# Using many objective bat algorithm for solving many-objective nonlinear functions 

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#### Abstract

Despite the fact that algorithmic strategies for dealing with Combinatorial Optimization (CO) have been available for a long time, the further application of Evolutionary Algorithms (EAs) to such problems provides a vehicle for dealing with MOPs of tremendous scope. BAT Algorithm with Many Objectives several BAT algorithms based on R2 Distance (MaBAT/R2) are described, which blend the predominance notion with the R2 marker technique. While the R2 Indicator simplifies the multi-objective problem (MOP) by rewriting it as a series of Tchebycheff Approach problems, since this leader decision making uses the Tchebycheff Approach as a criterion, tackling these issues at the same time inside the BAT framework may lead to early converging. Predominance is important in constructing the leader's collection because it allows the chosen leaders to encompass fewer dense regions, avoiding local optima and producing a more diverse approximated Pareto front. 9 non-linear standard functions yielded this result. MaBAT/R2 appears to be more efficient than MOEAD, NSGAII, MPSOD, and SPEA2. MATLAB was used to generate all of the findings (R2020b).


Keywords: Many Objective Problems, Bat Algorithm, Inverted Generational Distance
2020 MSC: 90X90

## 1 Introduction

The current study is addressed in this section. It considers the authors of the study context; it states the research aims, research question, significance of the study, breadth, and constraints of the investigation. There is frequently no one perfect solution in multi-objective optimization, but rather a set of Pareto optimal options. Consequently, cluster sampling is critical in the number of co optimization procedure for an algorithm to generate a comprehensive and diversified approximation of the Pareto front [2, 6].

By altering weights, the Multi-Objective Bat Algorithm (MOBAT) is presented to determine the Pareto optimal set for Multi-Objective (MO) functions. [12] also presents an expanded BAT for solving multi-objective issues, as well as a Multi-Objective Bat Algorithm (MOBAT). We'll check it against a subset of many-objective test functions first. Now we'll use it to address engineering design optimization challenges like macro - and micro steel beam. The multi-objective BAT algorithm (MOBAT), a biologically inspired meta-heuristic, was successfully employed to tackle the problem floor planning in VSLI design in the approach of the publication [5]. [14] proposes a Multi-Objective

[^0]Optimization Problem (MOOP) to achieve both of the aforementioned goals. The MOOP was resolved using a new simple optimization algorithm called Bat Algorithm (BAT), which is based on the Weight Sum Method (WSM). Therefore, from the literature we can say here no study before combining between MOBAT and R2 Indicator.

Moreover, in another study, a comparison of algorithms for training feed forward neural networks is done. Two gradient descent algorithms (Backpropagation and Levenberg Marquardt), and three population-based heuristics: Bat algorithm, Genetic algorithm, and Particle Swarm optimization algorithm are used for testing. Bat algorithm outperforms all other algorithms in training feed forward neural networks [16]. Those findings support the use of BAT in more trials and contributes substantial. The advantage of employing the bat algorithm is that it allows you to find solutions using population and local search techniques. This work provided both global variety and local rigorous extraction, both of which are critical for heuristic methods. As a result, the Bat algorithm combines PSO and local search, as well as pulse rate control and loudness 4. The MOBAT are utilized for the MaOPs and give a good balance in diversity and confluence, which is really the key issue in the MaOPs, by adapting the approach of reference sets. Furthermore, in 2021, a paper titled Using Multi-Objective Bat Algorithm for Solving Multi-Objective Non-linear Programming Problem is published [15]. In addition, in 2020 [1], a metaheuristic hybrid method for solving multi-objective optimisation problems is proposed.

The major goal of this study is to improve performance of many-objective algorithms by developing a new batinspired algorithm for many-objective optimization problems that uses a during forward technique to achieve organization to achieve its goals and diversification. Our present an augmentation method which is based on the R2 indicator distance algorithm to reduce processing efforts in the realm of various objective challenges in this research.

## 2 Definition and Basic Concept

The goal of this research is to tackle the following categories of problems (without sacrificing applicability, the current research will presuppose solely hindered by the lack):

$$
\begin{equation*}
\text { Minimize } f_{i}(x)=\left[f_{1}(x), f_{2}(x), \ldots, f_{k}(x)\right] \tag{2.1}
\end{equation*}
$$

subject to:

$$
\begin{gather*}
g_{i}(x) \leq 0, i=1, \ldots, m  \tag{2.2}\\
h_{j}(x)=0, j=1, \ldots, p \tag{2.3}
\end{gather*}
$$

where $x=\left[x_{1}: x_{2}, \ldots, x_{n}\right]^{T}$ is the vector of decision variables $f_{i}: R^{n} \rightarrow R ; ; i=1, \ldots, k$ are the objective functions and $g_{i} ; ; h_{j}: R^{n}$ to $R, i=1, . ., m$, and $j=1, \ldots, p$ are the constraint functions of the problem? To describe the objective concept of optimality, the researcher introduces the following definitions:

Definition 2.1. 10]: (Multi-objective Optimization Problem (MOP)). A MOP is made up of a number of parameters (decision variables), a number of optimization techniques ( m ), and a number of constraints ( k ). The determination variables' functions and constraints are functions of the optimization algorithms and requirements. The purpose of optimization is to:

$$
\begin{align*}
& \text { Minmize } y=f(x)=\left(f_{1}(x) ; f_{2}(x) ; \ldots ; f_{m}(x)\right) \\
& \text { subject to } e(x)=\left(e_{1}(x) ; e_{2}(x) ; \ldots ; e_{k}(x)\right) \leq 0 \tag{2.4}
\end{align*}
$$

t where $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right) \in X$ and $y=\left(y_{1}, y_{2}, \ldots, y_{m}\right) \in Y$ and x The choice pattern is called the decision vector, the ambition velocity is called the objective vector, the determination space is called the decision sector, and the object space is called the subjective space. The constraints $e(x) \leq 0$ determine the set of feasible solutions.

Definition 2.2. 10]: (Pareto-dominance). If a and b is some two choice templates, $a>b$ (a dominates b) if and only if $f(a)<f(b)$
$a>b$ ( $a$ weakly dominates $b$ ) if and only if $f(a) \leq f(b)$
$a \sim b(a$ is indifferent tob) if and only if $f(a) \nsupseteq f(b) \wedge f(b) \nsupseteq f(a)$.

Definition 2.3. [10]:(Allocative efficiency-optimality) a dimension of choice $x \in X_{f}$ When it comes to a set, it's said to be completely non $A \subseteq X_{f}$ iff $\nexists a \in A: a>x$. If it is evident from the circumstances whichever set A is wanted, the following will simply be omitted. Furthermore, x is described as Allocative Efficiency-Optimal $i f f x$ is non-dominated regarding $X_{f}$.

The Allocative performance set is made up of all Total factor productivity points, whereas the Amponsah efficiencyoptimal front or interface is made up of the equivalent goal vectors.

Definition 2.4. [10]: (Allocative efficiency frontier) The Allocative efficiency frontier, $\mathrm{P}(\mathrm{Y})$ outlined in the following manner. Contemplate a system that benefits a purpose $f: R n \rightarrow R m$, where X is Meanwhile, Y is the practicable set of criteria vectors in Rm, and a compact set of viable judgments in the mapping function Rn, such that $Y=\{y \in$ $R m: y=f(x), x \in X\}$.
The recommended inclinations of parameters are approximated. A point " $\in \mathrm{Rm}$ is preferred to (strictly dominate) another point $y^{\prime} \in \operatorname{Rm}$, as well as $\mathrm{y}^{\prime \prime}>\mathrm{y}^{\prime}$. The Allocative efficiency frontier is thus as well as: $P(Y)=$ $\left\{y^{\prime} \in Y:\left\{y^{\prime \prime} \in Y: y^{\prime \prime}>y^{\prime}, y^{\prime} \neq y^{\prime \prime}\right\}=\varnothing\right\}$.

