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## Reliability-Based Optimum Design of Dome Truss Structures through Enhanced Vibration Particle System

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### ABSTRACT

Recent years have seen a significant increase in structural engineers' interest in the assessment of reliability and structural safety. The Reliability-Based Design Optimization (RBDO) method has been utilized to create the most efficient and safe design of structures. Although there have been several theoretical advances in reliability analysis, computational barriers still occur in realistic problems. The purpose of this paper is to provide a process for the optimization of dome truss structures based on reliability. For this purpose, a flowchart including the process of Deterministic Design Optimization (DDO) and RBDO was presented. An evaluation of the reliability of the structure is made by using random variables to represent uncertain parameters. Throughout this study, random variables such as the module of elasticity, material density, and the cross-sectional area of the elements are considered. The deterministic constraints for DDO are the vertical displacement of free nodes and the demand-capacity ratio of all members. Also, reliability index 3 is set as the minimum target reliability index. Meta-heuristic algorithms can be used to achieve optimal design and appropriate safety since mathematical calculations are time-consuming. As part of this study, the Enhanced Vibration Particle System (EVPS) and Vibration Particle System (VPS) have been applied to DDO (incorporating reliability assessment) and RBDO of three dome trusses. The results were obtained using the processes of RBDO and DDO without any deviation in the acceptable space. The solution of RBDO will increase the weight and safety of structures.

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## 1. Introduction

Since construction cost along with providing safety and necessary strength of the structure is of high importance, the reliability assessment of a system is a subject that has been recently considered by structural engineers. The elements of the structure and altogether the whole system should be designed in a way to be reliable enough for an expected duration against destruction. It should be emphasized that despite the best selection of quantities, there is still the possibility of failure, and the safety of the structure is not fully guaranteed. The reason is that the existence of uncertainties such as external loads, material properties, the cross-sectional area of elements, and geometry of members and, etc. which must be taken into account carefully. For better structural performance, considering these uncertainties is necessary, and one of the best way to determine the effect of uncertainties is to perform Reliability-Based Design Optimization (RBDO). In recent years, efficient optimization formulations based on probability have been developed to account for the various uncertainties. The RBDO methods reduce not only the probability of failure but also the weight of the structure and constructional costs. For the first time, Cornell in 1969 proposed a method for calculating the reliability index [1]. In this method, the limit state function was approximated by the first sentence of the Taylor series at the mean point. The limit state function is converted to a linear function, and the reliability index is obtained by calculating the mean and standard deviation of the limit state function. Lind and Hasofer in 1974 [2] fixed the flaws of the

Cornell method and suggested a new approach to calculate the reliability index. The shortest distance from the origin of the standard normal space to a point on the failure surface was defined as a reliability index by Hasofer-Lind. And then, for more complex problems, new methods of RBDO are applied to reach the optimal design of the structure. Keshtgar et al. (2021) proposed an adaptive modified chaos control (AMC) method in RBDO to achieve robust and efficient results [3]. Abid et al. (2021) used a new robust hybrid method (RHM) method to solve the defects of the classical hybrid method (HM) because the HM method has only been applied to linear material, but the RHM method can be used for nonlinear materials such as shape memory alloy [4]. Zhu et al. (2021) proposed a new method called hybrid conjugate mean value (HCMV) to enhance of robustness and efficiency of RBDO. The conjugate gradient analysis (CGA) and advanced mean value (AMV) are dynamically applied for easy convex/concave constraints using the descent criterion in the new approach [5]. For buckling analysis and design of composite variable stiffness, Hao et al. (2021) established isogeometric analysis (IGA) to reduce computational efforts to reach RBDO. They used the augmented step size adjustment (ASSA) algorithm to enhance the efficiency of the RBDO process [6]. To obtain the optimal design of truss structures using dynamic and static constraints for multi-objective problems, Duy et al. (2020) investigated an effective couple method to solve reliability-based optimization. A new method SLDM-NSGA-II was created by integrating two different algorithms called the non-dominated sorting

genetic algorithm II (NSGA-II) to the single-loop deterministic method (SLDM) algorithm [7].

As a result, engineers are recommended to take many uncertainties into account. Although it is possible to create perfect and flawless structural models using computers, there are still imperfections in reality that cause deviations from the structure's nominal state. Metaheuristic algorithms help engineers and designers achieve the best results that can be performed in the real world. Recently, metaheuristic algorithms have been used successfully to solve structural optimization problems, and also their application in RBDO has had effective and acceptable results. Different types of algorithms have been developed over the last decades. Das and Saha studied the structural safety of real structures employing Eagle Perching Optimization, Dragonfly Algorithms, Bird Swarm Algorithms, Whale Optimization Algorithm, Firefly Algorithms, and Flower Pollination Algorithms. [8]. Using Lévy flight distribution (LFD), Houssein et al. (2021) solved real-world optimization problems [9]. A study by Ficarella et al. (2021) examined three metaheuristic algorithms to minimize the weight of four skeletal structures based on Big Bang-Big Crunch (BBBC), Simulated Annealing (SA), and Harmony Search (HS) [10].

Due to the nature of some optimization problems, they cannot be solved by conventional gradient based methods, so metaheuristic algorithms are used to provide more precise results in an efficient time.

Nature and physical phenomena inspired researchers to invent metaheuristic algorithms; there are three types of metaheuristic algorithms:

Physical algorithms, Evolutionary algorithms and Swarm [11-20].

Reliability is a characteristic of an element or component that indicates the probability of performing certain operations under certain conditions at a particular time. In other words, reliability means the ability of a system or component to repeat with the same results in certain time intervals continuously. The reliability index can be used to provide an optimal design for structures, and there are three categorized methods to solve RBDO problems: single-loop, double-loop, and decoupled.

The single-loop method is utilized to reach reliability-based design optimization; in this method, the probable constraints are converted to determined restrictions, and instead of RBDO to solve the problems, deterministic design optimization (DDO) is used [21]. In the double-loop, a nested optimization loop is required. The upper-level loop contains design optimization that calls reliability analysis from the lower loop repeatedly [22].

The optimal design based on reliability can be applied for dome truss structures by using metaheuristic algorithms. Dome trusses have many and varied applications. They cover large spans and can be easily implemented. Other advantages of constructing dome trusses are the minimal material usage, lightweight, strength, and no need for internal columns. Long life and no need for

special maintenance are the main features of such structures. Such structures are among the desired systems for design and construction, both economically and environmentally compatible and beautiful in architecture. There are various forms of dome trusses in single or double-layers, such as stadiums, skylight roofs, exhibition halls, greenhouses, etc. Truss optimization is done in three ways:

- 1) Optimization based on size, which reaches the optimal weight and cost by varying the cross-section of the elements.
- 2) Shape optimization that examines the geometry of a structure in which the coordinates of the nodes are variables to achieve optimal weight and price.
- 3) Topology optimization in which the ways of the connection of the elements are as variables to achieve the optimal structure in terms of weight and cost.

In this paper, based on RBDO, size optimization of dome trusses is applied to minimize the truss weight through changing the cross-sectional area of the elements considering the displacement and stress constraints.

