.7946228	0.2	1.67
249	37	279.6697474	0.2	1.67
250	287	192.9874109	0.2	1.67
251	693	156.0455185	0.2	1.67
252	343	-144.0541046	0.2	1.67
253	138	262.9879689	0.2	1.67
254	153	-245.9700886	0.2	1.67
255	204	477.1562173	0.2	1.67
256	745	-9.005582762	0.2	1.67
257	770	-24.21005615	0.2	1.67
258	739	159.1306345	0.2	1.67
259	181	102.1182391	0.2	1.67
260	467	196.508958	0.2	1.67
261	477	84.90081124	0.2	1.67
262	81	121.2322895	0.2	1.67
263	491	1368	0.2	1.67
264	209	1313	0.2	1.67
265	786	859	0.2	1.67
266	702	1366	0.2	1.67
267	242	936	0.2	1.67
268	300	344	0.2	1.67
269	55	949	0.2	1.67
270	282	954	0.2	1.67
271	152	564	0.2	1.67
272	374	943	0.2	1.67
273	285	543	0.2	1.67
274	105	850	0.2	1.67
275	336	1238	0.2	1.67
276	442	1387	0.2	1.67
277	488	1458	0.2	1.67
278	761	1576	0.2	1.67
279	248	808	0.2	1.67
280	631	1168	0.2	1.67
281	735	565	0.2	1.67

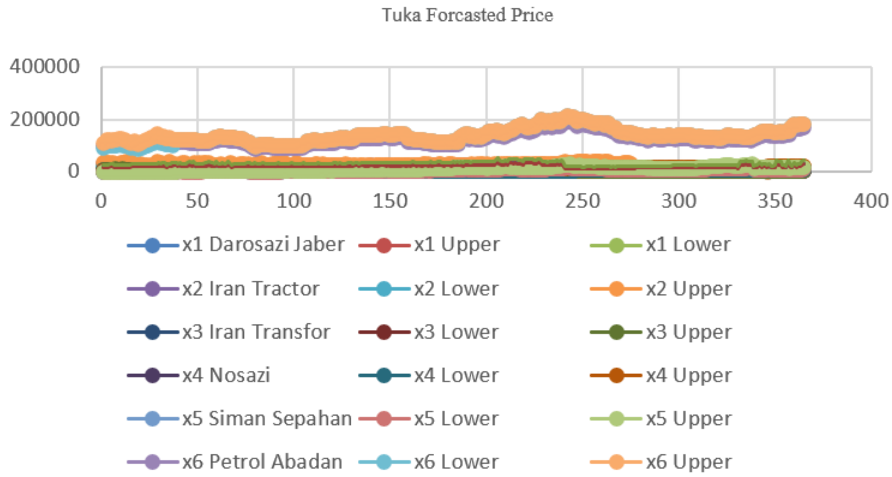
282	405	725	0.2	1.67
283	686	320	0.2	1.67
284	457	1043	0.2	1.67
285	511	1377	0.2	1.67
286	91	687	0.2	1.67
287	234	962	0.2	1.67
288	417	1009	0.2	1.67
289	182	1272	0.2	1.67
290	356	521	0.2	1.67
291	339	1363	0.2	1.67
292	725	1059	0.2	1.67
293	580	1317	0.2	1.67
294	769	1071	0.2	1.67
295	252	500	0.2	1.67
296	120	352	0.2	1.67
297	512	1113	0.2	1.67
298	147	835	0.2	1.67
299	109	856	0.2	1.67
300	352	1324	0.2	1.67
301	739	790	0.2	1.67
302	362	389	0.2	1.67
303	622	1345	0.2	1.67
304	735	350	0.2	1.67
305	520	874	0.2	1.67
306	319	364	0.2	1.67
307	725	462	0.2	1.67
308	291	892	0.2	1.67
309	275	941	0.2	1.67
310	68	510	0.2	1.67
311	193	1149	0.2	1.67
312	148	670	0.2	1.67
313	646	1457	0.2	1.67
314	779	703	0.2	1.67
315	628	497	0.2	1.67
316	60	1521	0.2	1.67
317	2	848	0.2	1.67
318	655	309	0.2	1.67
319	442	545	0.2	1.67
320	176	1528	0.2	1.67
321	220	650	0.2	1.67
322	744	636	0.2	1.67
323	651	371	0.2	1.67
324	298	1349	0.2	1.67
325	203	1517	0.2	1.67
326	489	341	0.2	1.67
327	295	366	0.2	1.67
328	407	1202	0.2	1.67
329	586	1054	0.2	1.67
330	420	1098	0.2	1.67
331	453	846	0.2	1.67
332	39	799	0.2	1.67
333	49	1251	0.2	1.67
334	749	607	0.2	1.67
335	722	811	0.2	1.67