Definition 2.5. 10]: A set of controller parameters in a scalar $x_{1} \in X \subset R^{n}$ is non-dominant when it comes to $X$, if no $x_{2} \in X$ appears in the sense that $f\left(x_{2}\right)<f\left(x_{1}\right)$.

Definition 2.6. 10]: A vector of decision variables $x_{1} \in X \subset R^{n}$ is non-dominated with respect to $X$, if no $x_{2} \in X$ exists, such that $f\left(x_{2}\right)<f\left(x_{1}\right)$.

Definition 2.7. 10]: A vector of decision factors $x^{*} \in F \subset R^{n}$ ( $F$ is the feasible region) is Pareto optimal if it is non-dominated with respect to $F$.

Definition 2.8. [10]: The Pareto optimal set $\mathrm{P}^{*}$ is defined as follows: $\mathrm{P}^{*}=\left\{x_{1} \in F: x_{1}\right.$ is Pareto optimal $\}$.
Definition 2.9. [10]: The Allocative efficiency optimal set $\mathrm{P}^{*}$ is characterized as follows: $\mathrm{P}^{*}=\left\{x_{1} \in F: x_{1}\right.$ is Allocative efficiency optimal \}.

## 3 The Proposed Method

Within that section, we'll go through a few terminologies that have been used in MOPs for a while. At that moment, we present the recommended calculation's system. The procedure of completing a healthcare task is then depicted. Finally, the procedures for mating and natural selection are discussed.

Bats are mammals with wings and the ability to use echolocation. Approximately 996 unique bat species were identified worldwide, accounting for almost $20 \%$ of all well-evolved mammal species 9 . Another improved computation dubbed as BAT [13] is based on swarm understanding and bat perceptions to tackle the described difficulty. Using the BAT, one may reenact some of the echolocation features of a smaller level bat. The benefits of this approach include its ease of use, versatility, and simplicity of execution. Furthermore, the approach effectively handles a wide range of challenges, such as extremely nonlinear issues. BAT also provides promising perfect arrangements quickly and works brilliantly with complex issues. The trying to follow are some of the disadvantages of this estimation: conjunction occurs quickly at first, and the rate of accompanying decreases. Furthermore, no scientific study has linked the factors to assorted variety rates. To obtain a superior ideal approach for multi-target possibilities using BAT, the expert develops a calculation known as MOBA by including two additional sectors, file and innovator, as found in the MOPSO calculation presented by [17]. The narrative is in charge of preserving and re-establishing the most significant non-overwhelmed also non-controllable Pareto ideal arrangements that have been discovered to date. A primary unit, which is the file's central processing unit, is also depicted in the chronicle. When there are fresh no governing arrangement, that unit regulates the number of no managing provisions. The most reasonable arrangement obtained is used in the MaBAT/R2 computation. This pioneer leads people through the transition zone in order to find a solution near the best arrangement. However, in contradiction to Pareto's best suggestions, arrangements can't be in a more objective inquiry about space. The pioneers option component is designed to address the problem. The most reasonable non-prevailing arrangements are recorded in a journal. The inventor selects a segment from among the crammed fragments of the space layout and proposes one of the non-dominant options. The random wheel is used to make the decision, together with the opportunities for each hyper:

```
MaBAT/ R2 Follow the Steps With R2 Indicator
Set \(\mathrm{k}:=0\) and velocity \(=0 \mu=0.1, \mathrm{r} 0=0.5, \mathrm{~A}=0.6\).
Reload Point at arbitrarily. \(\mathrm{P}_{\mathrm{i}}\) for n.population
Determine the starting Nation's model parameters: f (P ); ;
Discover non-dominated options and use them to start the storage.
WHILE (The requirements for withdrawal have not been met)
BAT Steps
\(\mathrm{Q}=\mathrm{Qmin}+(\mathrm{Qmin}-\mathrm{Qmax}) *\) rand (equation 1)
\(\mathrm{P}_{\text {leader } 1}=\) Choose a captain \((\operatorname{archive}) \mathrm{V}_{(\mathrm{t}+1)}=\mathrm{V}_{(\mathrm{t})}+\left(\mathrm{P}_{\text {leader } 1}-\mathrm{P}_{(\mathrm{t})}\right) * \mathrm{Q}\)
(equation 2)
\(\mathrm{P}_{\text {new }}=\mathrm{P}_{(\mathrm{t})}+\mathrm{V}_{(\mathrm{t}+1)}\) (equation 3)
If rand \(>\mathbf{r}\)
\(\mathrm{P}_{\text {leader2 }}=\) Choose a captain(archive)
\(\mathrm{P}_{\text {new }}=\mathrm{P}_{(\mathrm{t})}+\mathrm{rand} *\left(\mathrm{P}_{\text {leader2 }}-\mathrm{P}_{(\mathrm{t})}\right)\)
End
if \(P_{\text {new }}\) dominated on \(P_{(t)} \&(\) rand \(<A) \mathrm{P}_{(\mathrm{t})}=\mathrm{P}_{\text {new }}\)
End
If rand \(<\left(\frac{\mathbf{1 - ( k - 1 )}}{\text { Max iteration-1 }}\right)^{\mathbf{1 / \mu}}\)
\(\mathrm{S}=\) Mutation \(\left(\mathrm{P}_{(\mathrm{t})}\right)\)
if \(P_{\text {new }}\) dominated on \(P_{(t)} \&(\) rand \(<A) \mathrm{P}_{(\mathrm{t})}=\mathrm{S}\)
End
End
Look for options that aren't dominating.
Update the archive with the non-dominated alternatives that have
been found.
If the archive is full
To omit one of the current archive members, use the grid tech-
nique.
Make a note of the new solution in the database.
end if
How many of the new archived responses is outside of the hyper
cubes, update the grids to include the creative approach (s)
end if
Inc
rease \(\mathbf{r}\) and reduce \(\mathbf{A}\)
Set \(k:=k+1 ;\);
END WHILE
```


## 4 Experimental and Results

A total of 9 benchmark functions with diverse PF sizes were picked from the DTLZ [8] to assess the effectiveness of our proposed MaBAT/R2, especially on problems with irregular PF forms. The performance measures in this paper are the well-known HV [18] and inverted generational distance (IGD) [7]. Both HV and IGD are capable of reflecting the concentration and variety of the algorithms' perfect outcome set. A greater resemblance to the genuine PF is indicated by a larger HV value or a smaller IGD number. For many issues, a reference point dominated by the e true PF is carefully specified for determining HV.

### 4.1 Inverted Generational Distance (IGD)

Letting S denote the search result of a MOEA on a specific MOP. Should R be a set of PF representation points that are equally spaced. [1] can be used to determine S's IGD value in relation to $\mathrm{R}, \operatorname{IGD}(S, R)=\frac{\sum_{r \in R} d(r, S)}{|R|}$, when $|R|$ is the cardinality of $R$ and $d(r, S)$ is the minimum Euclidean distance between $r$ and the points in $S$. It is important to note that perhaps the elements in R should really be spread evenly, and $|\mathrm{R}|$ should be large enough to ensure that the points in $R$ fairly reflect the PF. This ensures that the IGD value of $S$ may accurately assess the solution set's confluence and diversification. S has a lower IGD value, which indicates that it is of higher quality [3]. A set R of
indicative points of the PF must be provided in this section to calculate the IGD value of a result set $S$ of a MOEA executing on a MOP.