Due to inherent uncertainties such as area of cross-section, the module of elasticity, external loading, material properties, and poor production quality of members, a structure may not meet the required functions. Through the theory of reliability of structural systems, uncertainties due to the probabilistic nature of the structural parameters can be solved in the form of mathematical relations. Since computational cost and time to attain reliability index is one

of the most crucial issues in the application of reliability-based design optimization RBDO for real problems, using metaheuristic algorithms is of greater importance than in the past. One of the metaheuristic algorithms proposed recently by some researchers to improve vibrating particle systems (VPS) is enhanced vibration particle systems (EVPS). Kaveh and Ghazaan used a new approach called vibrating particle systems VPS, using viscous damping for free vibration of a single degree of freedom systems. This method investigates the gradual movement of particles toward their equilibrium position [23]. As a solution to optimization problems, Kaveh and Khosravian proposed VPS [24]. To demonstrate the performance of VPS and EVPS algorithms in the optimization of truss structures, Kaveh and Hoseini Vaez analysed two different types of trusses [25]. A series of different types of trusses were used by Kaveh and Hoseini Vaez in order to demonstrate the performance of VPS and EVPS algorithms. Natural frequencies and mode shapes served as the objective functions throughout the experiment. As a result, they realized that EVPS provides better answers than VPS [26]. To evaluate EVPS and other metaheuristic algorithms, Kaveh and Hoseini Vaez used the Modified Dolphin Monitoring (MDM) operator [27]. To demonstrate the efficiency of the two-step approach for optimal design, Hoseini Vaez et al. used EVPS [28]. Kaveh et al. determined frequency constraints for large-scale dome trusses to reach optimization using the EVPS algorithm [29].

Based on the double-loop method, EVPS and VPS algorithms, this study investigated 52-bar, 120-bar, and 264-bar dome trusses in

order to reach an optimal weight based on reliability.

Each uncertainty parameter can be viewed as a random variable that must provide the desired constraints for reliability to be achieved. In this study, random variables are the module of elasticity and the cross-sectional area of the elements. Also, displacement and stress ratios were defined as probability constraints. The Monte Carlo simulation was used in this study as a convenient method for calculating the reliability index of structures. It is the intention of this paper to provide some examples of the optimization of dome truss structures based on reliability. The EVPS and VPS algorithms have been applied to deterministic design optimization (incorporating reliability assessment) and reliability based design optimization RBDO of three dome truss structures. In the final stage, the results are compared with each other

## 2. Optimization Based on Reliability

In this study, the EVPS and VPS algorithms are used to reach the optimal design of dome trusses. EVPS was successfully applied by some researchers to solve optimization problems of structures [25-29]. The answers obtained by this algorithm are sent for the objective function. In this study, minimizing the weight of dome trusses as an objective function is the main goal for the optimization of these structures. The objective function is investigated in two different ways called DDO and RBDO. In the DDO method, the displacement and strength constraints are the requirements of structural design that should

be satisfied. The members of the structure are designed according to AISC360-16 [30] in order to satisfy the strength constraints.

The design tensile strength,  $\phi_t P_n$  is as follows:

$$\phi_t = 0.9 \quad P_n = F_y A_g \quad (1)$$

where  $F_y$  and  $A_g$  are yield stress and gross section, respectively, and the design compressive strength,  $\phi_c P_n$  is determined as follows:

$$\phi_c = 0.9 \quad P_n = F_{cr} A_g \quad (2)$$

The critical stress,  $F_{cr}$ , is calculated based on the slenderness ratio as follows:

$$F_{cr} = \begin{cases} \left(0.658 \frac{F_y}{F_e}\right) F_y; & \left(\frac{L_c}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}\right) \\ 0.877 F_e; & \left(\frac{L_c}{r} > 4.71 \sqrt{\frac{E}{F_y}}\right) \end{cases} \quad (3)$$

where elastic buckling stress,  $F_e$ , is determined as Eq. (4):

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \quad (4)$$

$$L_c = KL \quad (5)$$

In Eq. (9),  $L_c$  represents the effective length of the member is calculated according to Eq. (5), in which K (effective length coefficient) is equal to 1 for each problem in this study and  $r$  the radius of gyration is calculated as Eq. (6), where, a and b are constants for the pipe sections ( $a = 0.4993$  and  $b = 0.6777$ ) [31].

$$r_i = aA^{ab} \quad (6)$$

The slenderness ratio ( $L_c/r$ ) of members in tension and compression should not exceed 300 and 200, respectively.

As mentioned above, the objective function is to minimize the weight of dome trusses. The formulation of the optimization problem is as follows:

Find:  $\{x\}$  (7)

To minimize:  $F(\{x\}) = \sum_{i=1}^{nm} \rho_i A_i L_i$

\* The constraints of the problem should be taken into consideration according the flowchart of Figure 1.

where  $\{x\}$  is a set of design variables of the pipe members' cross-sectional area; the net weight of dome trusses is  $F(\{x\})$ ; the number of elements is represented by  $nm$ ;  $\rho_i$  is the density of material for  $i$ th member;  $A_i$  is the cross-sectional area of the  $i$ th member, and  $L_i$  is the length of the  $i$ th member. All problems in this study are continuous optimization problems. For controlling each problem to satisfy requirements, a penalty approach is considered as follows:

$$Z(x) = F(x) + \alpha P(x), \quad (8)$$

$$P(x) = \sum_{i=1}^n \max(g_i, 0)$$

where  $Z(x)$  is the penalized objective function,  $F(x)$  represent the net weight (more information is provided in Figure 1.),  $\alpha$  is the magnification coefficient which is considered 0.05 for these problems,  $P(X)$  is deemed to be as penalized weight,  $n$  is the number of constraints and  $g_i$  is the violation of the  $i$ th constraint (the constraints such as: node displacement, demand capacity ratio, reliability index, etc). It should be noted that the deterministic constraints for deterministic design optimization are the vertical displacement of free nodes and the demand-capacity ratio of all members. The violation of the mentioned deterministic constraints is shown as  $V(x)$  in the flowchart of Figure 1.

In addition to DDO, this study investigated RBDO. To evaluate the reliability of the

structure, one should calculate the probability of failure by considering many uncertainties which are the module of elasticity and cross-sectional area of elements as random variables in this study. Since these variables have random behaviors, the reliability and safety of the structure are determined by the possibility of failure. The probability of failure,  $P_f$ , is calculated as follows:

$$P_f = \int_{G(X) \leq 0} f_x(X) dX \quad (9)$$

where  $X = (X_1, X_2, \dots, X_n)^T$  is a vector of random variables which represent the uncertainties of a structure;  $(X)$ , and  $G(X)$  are probability density function and limit state function, respectively. The reliability of a system is modeled by its limit state function which divides the space of variables into safe and unsafe domains. In this study, the limit state function is defined as Eq. (10):

$$G(X) = (\Delta_{all} - \Delta_{i,j}) \quad (10)$$

If  $G(X) > 0$ , a structure is in the safe domain, and if  $G(X) \leq 0$ , a structure is in an unsafe area and failure is probable. In Eq. (14),  $\Delta_{all}$  shows the allowable vertical displacement of free nodes of a structure, and  $\Delta_{i,j}$  represents the displacement of  $j^{\text{th}}$  free nodes and  $i^{\text{th}}$  sample of a structure. Calculating the above integral is hard due to the complication of the limit state function in Eq. (9). So the reliability index is used to obtain the value of the safety of a structure.

The Monte Carlo simulation is a practical method to calculate the reliability index. This simulation technique is based on the generation of a series of random samples, and then for each sample, the limit state function is calculated. The probability of a failure using this method is calculated by dividing the number of failed samples by the

total number of all samples as defined in Eq. (11)

$$P_f = \frac{1}{N} \sum_{i=1}^N \mathbf{I}(G_i) = \frac{N_f}{N} \quad (11)$$

In the above equation,  $I$  represent an index function. The value of  $I$  is equal to zero if for the  $i$ th sample of random variables, the  $G(X) > 0$ . Then, the reliability index is calculated by Eq. (12).

$$\beta = -\Phi^{-1}(P_f) \quad (12)$$

where  $\Phi$  is the normal cumulative distribution function.