336	578	771	0.2	1.67
337	287	1324	0.2	1.67
338	738	1188	0.2	1.67
339	190	669	0.2	1.67
340	267	302	0.2	1.67
341	222	583	0.2	1.67
342	710	1442	0.2	1.67
343	64	1253	0.2	1.67
344	192	1563	0.2	1.67
345	26	1104	0.2	1.67
346	225	855	0.2	1.67
347	780	1070	0.2	1.67
348	782	1473	0.2	1.67
349	469	558	0.2	1.67
350	128	394	0.2	1.67
351	634	1354	0.2	1.67
352	213	776	0.2	1.67
353	387	433	0.2	1.67
354	746	557	0.2	1.67
355	63	809	0.2	1.67
356	537	611	0.2	1.67
357	578	1174	0.2	1.67
358	521	328	0.2	1.67
359	136	304	0.2	1.67
360	414	1356	0.2	1.67
361	761	848	0.2	1.67
362	746	336	0.2	1.67
363	114	1131	0.2	1.67
364	351	423	0.2	1.67
365	443	374	0.2	1.67

Table 7: Part of simulation result for validation Case A and Case B for number of stock 3

W1	W2	r1x1	r2x2	x1	X2	w1x1r1	w2x2r2	$\mu^* \sigma_{ij}$	Profit	Lower	Upper	Profit Case A	Profit Case B
1000	11000	-0.00048	0.011194	0.1	0.9	-4.82486	123.1343	1.4035	116.9063	0.2	1.2	93.52501	140.2875
1000	11000	-0.00018	0.015292	0.1	0.9	-1.76991	168.2159	1.4035	165.0425	0.2	1.2	132.034	198.51
1000	11000	0.001861	0.032525	0.1	0.9	18.60987	357.7723	1.4035	374.9786	0.2	1.2	299.9829	499.9744
10000	11000	-0.00128	-0.02368	0.1	0.9	-12.7838	-260.526	1.4035	-274.714	0.2	1.2	-219.771	-329.656
10000	11000	0.000931	0.000427	0.1	0.9	9.305493	4.694168	1.4035	12.59616	0.2	1.2	10.07693	15.11539
10000	11000	-0.00016	0.008322	0.1	0.9	-1.57303	91.53953	1.4035	88.56299	0.2	1.2	70.8504	106.2756
10000	11000	-0.00032	-0.03669	0.1	0.9	-3.18182	-403.545	1.4035	-408.13	0.2	1.2	-326.504	-489.756
10000	11000	0.001245	0.018281	0.1	0.9	12.44813	201.0864	1.4035	212.1311	0.2	1.2	169.7049	254.5573
10000	11000	0.000711	0.016132	0.1	0.9	7.106832	177.4528	1.4035	183.1562	0.2	1.2	146.5249	219.7874
10000	11000	0.00018	0.014029	0.1	0.9	1.802208	154.3222	1.4035	154.7209	0.2	1.2	123.7767	185.665
10000	11000	0.001068	0.03253	0.1	0.9	10.68182	357.8313	1.4035	367.1096	0.2	1.2	293.6877	440.5316
10000	11000	-0.00212	0.03253	0.1	0.9	-21.1547	357.8313	1.4035	335.2731	0.2	1.2	268.2185	402.3278
10000	11000	-0.00063	0.013446	0.1	0.9	-6.2514	147.9036	1.4035	140.2487	0.2	1.2	112.199	168.2985
10000	11000	-0.00047	-0.01347	0.1	0.9	-4.69274	-148.176	1.4035	-154.272	0.2	1.2	-123.418	-185.127
10000	11000	0.000754	0.028028	0.1	0.9	7.542857	308.3045	1.4035	314.4439	0.2	1.2	251.5551	377.3326
10000	11000	0.000547	0.032638	0.1	0.9	5.473204	359.0164	1.4035	363.0861	0.2	1.2	290.4689	435.7033
10000	11000	0.000295	0.030849	0.1	0.9	2.948514	339.3443	1.4035	340.8893	0.2	1.2	272.7114	409.0671
10000	11000	0	0.031791	0.1	0.9	0	349.7015	1.4035	348.298	0.2	1.2	278.6384	417.9576
10000	11000	-0.00027	0.032255	0.1	0.9	-2.66667	354.8034	1.4035	350.7332	0.2	1.2	280.5866	420.8799
10000	11000	-0.00082	0.032255	0.1	0.9	-8.16417	354.8034	1.4035	345.2357	0.2	1.2	276.1886	414.2829
10000	11000	-0.00073	0.032255	0.1	0.9	-7.30088	354.8034	1.4035	346.099	0.2	1.2	276.8792	415.3188
10000	11000	0.000156	0.032255	0.1	0.9	1.5625	354.8034	1.4035	354.9624	0.2	1.2	283.9699	425.9549
10000	11000	-0.00049	0.032255	0.1	0.9	-4.85651	354.8034	1.4035	348.5434	0.2	1.2	278.8347	418.252
10000	11000	-0.00026	0.032255	0.1	0.9	-2.62066	354.8034	1.4035	350.7792	0.2	1.2	280.6234	420.9351