### 4.2 Hyper volume Indicator

The hyperbolic quantity indicator $I_{\text {hyp }}(\mathcal{A})$ calculates the volume of a territory H that is composed of a set of points A and a set of reference points N .

$$
\begin{equation*}
I_{\mathrm{hyp}}(\mathcal{A})=\text { volume }\left(\bigcup_{\forall \boldsymbol{a} \in \mathcal{A} ; \forall \boldsymbol{n} \in \mathcal{N}} \text { hypercube (a, n) }\right) \tag{4.1}
\end{equation*}
$$

As a result, higher indicative values correspond to better solutions. The S metric or the Lebesgue measure are other names for the hyper density indicator. It has a number of appealing attributes that have aided in its adoption and success. It is, in example, the only marker with metric features and the only one that is strictly Pareto monotonic [7. Because of these characteristics, this indicator has been employed in a variety of applications, including measuring performance and evolutionary programming.

## 5 Procedure for Analysis

The testing screen will be presented cantered on the mean, standard deviation (SD), MaBAT/ R2 point, and Wilcoxon marked placement test measures of the competence statistics.
(a) Mean ( $\mathbf{x}$ ) will be calculated by multiplying the total quantity of observed results from the example by the maximum population of these outcomes $\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i}$.
(b) SD: is a talent esteems tool that assesses the variety or scattering of a group of data $S D=\sqrt{\frac{1}{N-1} \sum_{i=1}^{N}\left(x_{i}-\bar{x}\right)^{2}}$.
(c) Wilcoxon marked: positional evaluation The Wilcoxon marked positioning test determines the difference between two illustrations [11] and provides an optional territory trial that is influenced by the sizes and indications of these distinctions. The following theories are addressed by this test:

$$
\begin{equation*}
\mathrm{H} 0: \text { mean }(\mathrm{A})-\text { mean }(\mathrm{B}), \quad \mathrm{H} 1: \text { mean }(\mathrm{A})-\text { mean }(\mathrm{B}), \tag{5.1}
\end{equation*}
$$

The solutions to the first and second equations are denoted by the letters A and B, correspondingly. Furthermore, this metric determines if one prediction outperforms the other. Let di denote the gap between the presentation scores of two calculations when it comes to dealing with the ith out of n difficulties. Enable $R^{+}$to represent the number of sites for instances where the main computation beats the second. Finally, let R-deal with the number of places for the instances where the next estimate outperforms the previous. Several 0's are equitably spread across the entireties. If any of these totals have an odd number, one of them has been discarded.

$$
\begin{equation*}
R^{+}=\sum_{d_{i}>0} \operatorname{rank}\left(d_{i}\right)+\frac{1}{2} \sum_{d_{i}=0} \operatorname{rank}\left(d_{i}\right), \quad R^{-}=\sum_{d_{i}<0} \operatorname{rank}\left(d_{i}\right)+\frac{1}{2} \sum_{d_{i}=0} \operatorname{rank}\left(d_{i}\right) \tag{5.2}
\end{equation*}
$$

We utilize MATLAB to find $p$ self-worth in order to contrast the equations at a large degree of alpha $=0.05$. The invalid hypothesis is rejected when the p-esteem is not exactly the essential part. $\mathrm{R}+$ deals with a high mean estimate that demonstrates predominance over processes of planning using a variety of test setups. This method outperforms all other algorithms in all tests. While $R^{+}=\frac{n *(n+1)}{2}$ surpasses all other techniques in all of Adventure.

## 6 Results and Discussion

This section is dedicated to the description of the intended calculation's confirmation. The proposed manyobjective bat calculation (MaBAT/R2) with decay is implemented in Matlab, and the registry time is within a few seconds to not quite a second, depending on the intrigue issue. We tested it with a variety of elements, including population size ( n ), din reduction, and tachycardia reduction rate $\beta$. The experiment findings applied the proposed methodology's suitability for adjusting closeness and variety. Scientists, on the other hand, have designed a set of decay-based algorithms specifically for multi-objective streamline. We compared MaBAT/ R2 to two multi-objective PSO calculations that are representative of the cream of the crop to see how severe it was. MOPSO [17], MOEA/D [13] are the different methods. Each computation is repeated several times in order to achieve the metrics (IGD) and (HV) for each test work. Tables 1 and 2 show the mean quality and standard deviation and variance of the outcomes. Figures (1) and (2) show the non-commanded fronts that follow (2).