A flowchart of the step-by-step process for RBDO using the metahuristic algorithm is provided in Fig.1. According to the flowchart of Figure 1, in the first step, the violations of deterministic constraints including the vertical displacement of free nodes and the demand-capacity ratio of all members are calculated, if the sum of these violations is zero, it enters the next step, which is the assessment of the reliability indices based on the limit state function (Eq. 10). In this step, reliability index 3 is set as the minimum target reliability index, and a violation of it is calculated.

The dashed part of the flowchart in Figure 1, which includes the first step, is shown DDO process.

### 3. Numerical Examples

This section presents three numerical examples of dome truss structures for the purpose of determining the optimal design based on reliability using two algorithms. During the optimization process, some

independent runs are conducted for each example. For the optimization algorithm in this study, a population size of 30 for two trusses and the other one population size of 40 and 400 iteration numbers for DDO and 150 iteration numbers for RBDO are selected. The number of samples is considered to be  $2 \times 10^5$ . The random variables for these three examples are the area of elements and modulus of elasticity with the 5% coefficient of variation. It is assumed that the modulus of elasticity for each example is  $2.1 \times 10^{11}$  (N/m<sup>2</sup>). The objective function is defined as penalized weight, and the constraints for this problem are allowable displacement and Demand Capacity Ratio (DCR).

#### 3.1. 52 Bar Dome Truss

A 52-bar dome truss is investigated for size optimization, and its shape is depicted in two top, and side views Fig. 2. This truss has 52 elements with 21 nodes and 39 degrees of freedom.

There are many types of research about this truss that have been done previously [32-34]. The elements grouping for this truss is according to Table 1. The yield strength of steel and material density ( $\rho$ ) are defined 400 (MPa) and 7800 (kg/m<sup>3</sup>) respectively as random variables in Table 2. The free nodes of this truss are subjected to vertical loading of -60 kN at node 1, -30 kN at nodes 2-5, and -10 kN at nodes 6-13. The allowable displacement of all free nodes is 5mm, and there is a boundary between  $1 \times 10^{-4}$  m<sup>2</sup> and  $1 \times 10^{-3}$  m<sup>2</sup> in the cross-sectional area.

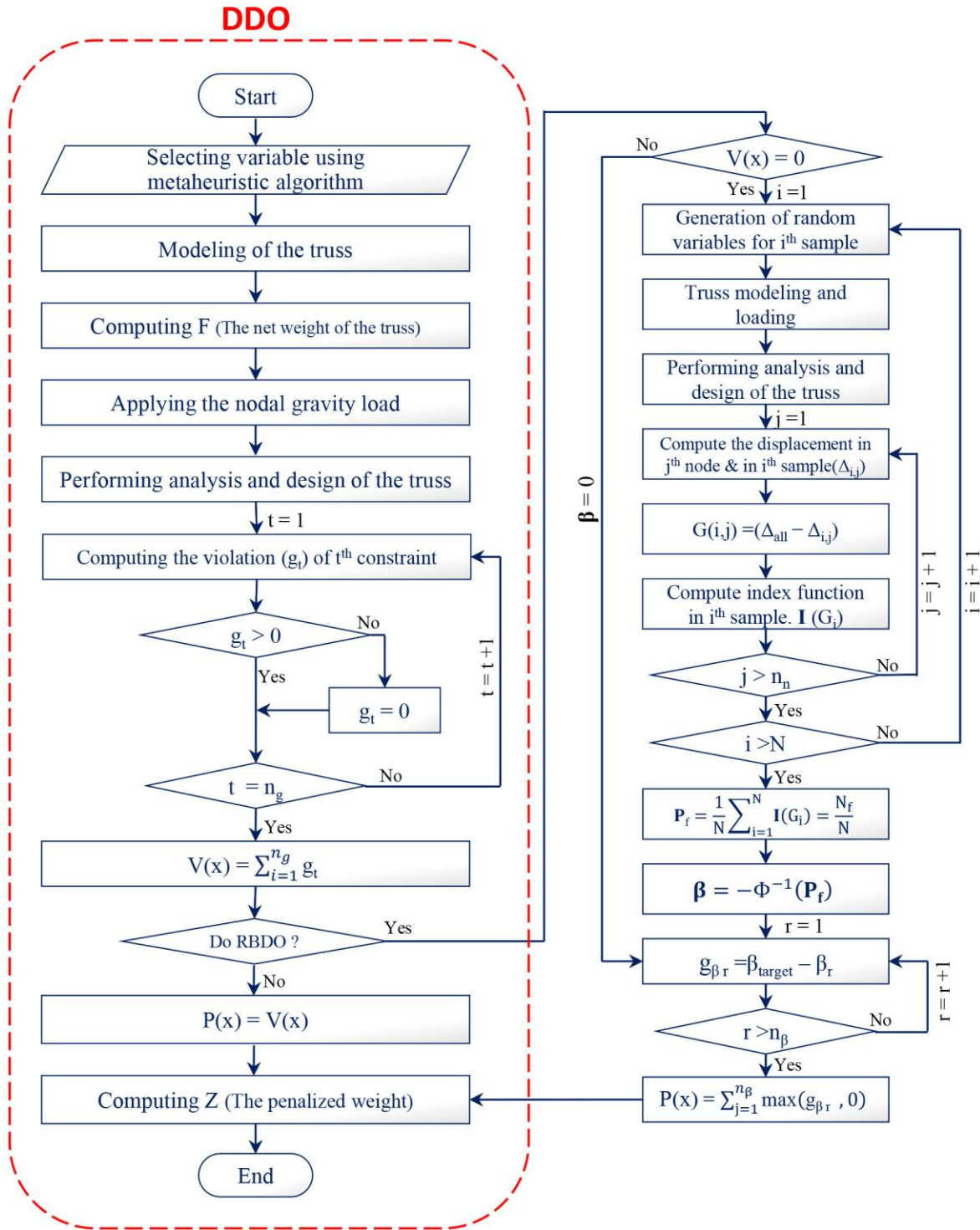


Fig 1. Flowchart of DDO & RBDO process.