7 Validation result

For the validation of the results of proposed novel bilevel model with IGDT method the Mont Carlo simulation technique is applied that shows the proposed model is efficient in robust portfolio selection optimization problem. For this end, the optimal robust operational decision for $\alpha = 0.20$ and $\beta = 0.35$ is named as Case A and $\alpha = 0.10$ and $\beta = 0.55$ is named as Case B as the best compromise solution used for robustness verification. For this problem without loss of generality suppose that prices and returns follow normal probability distributions. The simulations were performed on a personal computer with 6 GB of RAM and Intel Core 7 due 2.50 GHz processor using CPLEX solver in the generalized algebraic modelling systems (GAMS) environment. The optimality gap for solving novel bilevel IGDT problems is set to 10^{-5}. The computational results of simulation method of the proposed model are affected by the number of segments used to approximating nonlinear terms by means of the SOS2 technique. Few numbers of segments may cause not exact results in another wise many segments may make the problem computationally skills so as a result a trade-off is needed. The final simulation result for validation of proposed model is showed in table 7. Profit of the versus the number of contributions is shown in figure 12.

As can be seen in figure 12, with increasing the number of contributions the results become more accurate, nevertheless the computational burden is increased.

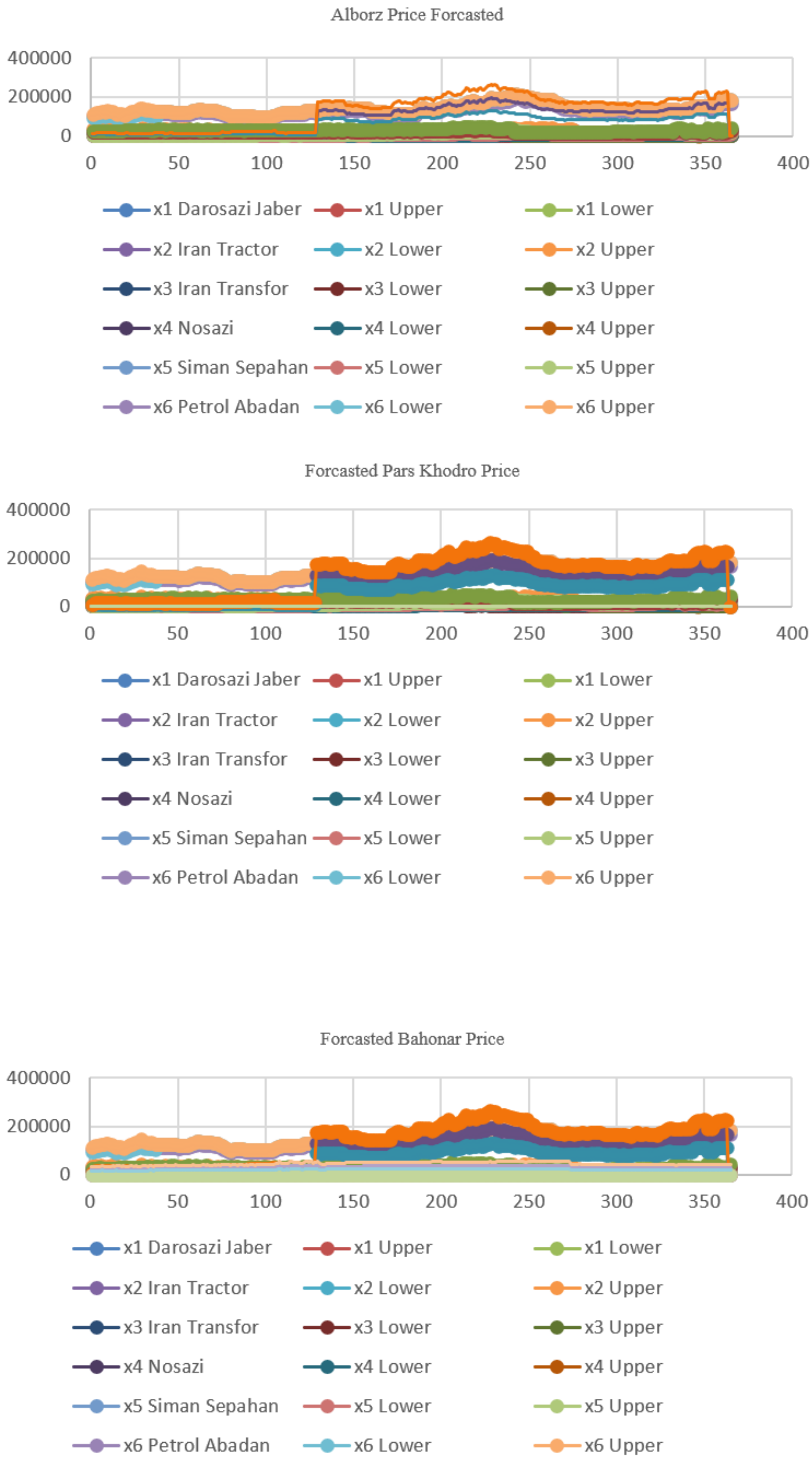


Figure 4: Time series learning prediction method

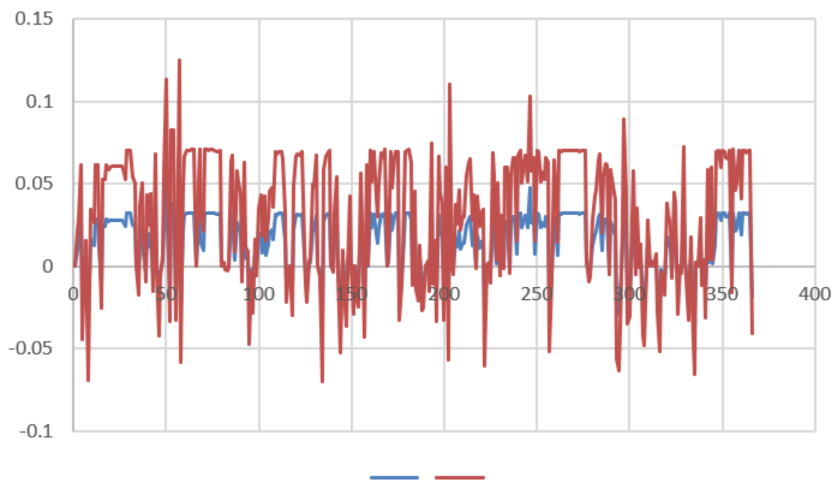


Figure 5: Forecasted return for Iran Traktor

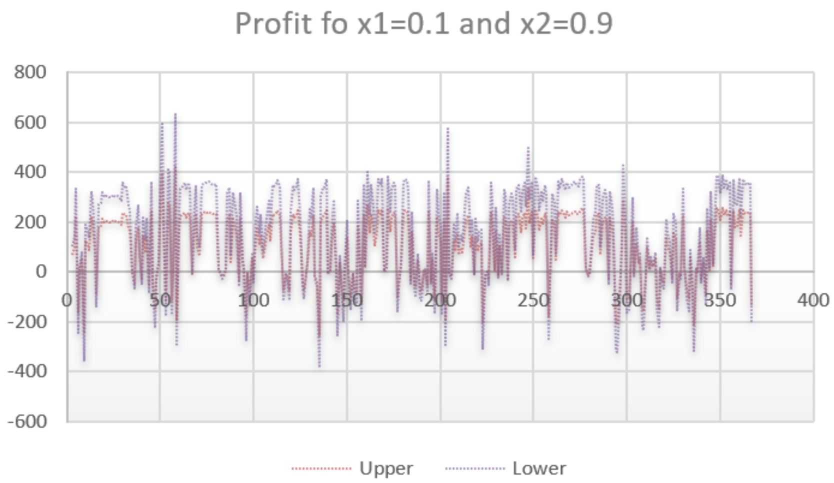


Figure 6: Target profit for all tj for $x1 = 0.1$ and $x2 = 0.9$

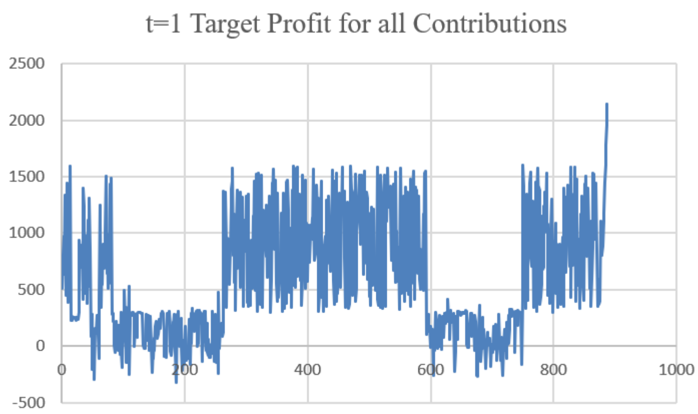


Figure 7: Target profit for all contribution $t = 1$

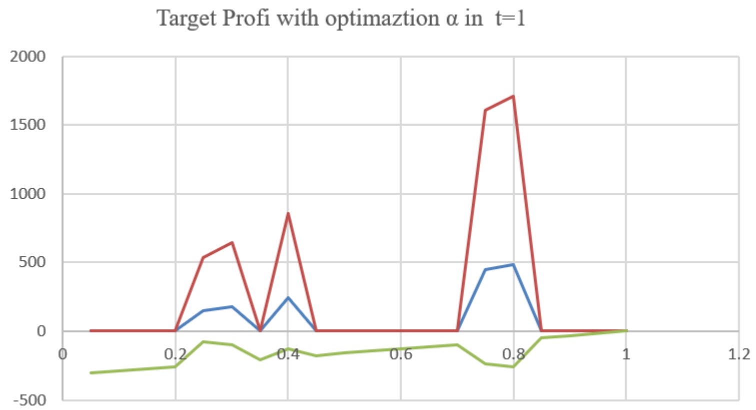