| Problem | N | M | D | FEs | IBEA | BiGE | MOMBIII | MODEMR | KnEA | RVEA | MaBAT/R2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DTLZ1 | 150 | 5 | 9 | 150000 | $6.8257 \mathrm{e}-1$ (6.33e-2) - | $8.9840 \mathrm{e}-1$ (5.53e-2) - | $9.7416 \mathrm{e}-1$ (3.07e-3) - | 8.4964e-1 (7.23e-2) - | $6.8355 \mathrm{e}-1$ (1.34e-1) - | $9.6374 \mathrm{e}-1$ (3.02e-4) - | $9.7496 \mathrm{e}-1(1.40 \mathrm{e}-4)$ |
|  | 200 | 10 | 14 | 200000 | $9.0336 \mathrm{e}-1(3.58 \mathrm{e}-2)$ - | $2.4403 \mathrm{e}-1$ (1.52e-1) - | $8.5471 \mathrm{e}-1$ (6.51e-2) - | $2.4234 \mathrm{e}-1(7.42 \mathrm{e}-2)-$ | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $9.2711 \mathrm{e}-1(3.91 \mathrm{e}-2)$ - | $9.9749 \mathrm{e}-1(2.40 \mathrm{e}-4)$ |
|  | 250 | 15 | 19 | 250000 | $9.3565 \mathrm{e}-1(2.91 \mathrm{e}-2)$ - | $2.7192 \mathrm{e}-1(1.98 \mathrm{e}-1)$ - | $9.3302 \mathrm{e}-1$ (4.41e-2) - | $9.8715 \mathrm{e}-1(5.00 \mathrm{e}-2)$ - | $1.8867 \mathrm{e}-6$ (1.03e-5) - | $9.5880 \mathrm{e}-1(2.90 \mathrm{e}-2)$ - | $9.9989 \mathrm{e}-1(5.96 \mathrm{e}-5)$ |
|  | 300 | 20 | 24 | 300000 | $9.5413 \mathrm{e}-1(2.15 \mathrm{e}-2)$ - | $2.2710 \mathrm{e}-1(2.53 \mathrm{e}-1)$ - | $9.1624 \mathrm{e}-1(4.84 \mathrm{e}-2)$ - | $2.6223 \mathrm{e}-1$ (6.61e-2) - | $3.0867 \mathrm{e}-3$ (1.67e-2) - | $9.6537 \mathrm{e}-1(2.70 \mathrm{e}-2)$ - | $1.0000 \mathrm{e}+0(1.63 \mathrm{e}-6)$ |
| DTLZ2 | 150 | 5 | 14 | 150000 | $7.9481 \mathrm{e}-1$ (4.08e-4) | $7.6948 \mathrm{e}-1(5.95 \mathrm{e}-3)$ - | $7.9450 \mathrm{e}-1(3.89 \mathrm{e}-4)$ - | $7.6448 \mathrm{e}-1$ (5.33e-3) - | $7.8019 \mathrm{e}-1$ (3.76e-3) - | $7.9323 \mathrm{e}-1(4.92 \mathrm{e}-4)$ - | $7.9836 \mathrm{e}-1$ (1.27e-3) |
|  | 200 | 10 | 19 | 200000 | $9.4386 \mathrm{e}-1(2.21 \mathrm{e}-4)$ | $9.4840 \mathrm{e}-1(3.12 \mathrm{e}-3)-$ | $8.2676 \mathrm{e}-1(1.02 \mathrm{e}-1)$ - | $9.2320 \mathrm{e}-1(9.91 \mathrm{e}-3)-$ | $9.5777 \mathrm{e}-1(3.02 \mathrm{e}-3)-$ | $9.4295 \mathrm{e}-1(3.54 \mathrm{e}-4)$ - | $9.6815 \mathrm{e}-1(6.75 \mathrm{e}-4)$ |
|  | 250 | 15 | 24 | 250000 | $9.9124 \mathrm{e}-1(1.05 \mathrm{e}-4)$ - | $9.8878 \mathrm{e}-1(9.11 \mathrm{e}-4)$ - | $8.7632 \mathrm{e}-1$ (7.83e-2) - | $9.9159 \mathrm{e}-1(2.00 \mathrm{e}-4)-$ | $6.4009 \mathrm{e}-1(4.45 \mathrm{e}-1)=$ | $9.8956 \mathrm{e}-1(7.75 \mathrm{e}-4)$ - | $9.9315 \mathrm{e}-1$ (2.37e-4) |
|  | 300 | 20 | 29 | 300000 | $9.9819 \mathrm{e}-1(1.68 \mathrm{e}-4)$ - | $9.9712 \mathrm{e}-1(3.54 \mathrm{e}-4)$ - | $8.9309 \mathrm{e}-1$ ( $5.76 \mathrm{e}-2)$ - | $9.9327 \mathrm{e}-1$ (1.47e-3) - | $4.4353 \mathrm{e}-1$ (4.94e-1) - | $9.9796 \mathrm{e}-1(3.07 \mathrm{e}-4)$ - | $9.9860 \mathrm{e}-1(2.91 \mathrm{e}-4)$ |
| DTLZ3 | 150 | 5 | 14 | 150000 | $3.7945 \mathrm{e}-1$ (2.18e-3) - | $4.8529 \mathrm{e}-1(1.31 \mathrm{e}-1)$ - | $7.9228 \mathrm{e}-1(1.99 \mathrm{e}-3)$ - | $4.2129 \mathrm{e}-1(2.32 \mathrm{e}-1)$ - | $5.0144 \mathrm{e}-1(1.48 \mathrm{e}-1)$ - | $2.4283 \mathrm{e}-1(3.24 \mathrm{e}-1)$ - | $7.9323 \mathrm{e}-1(1.70 \mathrm{e}-3)$ |
|  | 200 | 10 | 19 | 200000 | $6.1617 \mathrm{e}-1$ (1.22e-2) - | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)$ - | $4.8355 \mathrm{e}-1$ (6.43e-2) - | $1.2788 \mathrm{e}-1$ (7.58e-2) - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $7.7229 \mathrm{e}-1(2.30 \mathrm{e}-1)$ - | $9.4241 \mathrm{e}-1$ (1.38e-3) |
|  | 250 | 15 | 24 | 250000 | $7.3150 \mathrm{e}-1(2.42 \mathrm{e}-2)$ - | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)$ - | $5.6194 \mathrm{e}-1$ (3.27e-2) - | $9.1721 \mathrm{e}-2(5.24 \mathrm{e}-3)-$ | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $5.5369 \mathrm{e}-1(2.77 \mathrm{e}-1)$ - | $9.9062 \mathrm{e}-1(4.53 \mathrm{e}-4)$ |
|  | 300 | 20 | 29 | 300000 | $7.8536 \mathrm{e}-1$ (4.53e-2) - | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)$ - | $5.5866 \mathrm{e}-1(4.58 \mathrm{e}-2)$ - | $9.0810 \mathrm{e}-2$ (9.74e-4) - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $8.1717 \mathrm{e}-1(2.40 \mathrm{e}-1)$ - | $9.9792 \mathrm{e}-1$ (3.62e-3) |
| DTLZ4 | 150 | 5 | 14 | 150000 | $7.9124 \mathrm{e}-1(2.45 \mathrm{e}-2)$ - | $7.7511 \mathrm{e}-1(4.60 \mathrm{e}-3)$ - | $7.8818 \mathrm{e}-1$ (2.30e-2) - | $7.7209 \mathrm{e}-1$ (4.42e-3) - | $7.8581 \mathrm{e}-1$ (3.86e-3) - | $7.9315 \mathrm{e}-1(5.55 \mathrm{e}-4)=$ | $7.9184 \mathrm{e}-1$ (1.67e-2) |
|  | 200 | 10 | 19 | 200000 | $9.6910 \mathrm{e}-1(2.03 \mathrm{e}-3)=$ | $9.4497 \mathrm{e}-1(2.40 \mathrm{e}-2)$ - | $9.2613 \mathrm{e}-1(2.58 \mathrm{e}-2)=$ | $9.4066 \mathrm{e}-1(3.10 \mathrm{e}-3)=$ | $9.5469 \mathrm{e}-1(3.05 \mathrm{e}-3)-$ | $9.4337 \mathrm{e}-1(3.21 \mathrm{e}-4)=$ | $9.3291 \mathrm{e}-1$ (1.80e-2) |
|  | 250 | 15 | 24 | 250000 | $9.9357 \mathrm{e}-1(2.21 \mathrm{e}-4)=$ | $7.8809 \mathrm{e}-1$ (2.55e-2) - | $9.8996 \mathrm{e}-1(3.34 \mathrm{e}-3)=$ | $9.9044 \mathrm{e}-1$ (1.65e-4) - | $9.9298 \mathrm{e}-1(4.29 \mathrm{e}-4)$ - | $9.9102 \mathrm{e}-1(1.00 \mathrm{e}-4)$ - | $9.9126 \mathrm{e}-1$ (1.11e-4) |
|  | 300 | 20 | 29 | 300000 | $9.9858 \mathrm{e}-1$ (7.05e-5) - | $8.2491 \mathrm{e}-1$ (3.37e-2) - | $9.9841 \mathrm{e}-1(4.52 \mathrm{e}-4)$ - | $9.9122 \mathrm{e}-1(1.77 \mathrm{e}-3)-$ | $9.9845 \mathrm{e}-1$ (9.00e-5) - | $9.9866 \mathrm{e}-1(3.00 \mathrm{e}-5)=$ | $9.9865 \mathrm{e}-1(1.00 \mathrm{e}-4)$ |