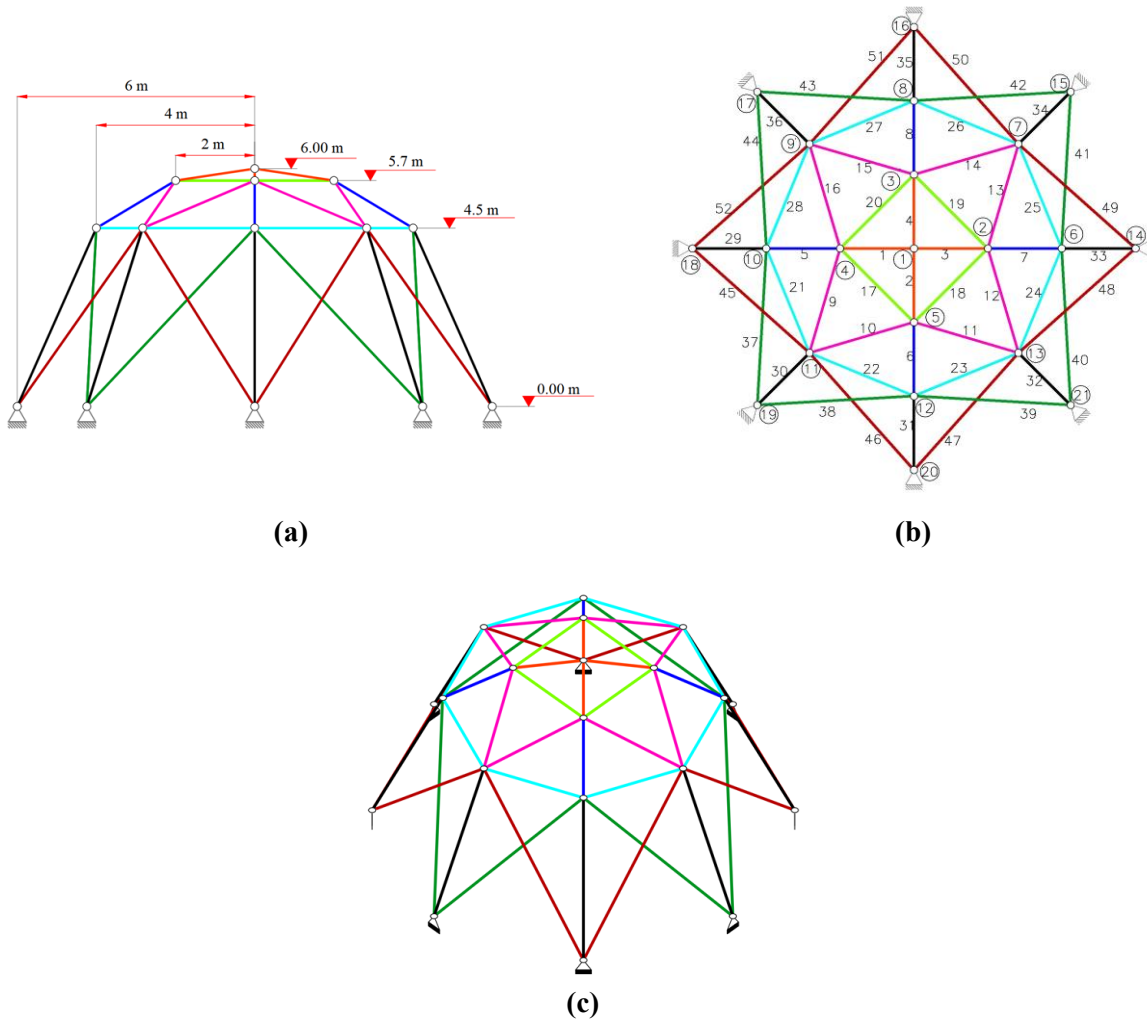


**Table 1.** 52-bar dome truss element grouping.

Group No.	Elements	Group No.	Elements
1	1-4	5	21-28
2	5-8	6	29-36
3	9-16	7	37-44
4	17-20	8	45-52

**Table 2.** Material properties for the 52-bar dome truss.

Random Variable, unit	Mean	COV
$E$ (Modulus of elasticity), $N/m^2$	$2 \times 10^{11}$	5%
$\rho$ (Material density), $kg/m^3$	7800	5%
$A$ (element group crosssection), $cm^2$	$A_1=1.0464$ $A_2=1.7295$ $A_3=1.6507$ $A_4=1.5059$ $A_5=1.7210$ $A_6=1.0020$ $A_7=1.7415$ $A_8=1.2555$	5%



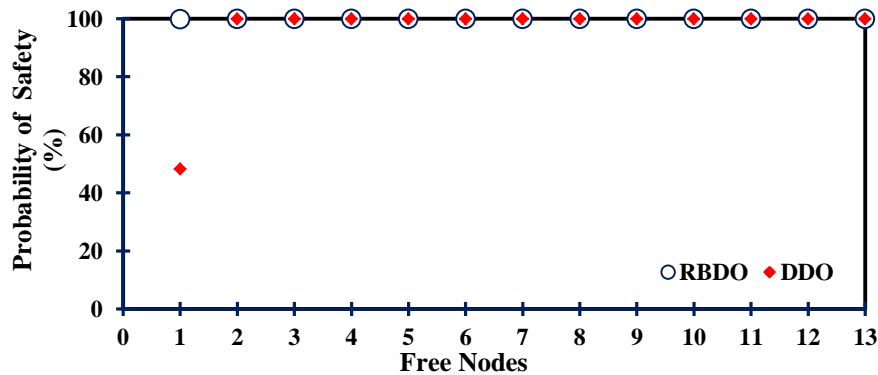
**Fig 2.** schematic view of 52-bar dome truss (a) View from the side, (b) View from the top, (c) View from an isometric perspective.

The results of optimization for the best cross-sectional area and weight of this truss for both RBDO and DDO are presented in Table 3. The probability of safety for all free nodes in RBDO and the reliability assessment for DDO is depicted in Fig. 3. The comparison

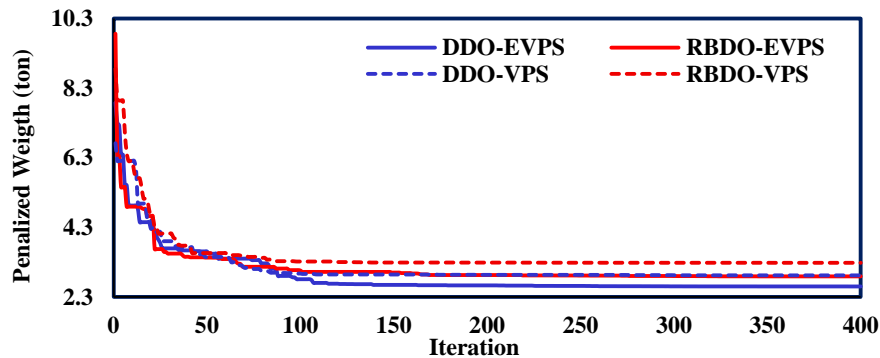
of results for optimum weight in both DDO and RBDO is depicted in Fig. 4. The comparison of results for allowable displacement and DCR is represented in Fig. 5 and Fig. 6, respectively.

**Table 3.** The results of optimization for the 52-bar dome truss.

Design variables	EVPS		VPS	
	RBDO	DDO	RBDO	DDO
$A_1(\text{cm}^2)$	65.8782	55.2793	54.91803	51.81405
$A_2(\text{cm}^2)$	14.9139	11.4971	32.46896	20.2625
$A_3(\text{cm}^2)$	12.7741	10.4852	25.02981	9.01765
$A_4(\text{cm}^2)$	28.1739	24.0730	33.9424	20.66141
$A_5(\text{cm}^2)$	6.3903	5.6057	6.02382	5.92697
$A_6(\text{cm}^2)$	23.3621	20.1294	23.15953	17.23042
$A_7(\text{cm}^2)$	13.2727	13.2292	13.74494	13.23
$A_8(\text{cm}^2)$	13.2337	13.2285	14.06334	24.06284
Best weight(kg)	2890.16	2600.73	3280.078	2918.79
Mean weight(kg)	3129.69	2743.04	3503.06	3126.15
Worst (kg)	3458.16	3257.29	3947.45	3810.47



**Fig 3.** Probability of safety for 52bar dome truss (EVPS).



**Fig 4.** Comparison of the convergence curves obtained by the algorithms for the 52-bar dome truss.

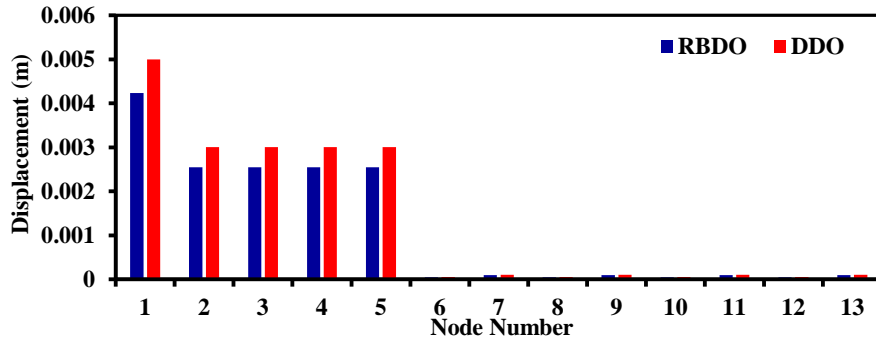


Fig 5. Comparison of allowable displacement of 52-bar dome truss (EVPS).