Figure 8: Results of calculating α and β for $t = 1$

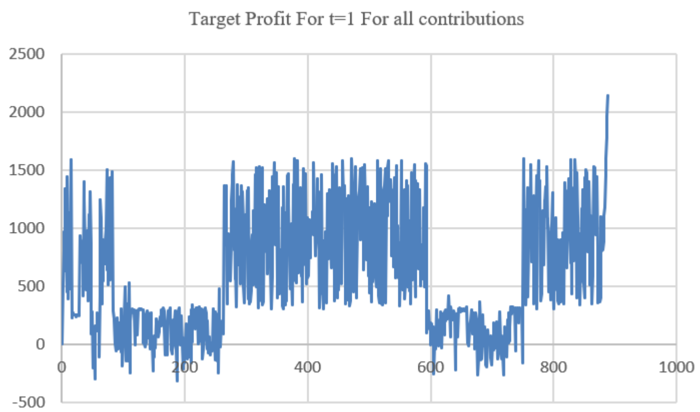


Figure 9: Target profit for t=1 for all contributions

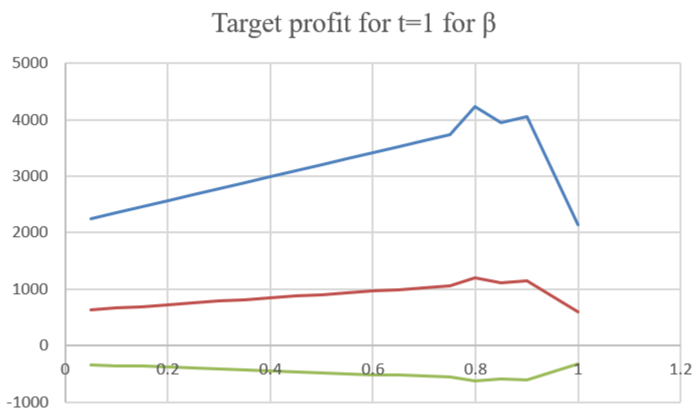


Figure 10: Target profit for $t = 1$ for β

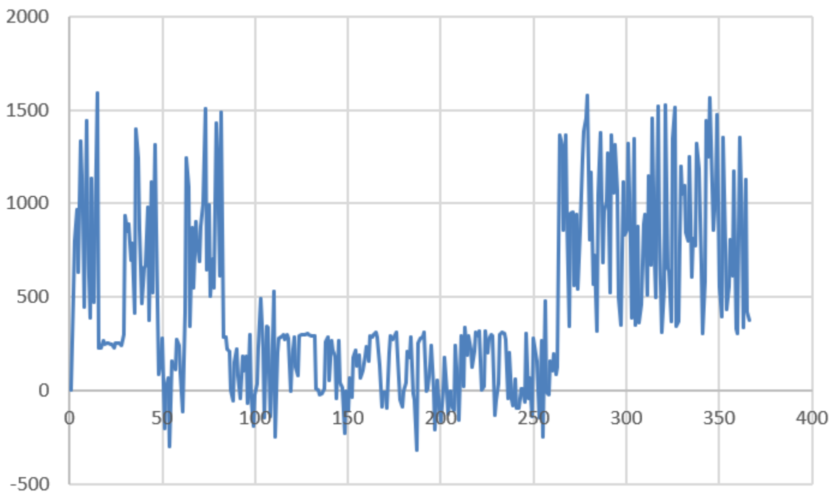


Figure 11: final results

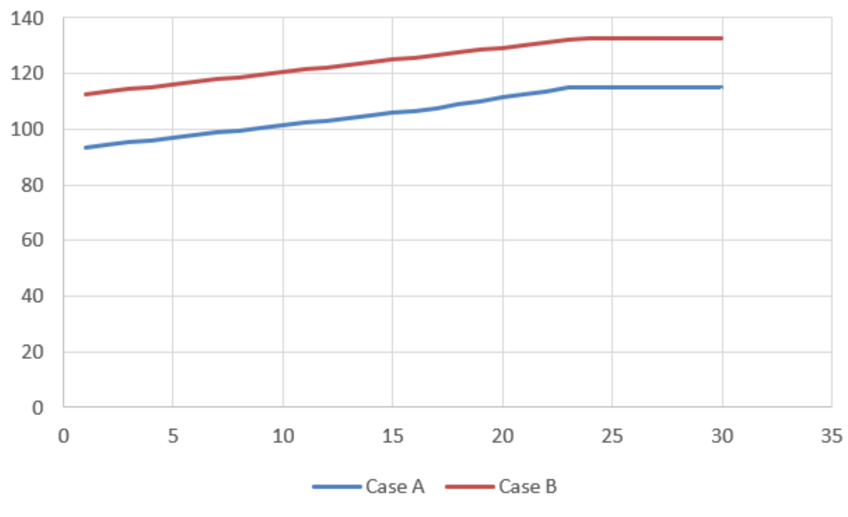


Figure 12: Simulation profit for case A and case B

8 Conclusion

Today portfolio optimization in finance is more than a mathematical problem for improving performance under risk constraints. Practically all organizations seek to create value by selecting the best portfolios that consume least resources and obtaining high expected portfolio return and controlling risk. Typically, in the context of portfolio selection problem severe uncertainties (imprecise probabilistic information) would significantly affect the technical and financial aspects. This paper presents a risk aversion bi-level information gap decision theory (IGDT) decision making tool to help organizations or investors for managing their portfolios and finding the best transactions with severe uncertainty variables (price and return) to process the forecast data generated by the prediction method in order to construct the optimal stock portfolios that a target profit for the risk averse investors is guaranteed.The results from Mont Carlo simulation method for validation shows the power of the model for controlling uncertainty in portfolio selection and also it can be generalized and can be used for another practical problems. The bilevel model based on IGDT for severe uncertainty compare with traditional scenario-based methods shows that it is more accurate because of it does not need PDF of uncertain variables that are difficult to estimate. The novel bilevel model is applied in Iran Stock Market that for the future research this model can be applied in other specs such as electricity market.

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