| DTLZ5 | 150 | 5 | 14 | 150000 | $1.1598 \mathrm{e}-1(2.85 \mathrm{e}-3)=$ | $1.1421 \mathrm{e}-1$ (4.17e-3) - | $9.1022 \mathrm{e}-2$ (2.34e-4) - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $8.8917 \mathrm{e}-2$ (1.17e-2) - | $1.1535 \mathrm{e}-1(2.98 \mathrm{e}-3)$ - | $1.0512 \mathrm{e}-1(1.10 \mathrm{e}-3)$ |
|  | 200 | 10 | 19 | 200000 | $8.9600 \mathrm{e}-2$ (2.00e-3) - | $9.0956 \mathrm{e}-2$ (1.95e-4) - | $9.1293 \mathrm{e}-2(3.09 \mathrm{e}-4)$ - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $6.1102 \mathrm{e}-2$ (2.51e-2) - | $9.1144 \mathrm{e}-2$ (1.13e-3) - | $9.2377 \mathrm{e}-2$ (1.38e-3) |
|  | 250 | 15 | 24 | 250000 | $8.8414 \mathrm{e}-2$ (4.13e-3) - | $9.0898 \mathrm{e}-2$ (1.19e-4) - | $9.1254 \mathrm{e}-2(3.87 \mathrm{e}-4)=$ | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $1.8175 \mathrm{e}-2$ (3.03e-2) - | $9.1038 \mathrm{e}-2(5.22 \mathrm{e}-4)=$ | $9.1199 \mathrm{e}-2$ (5.38e-4) |
|  | 300 | 20 | 29 | 300000 | $8.7890 \mathrm{e}-2$ (5.02e-3) - | $9.0899 \mathrm{e}-2$ (7.79e-5) - | $9.0998 \mathrm{e}-2(1.25 \mathrm{e}-4)=$ | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $1.0933 \mathrm{e}-2$ (2.51e-2) - | $9.0972 \mathrm{e}-2(3.31 \mathrm{e}-4)=$ | $9.0949 \mathrm{e}-2$ (2.17e-4) |
| DTLZ6 | 150 | 5 | 14 | 150000 | $1.1293 \mathrm{e}-1(2.21 \mathrm{e}-3)$ - | $9.3905 \mathrm{e}-2$ (4.23e-3) - | $9.0933 \mathrm{e}-2(2.81 \mathrm{e}-4)$ - | $4.5455 \mathrm{e}-2$ (4.62e-2) - | $9.1913 \mathrm{e}-2$ (3.59e-3) - | $1.1679 \mathrm{e}-1(3.64 \mathrm{e}-3)=$ | $1.0630 \mathrm{e}-1(3.78 \mathrm{e}-3)$ |
|  | 200 | 10 | 19 | 200000 | $9.0908 \mathrm{e}-2$ (2.40e-4) - | $9.2343 \mathrm{e}-2(1.35 \mathrm{e}-3)=$ | $9.1571 \mathrm{e}-2$ (7.52e-4) - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $4.6187 \mathrm{e}-3$ (1.85e-2) - | $9.0956 \mathrm{e}-2(1.12 \mathrm{e}-4)$ - | $9.2194 \mathrm{e}-2$ (1.07e-3) |
|  | 250 | 15 | 24 | 250000 | $9.0872 \mathrm{e}-2$ (2.07e-4) - | $9.1583 \mathrm{e}-2$ (6.45e-4) - | $9.1016 \mathrm{e}-2$ (3.10e-4) - | $0.0000 \mathrm{e}-0$ (0.00e-0) - | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)$ - | $9.1012 \mathrm{e}-2(2.90 \mathrm{e}-4)=$ | $9.1619 \mathrm{e}-2(5.92 \mathrm{e}-4)$ |
|  | 300 | 20 | 29 | 300000 | $9.0894 \mathrm{e}-2(1.40 \mathrm{e}-4)=$ | $9.1428 \mathrm{e}-2$ (4.11e-4) - | $9.0895 \mathrm{e}-2$ (3.45e-4) - | $3.0303 \mathrm{e}-3$ (1.66e-2) - | $3.0303 \mathrm{e}-3$ (1.66e-2) - | $9.0940 \mathrm{e}-2(1.10 \mathrm{e}-4)=$ | $9.1439 \mathrm{e}-2$ (3.43e-4) |
| DTLZ7 | 150 | 5 | 24 | 150000 | $2.0044 \mathrm{e}-1(4.29 \mathrm{e}-3)$ - | $2.5704 \mathrm{e}-1(8.01 \mathrm{e}-3)$ - | $2.4080 \mathrm{e}-1$ (8.06e-3) - | $2.1064 \mathrm{e}-1(1.75 \mathrm{e}-2)$ - | $2.5636 \mathrm{e}-1(3.52 \mathrm{e}-3)-$ | $2.5006 \mathrm{e}-1(1.09 \mathrm{e}-2)$ - | $2.6610 \mathrm{e}-1$ (1.02e-2) |
|  | 200 | 10 | 29 | 200000 | $1.5862 \mathrm{e}-1(3.35 \mathrm{e}-3)$ - | $1.8510 \mathrm{e}-1(7.20 \mathrm{e}-3)$ - | $1.3265 \mathrm{e}-1(8.62 \mathrm{e}-4)$ - | $1.1934 \mathrm{e}-1$ (8.71e-3) - | $5.1148 \mathrm{e}-2$ (3.59e-2) - | $1.3907 \mathrm{e}-1(1.02 \mathrm{e}-2)$ - | $1.9343 \mathrm{e}-1$ (4.84e-3) |
|  | 250 | 15 | 34 | 250000 | $6.7593 \mathrm{e}-2$ (5.98e-2) - | $1.3131 \mathrm{e}-1(5.06 \mathrm{e}-3)$ - | $1.2089 \mathrm{e}-1(3.25 \mathrm{e}-3)=$ | $1.3923 \mathrm{e}-1(1.03 \mathrm{e}-2)$ - | $3.9839 \mathrm{e}-3$ (1.52e-2) - | $1.2061 \mathrm{e}-1(5.18 \mathrm{e}-3)=$ | $1.5651 \mathrm{e}-1$ (1.02e-2) |
|  | 300 | 20 | 39 | 300000 | $1.1274 \mathrm{e}-1(4.82 \mathrm{e}-3)$ - | $1.1749 \mathrm{e}-1$ (3.09e-3) - | $1.1573 \mathrm{e}-1$ (1.80e-3) - | $1.0497 \mathrm{e}-1$ (3.70e-3) - | $8.1985 \mathrm{e}-9$ (4.49e-8) - | $1.0912 \mathrm{e}-1(6.49 \mathrm{e}-3)$ - | $1.3393 \mathrm{e}-1$ (1.77e-2) |
| DTLZ8 | 150 | 5 | 50 | 150000 |  |  |  |  | $3.7228 \mathrm{e}-1(2.21 \mathrm{e}-2)=$ |  | $3.4512 \mathrm{e}-1(1.59 \mathrm{e}-2)$ |
|  | 200 | 10 | 100 | 200000 |  |  |  |  | $1.0257 \mathrm{e}-1(1.30 \mathrm{e}-2)=$ |  | $9.3087 \mathrm{e}-2$ (9.31e-3) |
|  | 250 | 15 | 150 | 250000 |  |  |  |  | $6.4673 \mathrm{e}-3$ (1.45e-3) - |  | $4.0978 \mathrm{e}-2$ (6.44e-3) |
|  | 300 | 20 | 200 | 300000 |  |  |  |  | $1.8794 \mathrm{e}-4$ (7.14e-5) - |  | $2.1706 \mathrm{e}-2$ (4.09e-3) |
| DTLZ9 | 150 | 5 | 50 | 150000 |  |  | $6.9214 \mathrm{e}-3(3.99 \mathrm{e}-3)=$ |  | $5.3080 \mathrm{e}-2(4.41 \mathrm{e}-2)=$ | $3.8851 \mathrm{e}-3$ (6.73e-3) $=$ | $7.0481 \mathrm{e}-2(2.70 \mathrm{e}-2)$ |
|  | 200 | 10 | 100 | 200000 |  | 8.6266e-4 (3.84e-3) - | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ |  | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ |  | $1.1122 \mathrm{e}-3$ (3.69e-3) |
|  | 250 | 15 | 150 | 250000 | $0.0000 \mathrm{e}-0.0 .00 \mathrm{e}-0)=$ | 1.3791e-6 (4.31e-6) - | $5.3719 \mathrm{e}-5$ (1.73e-4) - |  | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ | $9.8307 \mathrm{e}-3$ (2.10e-2) |
|  | 300 | 20 | 200 | 300000 | $0.0000 \mathrm{e}-0.0 .00 \mathrm{e}-0)=$ | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ |  | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ | $0.0000 \mathrm{e}-0(0.00 \mathrm{e}-0)=$ | $1.2884 \mathrm{e}-7$ (6.94e-7) |
| $+/-/=$ |  |  |  |  | 0/24/6 | 0/29/2 | 0/24/8 | 0/27/1 | 0/29/7 | 0/19/12 | 25/11/0 |