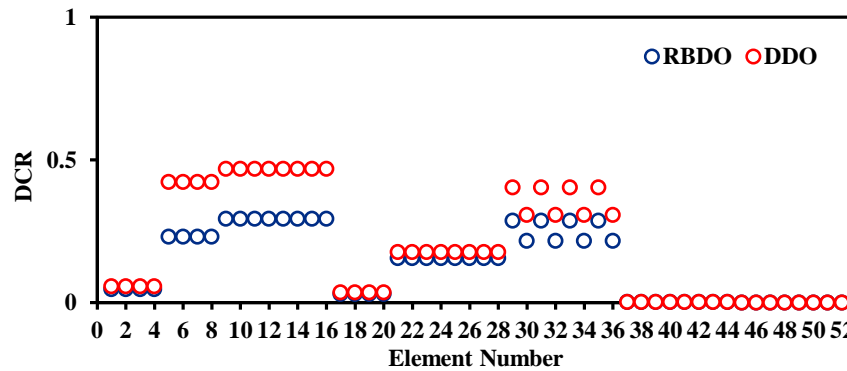


Fig 6. Comparison of allowable demand capacity ratio of 52-bar dome truss (EVPS).

The above results for the 52-bar dome truss illustrate that the weight of the structure has been optimized successfully and there are no violations of displacement and stress ratio constraints.

### 3.2. 120 Bar Dome Truss

In this section, the 120-bar dome truss structure is investigated and depicted as Fig. 7. This truss has 120 elements with 49 nodes and 111 degrees of freedom.

This example has already been studied by some researchers [35-39]. The elements

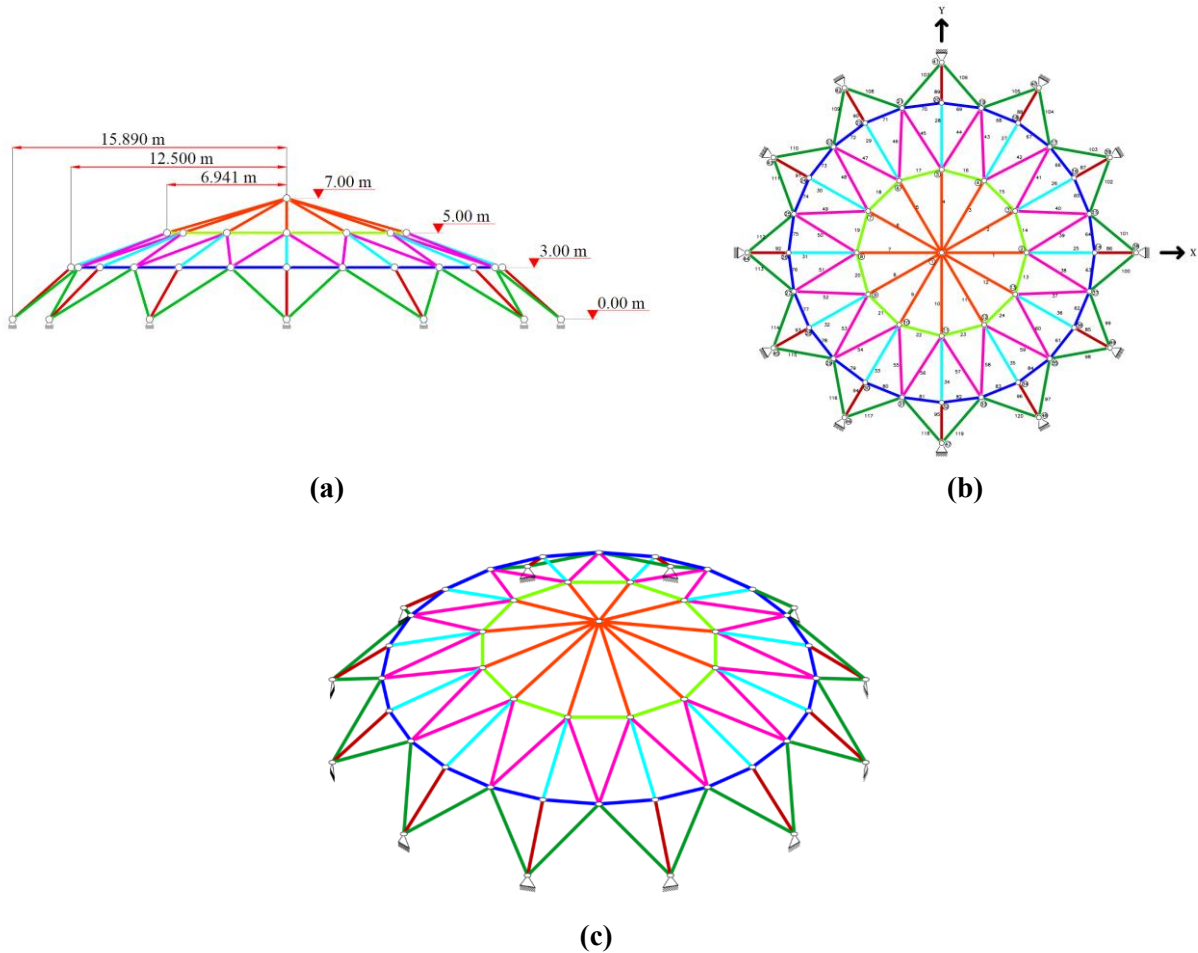
grouping for this truss is according to Table 4. The yield strength of steel and material density ( $\rho$ ) are defined 400 (MPa) and 7971.81 ( $\text{kg/m}^3$ ) respectively as random variables as in Table 5. The free nodes of the 120-bar dome truss are subjected to vertical loading of -60 kN at node 1, -30 kN at nodes 2-13, and -10 kN at nodes 14-37. The allowable displacement of all free nodes is 5mm, and there is a boundary between  $1 \times 10^{-4} \text{ m}^2$  and  $1 \times 10^{-3} \text{ m}^2$  in the cross-sectional area. The allowable strength is calculated like a 52-bar dome truss.

Table 4. 120-bar dome truss element grouping.

Group No.	Elements	Group No.	Elements
1	1-12	5	61-84
2	13-24	6	85-96
3	25-36	7	97-120
4	37-60		

**Table 5.** Material properties for the 120-bar dome truss.

Random Variable, unit	Mean	COV
$E$ (Modulus of elasticity), $N/m^2$	$2 \times 10^{11}$	5%
$\rho$ (Material density), $kg/m^3$	7971.81	5%
$A$ (element group crosssection), $cm^2$	$A_1=19.547$ $A_2=95.499$ $A_3=35.089$ $A_4=20.152$ $A_5=51.733$ $A_6=23.308$ $A_7=16.044$	5%

**Fig 7.** schematic view of 120-bar dome truss (a) View from the side, (b) View from the top, (c) View from an isometric perspective.

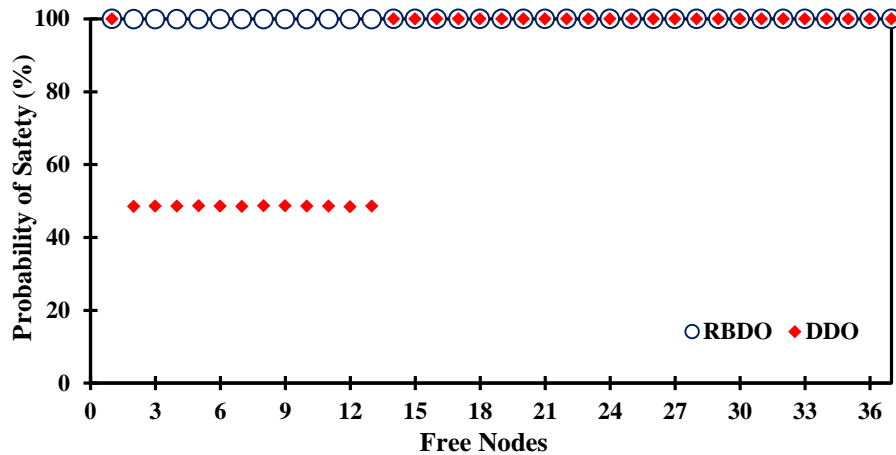
The results of optimization for the best cross-sections and weight of this truss for both RBDO and DDO are presented in Table 6. The probability of safety for all free nodes in RBDO and the reliability assessment for DDO is depicted in Fig. 8.

The comparison of results for optimum weight in both DDO and RBDO is depicted in Fig. 9.

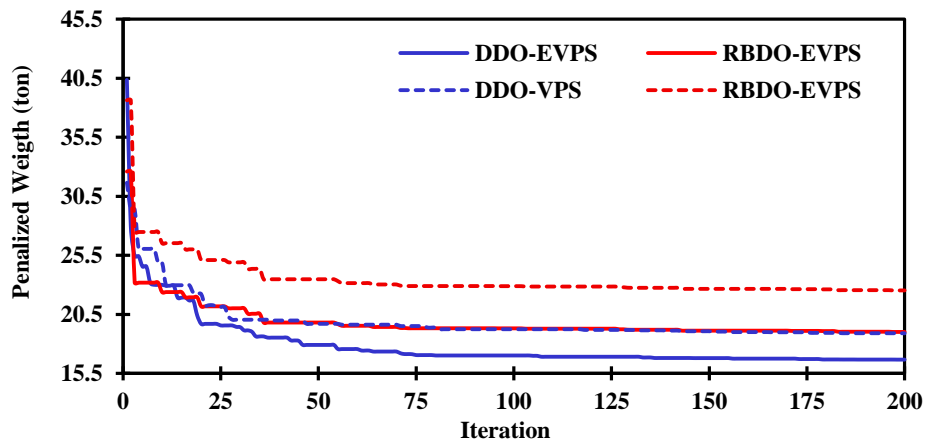
The comparison of results for allowable displacement and DCR are represented in Fig. 10 and Fig. 11, respectively.

**Table 6.** The results of optimization for the 120-bar dome truss.

Design variables	EVPS		VPS	
	RBDO	DDO	RBDO	DDO
A <sub>1</sub> (cm <sup>2</sup> )	30.2939	30.2234	54.4126	31.70977
A <sub>2</sub> (cm <sup>2</sup> )	114.3872	89.1481	103.33856	69.53239
A <sub>3</sub> (cm <sup>2</sup> )	38.3436	30.8779	33.42117	47.9345
A <sub>4</sub> (cm <sup>2</sup> )	25.5196	25.2144	38.1376	28.59625
A <sub>5</sub> (cm <sup>2</sup> )	66.7825	53.6975	71.84619	53.51381
A <sub>6</sub> (cm <sup>2</sup> )	23.5688	19.7777	21.48545	31.91932
A <sub>7</sub> (cm <sup>2</sup> )	22.1812	22.0483	29.1189	30.34902
Best weight(kg)	18976.01	16649.17	22527.85	18895.72
Mean weight(kg)	19117.93	16651.81	23451.74	19316.05
Worst (kg)	19272.13	16660.33	24551.74	19660.52



**Fig 8.** Probability of safety for 120bar dome truss (EVPS).



**Fig 9.** Comparison of the convergence curves obtained by the algorithms for the 120-bar dome truss.

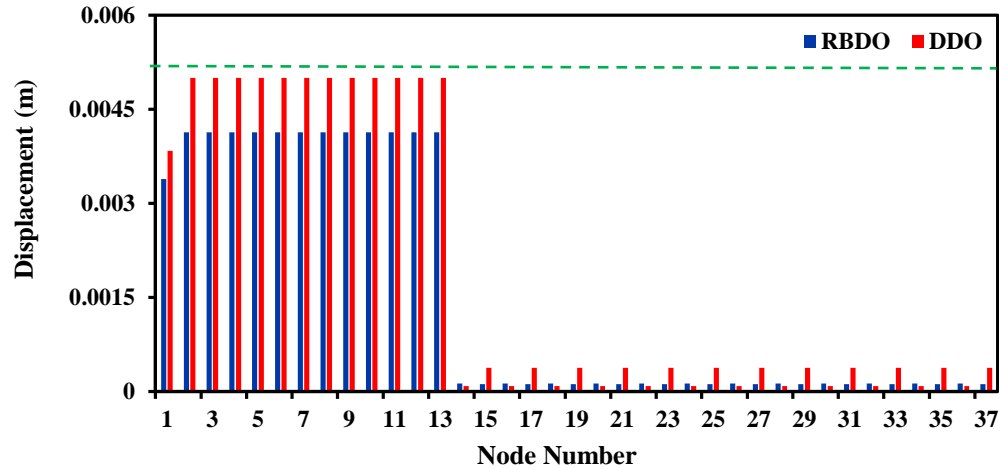


Fig 10. Comparison of allowable displacement of 120-bar dome truss (EVPS).

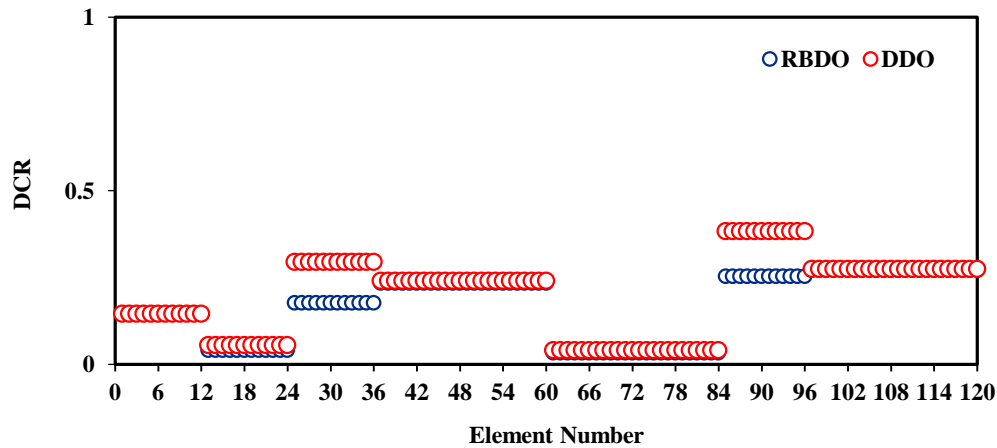


Fig 11. Comparison of allowable demand capacity ratio of 120-bar dome truss (EVPS).