| Table 2 Pairwise Comparisons of Algorithms By R2 Indicator Distance When ( $\mathrm{M}=5,10,15 \mathrm{and} 20$ ), $\mathrm{M}=$ No. of Objectives, $\mathrm{N}=$ No.of Populations and $\mathrm{D}=\mathrm{Dimensions}$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Problem | N | M | D | FEs | IBEA | BiGE | MOMBIII | MODEMR | KnEA | RVEA | MaBAT/R2 |
| DTLZ1 | 150 | 5 | 9 | 150000 | $2.1579 \mathrm{e}-1$ (2.86e-2) - | $1.1711 \mathrm{e}-1$ (2.83e-2) - | $6.3843 \mathrm{e}-2$ (1.97e-3) - | $1.0538 \mathrm{e}-1$ (3.09e-2) - | $1.7324 \mathrm{e}-1(5.79 \mathrm{e}-2)$ - | $8.3613 \mathrm{e}-2$ (4.07e-4) - | $6.3322 \mathrm{e}-2$ (2.21e-5) |
|  | 200 | 10 | 14 | 200000 | $2.6133 \mathrm{e}-1$ (1.96e-2) - | $4.8316 \mathrm{e}-1$ (1.39e-1) - | $2.6147 \mathrm{e}-1$ (2.13e-2) - | $4.3158 \mathrm{e}-1$ (3.90e-2) - | $7.1208 \mathrm{e}-0$ (1.08e-1) - | $2.1385 \mathrm{e}-1(2.88 \mathrm{e}-2)$ - | $1.1263 \mathrm{e}-1$ ( $5.54 \mathrm{e}-4$ ) |
|  | 250 | 15 | 19 | 250000 | $2.9297 \mathrm{e}-1$ (2.00e-2) - | $5.4748 \mathrm{e}-1(3.01 \mathrm{e}-1)$ - | $2.4827 \mathrm{e}-1(2.69 \mathrm{e}-2)$ - | $1.3452 \mathrm{e}-1(2.91 \mathrm{e}-2)=$ | $3.8117 \mathrm{e}-0$ (3.63e-0) - | $2.2947 \mathrm{e}-1(2.40 \mathrm{e}-2)$ - | $1.2676 \mathrm{e}-1(8.46 \mathrm{e}-4)$ |
|  | 300 | 20 | 24 | 300000 | $3.1456 \mathrm{e}-1$ (1.45e-2) - | $6.3545 \mathrm{e}-1(2.46 \mathrm{e}-1)$ - | $3.0430 \mathrm{e}-1(1.56 \mathrm{e}-2)$ - | $4.6764 \mathrm{e}-1(2.57 \mathrm{e}-2)$ - | $7.2352 \mathrm{e}-0$ (8.67e-0) - | $2.7957 \mathrm{e}-1$ (1.74e-2) - | $1.9829 \mathrm{e}-1(2.10 \mathrm{e}-3)$ |
| DTLZ2 | 150 | 5 | 14 | 150000 | $2.0807 \mathrm{e}-1$ (1.28e-3) - | $2.3521 \mathrm{e}-1(6.00 \mathrm{e}-3)$ - | $1.9520 \mathrm{e}-1(2.81 \mathrm{e}-4)$ - | $2.0379 \mathrm{e}-1$ (3.43e-3) - | $1.9932 \mathrm{e}-1(2.83 \mathrm{e}-3)-$ | $1.9497 \mathrm{e}-1$ (4.80e-5) - | $1.9489 \mathrm{e}-1(2.59 \mathrm{e}-6)$ |
|  | 200 | 10 | 19 | 200000 | $4.5452 \mathrm{e}-1$ (3.67e-3) - | $4.7877 \mathrm{e}-1$ (4.46e-3) - | $6.3301 \mathrm{e}-1(1.49 \mathrm{e}-1)$ - | $4.5164 \mathrm{e}-1$ (1.89e-2) - | $4.5356 \mathrm{e}-1(3.95 \mathrm{e}-3)-$ | $4.5443 \mathrm{e}-1$ (1.61e-3) - | $4.3767 \mathrm{e}-1$ (3.29e-4) |
|  | 250 | 15 | 24 | 250000 | $5.4964 \mathrm{e}-1$ (3.22e-3) - | $5.8116 \mathrm{e}-1(5.66 \mathrm{e}-3)$ - | $7.7670 \mathrm{e}-1$ (1.08e-1) - | $5.6091 \mathrm{e}-1$ (1.44e-2) - | $7.1794 \mathrm{e}-1$ (2.31e-1) - | $5.7423 \mathrm{e}-1$ (4.43e-3) - | $5.2810 \mathrm{e}-1(2.80 \mathrm{e}-4)$ |
|  | 300 | 20 | 29 | 300000 | $5.9505 \mathrm{e}-1(3.16 \mathrm{e}-3)=$ | $6.1461 \mathrm{e}-1$ (3.54e-3) - | $8.8575 \mathrm{e}-1$ (6.83e-2) - | $6.2033 \mathrm{e}-1$ (7.19e-3) - | $8.8491 \mathrm{e}-1(2.80 \mathrm{e}-1)=$ | $6.2479 \mathrm{e}-1(3.66 \mathrm{e}-3)=$ | $6.2422 \mathrm{e}-1$ (4.62e-3) |
| DTLZ3 | 150 | 5 | 14 | 150000 | $5.9589 \mathrm{e}-1(4.95 \mathrm{e}-3)$ - | $4.8713 \mathrm{e}-1(1.71 \mathrm{e}-1)$ - | $1.9566 \mathrm{e}-1(4.39 \mathrm{e}-4)$ - | $5.9740 \mathrm{e}-1$ (2.80e-1) - | $4.3342 \mathrm{e}-1$ (1.74e-1) - | $1.7647 \mathrm{e}-0$ (2.03e-0) - | $1.9505 \mathrm{e}-1(1.89 \mathrm{e}-4)$ |
|  | 200 | 10 | 19 | 200000 | $7.1970 \mathrm{e}-1$ (1.13e-2) - | $1.4893 \mathrm{e}-1(6.89 \mathrm{e}-0)$ - | $9.5178 \mathrm{e}-1(4.21 \mathrm{e}-2)$ - | $1.2122 \mathrm{e}-0$ (4.73e-2) - | $4.1396 \mathrm{e}-2$ (8.01e-1) - | $6.6584 \mathrm{e}-1(2.22 \mathrm{e}-1)$ - | $4.3776 \mathrm{e}-1$ (7.05e-4) |
|  | 250 | 15 | 24 | 250000 | $8.1169 \mathrm{e}-1$ (1.71e-2) - | $1.8619 \mathrm{e}-1(9.04 \mathrm{e}-0)$ - | $1.0303 \mathrm{e}-0$ (1.38e-2) - | $1.2878 \mathrm{e}-0$ (6.17e-3) - | $7.4785 \mathrm{e}-2$ (1.19e-2) - | $1.0691 \mathrm{e}-0$ (3.04e-1) - | $5.2791 \mathrm{e}-1$ (1.62e-3) |
|  | 300 | 20 | 29 | 300000 | $8.4742 \mathrm{e}-1$ (2.86e-2) - | $1.8720 \mathrm{e}-1(5.57 \mathrm{e}-0)$ - | $1.1204 \mathrm{e}-0$ (2.24e-2) - | $1.3354 \mathrm{e}-0$ (1.42e-3) - | $8.5538 \mathrm{e}-2$ (9.84e-1) - | $9.3564 \mathrm{e}-1(3.45 \mathrm{e}-1)$ - | $6.2679 \mathrm{e}-1(1.66 \mathrm{e}-2)$ |
| DTLZ4 | 150 | 5 | 14 | 150000 | $2.2728 \mathrm{e}-1(6.64 \mathrm{e}-2)$ - | $2.3255 \mathrm{e}-1(5.38 \mathrm{e}-3)$ - | $2.1063 \mathrm{e}-1(5.71 \mathrm{e}-2)$ - | $2.2055 \mathrm{e}-1(5.60 \mathrm{e}-3)-$ | $1.9721 \mathrm{e}-1(4.36 \mathrm{e}-3)=$ | $1.9656 \mathrm{e}-1(1.33 \mathrm{e}-3)=$ | $2.0243 \mathrm{e}-1$ (4.13e-2) |
|  | 200 | 10 | 19 | 200000 | $4.5376 \mathrm{e}-1(9.29 \mathrm{e}-3)=$ | $4.9267 \mathrm{e}-1$ (2.76e-2) - | $5.1485 \mathrm{e}-1(4.57 \mathrm{e}-2)$ - | $4.8855 \mathrm{e}-1$ (9.14e-3) - | $4.5809 \mathrm{e}-1$ ( $5.21 \mathrm{e}-3$ ) - | $4.9267 \mathrm{e}-1(2.59 \mathrm{e}-3)-$ | $4.6585 \mathrm{e}-1$ (4.56e-2) |
|  | 250 | 15 | 24 | 250000 | $5.5349 \mathrm{e}-1(3.60 \mathrm{e}-3)$ - | $7.7809 \mathrm{e}-1$ (1.49e-2) - | $5.9202 \mathrm{e}-1(1.17 \mathrm{e}-2)$ - | $6.2216 \mathrm{e}-1$ (3.78e-3) - | 5.3926e-1 (3.87e-3) - | $5.9580 \mathrm{e}-1(1.95 \mathrm{e}-3)$ - | $5.2772 \mathrm{e}-1.5 .19 \mathrm{e}-4)$ |
|  | 300 | 20 | 29 | 300000 | $5.9584 \mathrm{e}-1(2.07 \mathrm{e}-3)$ - | $8.1218 \mathrm{e}-1$ (8.25e-3) - | $6.2801 \mathrm{e}-1(7.63 \mathrm{e}-3)$ - | $6.5323 \mathrm{e}-1(5.91 \mathrm{e}-3)$ - | $5.7963 \mathrm{e}-1(1.53 \mathrm{e}-3)=$ | $6.2319 \mathrm{e}-1$ (7.56e-5) - | $6.2382 \mathrm{e}-1(2.23 \mathrm{e}-3)$ |