As can be seen, the obtained results for the 120-bar dome truss confirm the performance of the EVPS algorithm for optimization of this structure, and there are no violations of displacement and stress ratio constraints.

### 3.3. 264 Bar Dome Truss

Schwendeler's dome truss is a truss with 264 elements, 97 nodes, and 219 degrees of freedom, and it is shown in Fig. 12. This

truss has been investigated by some researchers previously [40-43].

The element grouping for this truss structure is according to Table 7. The yield strength of steel and material density ( $\rho$ ) are defined 400 (MPa) and 7971.81 ( $\text{kg/m}^3$ ) respectively as random variables as in Table 8. The free nodes of the 264-bar dome truss are subjected to vertical loading of -20 kN at node 1, and -5 kN at other nodes. The allowable displacement of all free nodes of this dome truss is 15mm, and there is a boundary between  $1 \times 10^{-4} \text{ m}^2$  and  $1 \times 10^{-3} \text{ m}^2$  in the cross-sectional area.

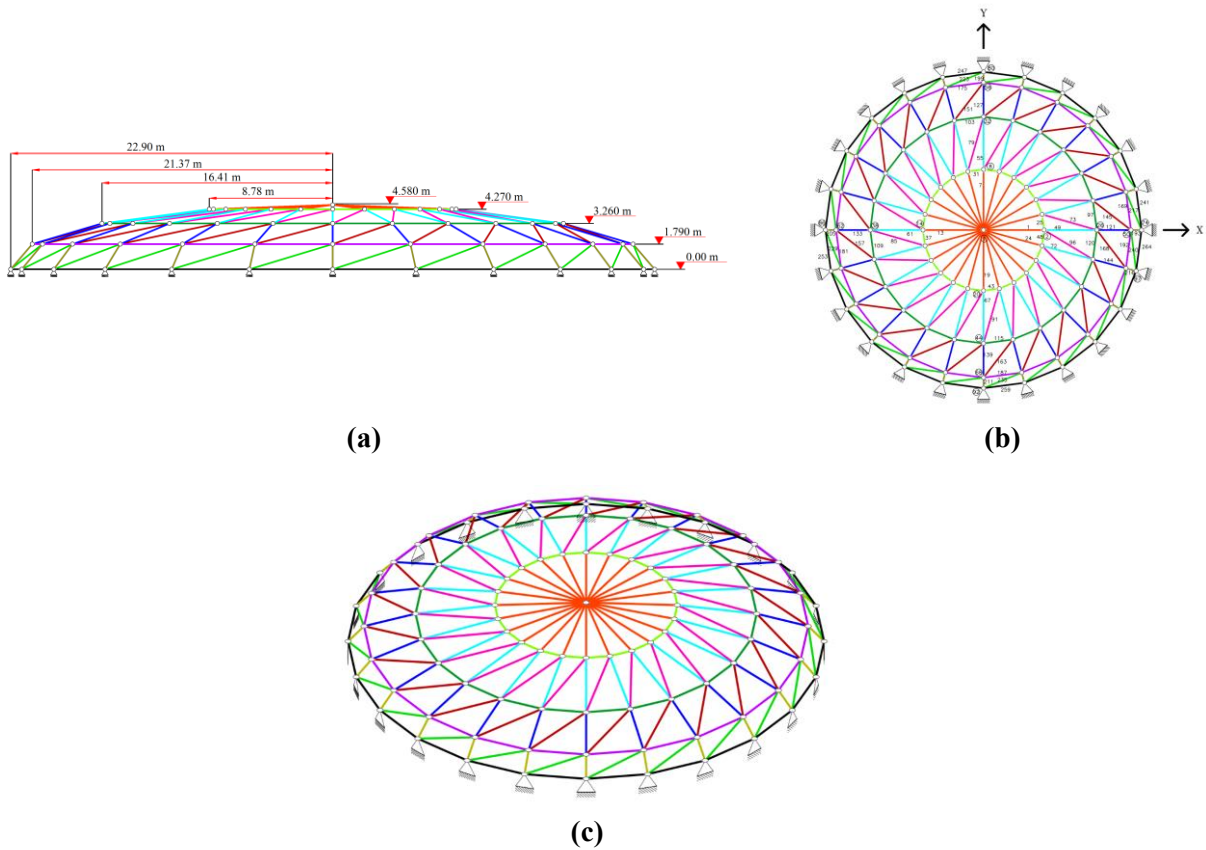
The allowable strength is calculated like other dome trusses in this work.

**Table 7.** 264-bar dome truss element grouping.

Group No.	Elements	Group No.	Elements
1	1-24	7	145-168
2	25-48	8	169-192
3	49-72	9	193-216
4	73-96	10	217-240
5	97-120	11	241-264
6	121-144		

**Table 8.** Material properties for the 264-bar dome truss.

Random Variable, unit	Mean	COV
$E$ (Modulus of elasticity), $N/m^2$	$2 \times 10^{11}$	5%
$\rho$ (Material density), $kg/m^3$	7971.81	5%
$A$ (element group crosssection), $cm^2$	$A_1=34.722$ $A_2=51.161$ $A_3=52.596$ $A_4=21.481$ $A_5=53.888$ $A_6=36.274$ $A_7=16.481$ $A_8=19.253$ $A_9=27.289$ $A_{10}=14.486$ $A_{11}=17.886$	5%



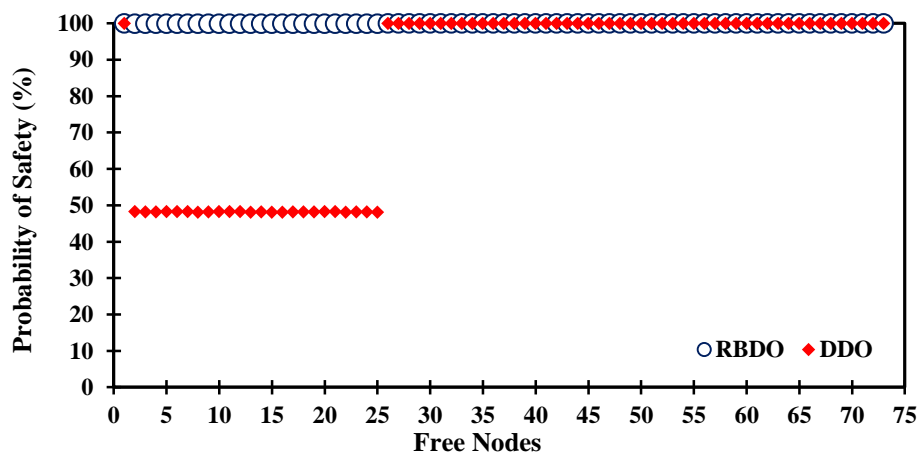
**Fig 12.** schematic view of 264-bar dome truss (a) View from the side, (b) View from the top, (c) View from an isometric perspective.