| DTLZ5 | 150 | 5 | 14 | 150000 | $1.8849 \mathrm{e}-1(1.64 \mathrm{e}-2)$ - | $1.0032 \mathrm{e}-1(1.63 \mathrm{e}-2)$ - | $2.7061 \mathrm{e}-1(5.00 \mathrm{e}-3)$ - | $8.9884 \mathrm{e}-1$ (3.59e-2) - | $1.6811 \mathrm{e}-1(4.75 \mathrm{e}-2)$ - | $7.2858 \mathrm{e}-2$ (1.14e-2) - | $5.6634 \mathrm{e}-2$ (1.24e-2) |
|  | 200 | 10 | 19 | 200000 | $4.7978 \mathrm{e}-1$ (2.41e-1) - | $3.3662 \mathrm{e}-1$ (8.61e-2) - | $7.1555 \mathrm{e}-1(1.45 \mathrm{e}-2)$ - | $1.6400 \mathrm{e}-0$ (1.17e-1) - | $3.4394 \mathrm{e}-1$ (7.25e-2) - | $7.1483 \mathrm{e}-1(1.25 \mathrm{e}-1)$ - | $1.2481 \mathrm{e}-1$ (3.61e-2) |
|  | 250 | 15 | 24 | 250000 | $6.4949 \mathrm{e}-1(1.75 \mathrm{e}-1)$ - | $4.1669 \mathrm{e}-1$ (3.25e-2) - | $7.1539 \mathrm{e}-1(2.25 \mathrm{e}-2)=$ | $1.7302 \mathrm{e}-0$ (1.28e-1) - | $5.4652 \mathrm{e}-1(2.14 \mathrm{e}-1)$ - | $7.1969 \mathrm{e}-1(1.01 \mathrm{e}-1)=$ | $2.2716 \mathrm{e}-1$ ( $5.99 \mathrm{e}-2)$ |
|  | 300 | 20 | 29 | 300000 | $2.9953 \mathrm{e}-1(1.62 \mathrm{e}-1)=$ | $3.9637 \mathrm{e}-1(2.88 \mathrm{e}-2)$ - | $7.3003 \mathrm{e}-1(1.10 \mathrm{e}-2)$ - | $1.8148 \mathrm{e}-0$ (3.06e-1) - | $7.8902 \mathrm{e}-1(3.47 \mathrm{e}-1)$ - | $7.3343 \mathrm{e}-1(4.02 \mathrm{e}-2)$ - | $2.3074 \mathrm{e}-1$ (6.57e-2) |
| DTLZ6 | 150 | 5 | 14 | 150000 | $8.2353 \mathrm{e}-2$ (1.61e-2) - | $6.7385 \mathrm{e}-1$ (8.36e-2) - | $3.1331 \mathrm{e}-1$ ( $5.15 \mathrm{e}-3)$ - | $8.3743 \mathrm{e}-1$ (9.93e-2) - | $2.8633 \mathrm{e}-1(7.83 \mathrm{e}-2)$ - | $6.1403 \mathrm{e}-2(7.39 \mathrm{e}-3)=$ | $1.1179 \mathrm{e}-1(1.84 \mathrm{e}-2)$ |
|  | 200 | 10 | 19 | 200000 | $3.4590 \mathrm{e}-1$ (1.72e-2) - | $6.7960 \mathrm{e}-1(5.35 \mathrm{e}-2)$ - | $7.0827 \mathrm{e}-1(2.95 \mathrm{e}-2)$ - | $1.2539 \mathrm{e}-0$ (1.99e-1) - | $1.1678 \mathrm{e}-0$ (3.28e-1) - | $7.3656 \mathrm{e}-1(8.15 \mathrm{e}-3)-$ | $2.5018 \mathrm{e}-1(1.06 \mathrm{e}-1)$ |
|  | 250 | 15 | 24 | 250000 | $3.5146 \mathrm{e}-1$ (2.96e-2) - | $6.7667 \mathrm{e}-1(7.62 \mathrm{e}-2)$ - | $6.8811 \mathrm{e}-1$ (4.03e-2) - | $1.5006 \mathrm{e}-0$ (7.53e-1) - | $1.5900 \mathrm{e}-0$ (3.45e-1) - | $7.3379 \mathrm{e}-1(1.50 \mathrm{e}-2)$ - | $2.2020 \mathrm{e}-1$ (6.40e-2) |
|  | 300 | 20 | 29 | 300000 | $3.5236 \mathrm{e}-1$ (4.21e-2) - | $6.4627 \mathrm{e}-1(8.64 \mathrm{e}-2)$ - | $6.8380 \mathrm{e}-1(3.73 \mathrm{e}-2)$ - | $1.2134 \mathrm{e}-0$ (1.54e-1) - | $1.5003 \mathrm{e}-0$ (3.95e-1) - | $7.3777 \mathrm{e}-1$ (8.15e-3) - | $1.9083 \mathrm{e}-1$ ( $7.53 \mathrm{e}-3$ ) |
| DTLZ7 | 150 | 5 | 24 | 150000 | $2.9866 \mathrm{e}-1$ (3.51e-2) - | $4.2235 \mathrm{e}-1(1.02 \mathrm{e}-1)$ - | $5.4978 \mathrm{e}-1(1.51 \mathrm{e}-1)=$ | $8.9328 \mathrm{e}-1(1.06 \mathrm{e}-1)$ - | $5.6062 \mathrm{e}-1(1.46 \mathrm{e}-2)$ - | $9.0166 \mathrm{e}-1(1.50 \mathrm{e}-1)$ - | $2.7793 \mathrm{e}-1$ (1.11e-2) |
|  | 200 | 10 | 29 | 200000 | $1.1532 \mathrm{e}-0$ (3.27e-1) - | $3.0629 \mathrm{e}-0$ (3.85e-1) - | $5.5615 \mathrm{e}-0$ ( $5.08 \mathrm{e}-2)$ - | $5.3866 \mathrm{e}-0(5.21 \mathrm{e}-1)$ - | $2.4594 \mathrm{e}-0(2.80 \mathrm{e}-1)$ - | $5.1838 \mathrm{e}-0$ (6.68e-1) - | $9.4318 \mathrm{e}-1(2.15 \mathrm{e}-2)$ |
|  | 250 | 15 | 34 | 250000 | $3.0003 \mathrm{e}-0$ ( $9.14 \mathrm{e}-1)$ - | $1.0083 \mathrm{e}-1(5.16 \mathrm{e}-1)$ - | $1.1001 \mathrm{e}-1(2.25 \mathrm{e}-1)$ - | $9.7764 \mathrm{e}-0$ (6.90e-1) - | $3.7625 \mathrm{e}-0$ (7.28e-1) - | $1.0680 \mathrm{e}-1(4.26 \mathrm{e}-1)$ - | $2.2694 \mathrm{e}+0$ (1.97e-1) |
|  | 300 | 20 | 39 | 300000 | $5.6229 \mathrm{e}-0$ (1.68e-0) - | $1.4931 \mathrm{e}-1(3.62 \mathrm{e}-1)$ - | $1.4870 \mathrm{e}-1(1.11 \mathrm{e}-1)$ - | $1.5629 \mathrm{e}-1$ (1.80e-1) - | $3.3721 \mathrm{e}-0$ (4.32e-1) - | $1.4900 \mathrm{e}-1(5.73 \mathrm{e}-1)$ - | $3.2242 \mathrm{e}+0$ (6.21e-1) |
| DTLZ8 | 150 | 5 | 50 | 150000 |  |  |  |  | $2.6793 \mathrm{e}-1(4.14 \mathrm{e}-2)=$ |  | $2.4844 \mathrm{e}-1$ (6.19e-2) |
|  | 200 | 10 | 100 | 200000 |  |  |  |  | $8.9074 \mathrm{e}-1(3.94 \mathrm{e}-2)$ - |  | $5.4102 \mathrm{e}-1(3.99 \mathrm{e}-2)$ |
|  | 250 | 15 | 150 | 250000 |  |  |  |  | $1.1777 \mathrm{e}-0$ (4.46e-2) - |  | $6.2401 \mathrm{e}-1$ (3.33e-2) |
|  | 300 | 20 | 200 | 300000 |  |  |  |  | $1.4650 \mathrm{e}-0$ (4.84e-2) - |  | $7.0367 \mathrm{e}-1(2.95 \mathrm{e}-2)$ |
| DTLZ9 | 150 | 5 | 50 | 150000 |  |  | $6.2969 \mathrm{e}-1(5.01 \mathrm{e}-3)=$ |  | $1.1823 \mathrm{e}-0(5.21 \mathrm{e}-1)$ - | $1.4127 \mathrm{e}-0(2.83 \mathrm{e}-1)=$ | $4.9685 \mathrm{e}-1$ (9.33e-2) |
|  | 200 | 10 | 100 | 200000 |  | $3.5635 \mathrm{e}-0$ (2.10e-0) - | $1.9979 \mathrm{e}-0$ (1.02e-1) - |  | $9.3883 \mathrm{e}-0(5.16 \mathrm{e}-1)$ - |  | $1.6384 \mathrm{e}+0(1.82 \mathrm{e}-1)$ |
|  | 250 | 15 | 150 | 250000 | $2.2339 \mathrm{e}-0$ (1.33e-1) - | $8.9054 \mathrm{e}-0$ (1.12e-0) - | $2.3968 \mathrm{e}-0$ (1.94e-1) - |  | $1.5394 \mathrm{e}-1$ (4.53e-1) - | $6.0172 \mathrm{e}-0$ (1.05e-0) - | $2.0085 \mathrm{e}+0(2.96 \mathrm{e}-1)$ |
|  | 300 | 20 | 200 | 300000 | $5.0188 \mathrm{e}-0$ (1.05e-0) - | $1.4692 \mathrm{e}-1(1.26 \mathrm{e}-0)$ - | $4.2373 \mathrm{e}-0$ (8.05e-1) - |  | $1.9929 \mathrm{e}-1(4.77 \mathrm{e}-1)$ - | $5.2702 \mathrm{e}-0$ (8.41e-1) - | $2.1655 \mathrm{e}+0$ (2.38e-1) |
| $+/-/=$ |  |  |  |  | 0/27/3 | 0/31/0 | 0/29/3 | 0/27/1 | 0/32/4 | 0/26/5 | 31/5/0 |