The results of optimization for the best cross-sections and weight of this truss for both RBDO and DDO are presented in Table 9. The probability of safety for all free nodes in RBDO and the reliability assessment for DDO is depicted in Fig. 13. The comparison

of results for optimum weight in both DDO and RBDO is shown in Fig. 14. The comparison of results for allowable displacement and DCR are represented in Fig. 15 and Fig. 16, respectively.

**Table 9.** The results of optimization for the 264-bar dome truss.

Design variables	EVPS		VPS	
	RBDO	DDO	RBDO	DDO
$A_1(\text{cm}^2)$	40.56991	40.35407973	40.40369	40.45478
$A_2(\text{cm}^2)$	100.66205	86.61624821	91.01986	56.88011
$A_3(\text{cm}^2)$	36.85453	33.59272443	51.2987	39.98452
$A_4(\text{cm}^2)$	37.85027	37.17120743	42.13349	43.56701
$A_5(\text{cm}^2)$	48.68723	39.73779135	36.65985	77.33904
$A_6(\text{cm}^2)$	22.31664	21.73672111	35.8979	29.57561
$A_7(\text{cm}^2)$	30.62781	29.57357909	43.95088	34.38748
$A_8(\text{cm}^2)$	70.98640	53.51254456	83.29205	58.75077
$A_9(\text{cm}^2)$	21.03352	9.20248649	16.94356	34.0759
$A_{10}(\text{cm}^2)$	24.74217	24.33430962	29.46138	25.4435
$A_{11}(\text{cm}^2)$	23.90110	22.89112181	29.64345	23.21901
Best weight(kg)	47258.47	42521.28	54137.44	49554.07
Mean weight(kg)	47906.87	42550.62	56481.12	51420.45
Worst (kg)	48433.46	42762.27	57015.48	52465.32



**Fig 13.** Probability of safety for 264-bar dome truss (EVPS).



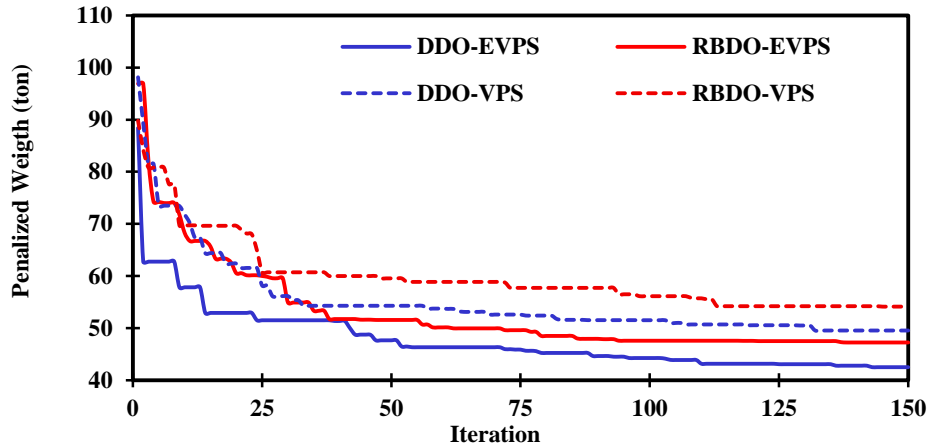


Fig 14. Comparison of the convergence curves obtained by the algorithms for the 264-bar dome truss.

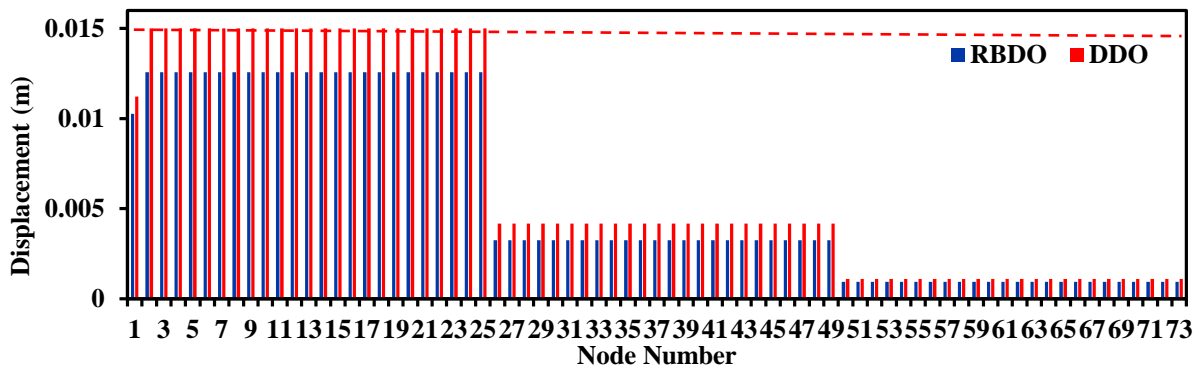


Fig 15. Comparison of allowable displacement of 264-bar dome truss (EVPS).

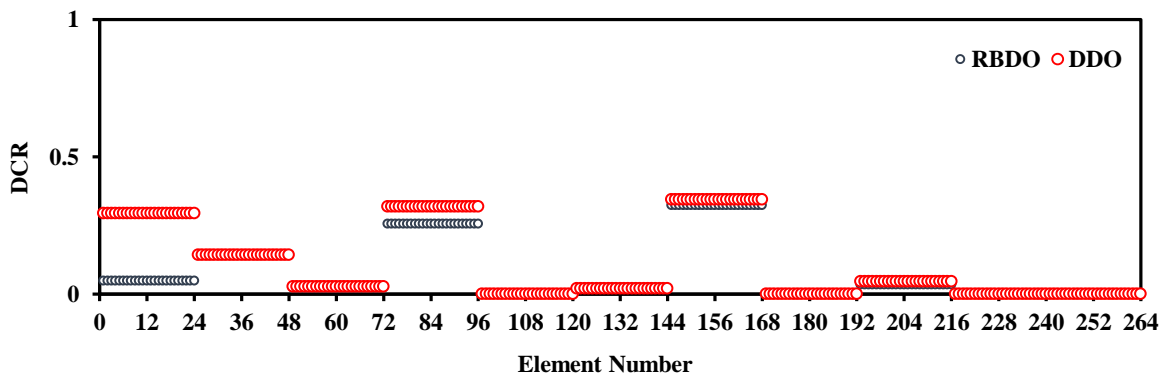


Fig 16. Comparison of allowable demand capacity ratio of 264-bar dome truss (EVPS).

Examining the results for the 264-bar dome truss, it can be concluded that the proposed algorithm has the necessary efficiency to reach optimal design with satisfying the displacement and stress ratio constraints.

#### 4. Conclusion

It is critical that a structure provides adequate safety and is economically viable. Due to the existence of some uncertainties, such as

material properties, external loads, the geometry of members, etc., it is necessary to consider these uncertainties. By using metaheuristic algorithms, it is possible to achieve a design that simultaneously considers safety and economics. A process for the optimization of dome truss structures based on reliability by introducing a flowchart including the process of Deterministic Design Optimization (DDO) and RBDO is provided. To optimize three types of dome truss structures based on reliability, EVPS and VPS were applied. By using the RBDO and DDO processes, no deviation of the feasible space was observed. One of the key goals of this article was to compare the reliability indices of DDO and RBDO, as well as their weights. The results indicate that when the reliability index is not taken into account, the structure weight is lower, but the structure's safety increases. To minimize the weight of the structure, the allowable displacement and stress ratios must be observed. For further research, it is recommended to perform a similar study for large-scale dome trusses.

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