## 7 Convergence Graphs

Again for datasets, converging graph have been created to show quickly quickly the fitness value converges with the number of iterations. For all datasets, 100000 iterations have been run. These graphics demonstrate the effectiveness of our suggested approach in obtaining the optimal value as quickly as possible. For this finding, the MOEA/D, MOPSO, NSGAII, and SPEA2 algorithms were analyzed. All five procedures have just had 100000 iterations of Hyper Volume (HV) and (IGD) run on them, and their divergence graphs have already been displayed.


Figure 1 Number of functions VS Fitness Value Graph for IGD


Figure 2 Number of functions VS Fitness Value Graph for HV

## 8 Conclusion and Future Work

Many objective bat algorithms based on deterioration subsystem (MaBAT/IGD) are proposed in this paper, in which MOPs is deteriorate into several scalar improvement sub-issues, and each sub-issue is enhanced by just using information from it's own few nearby sub-issues in a single run. It is clear from both performance metrics (IGD and HV) that MaBAT/R2 is quite serious and even outflanks the chosen MOBATs. In comparison to the chosen MOBATs, the numbers of Pareto battlefields suggest that MaBAT/ R2 can offer quite well Pareto lines.
Additional tests and examinations of the recommended are performed on a case-by-case basis. Later in the project, we will focus on parametric examinations for a broader range of test concerns, including discrete and blended aim of boosting. We aim to examine the various variations of the Pareto frontline it can generate in order to distinguish the methods for improving this computation to meet a range of difficulties. There are a few effective approaches for creating various Pareto fronts, and combining these procedures with others could considerably improve MaBAT/IGD. Further research can also show that this estimate has a strong association with some other well-known strategies for multi-target improvement. Furthermore, combining different estimates in a hybrid model could be beneficial.

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