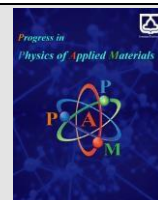




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# Effect of Barrier's Geometry on the Transport Properties of Gaussian Wave-Packet in the Presence of Rashba and Dresselhaus Spin-Orbit Interactions: Comparison of High-Energy and Low-Energy Wave-Packets

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## ABSTRACT

A Gaussian wave-packet quantum tunneling across a one-dimensional double-barrier structure has been explored in order to obtain the spin-based transport coefficients. We have used a split-step finite difference method to solve the resulting nonlinear coupled Schrodinger equations. The related behavior of scattering properties of the system as a function of the geometry of the barriers in the presence of Rashba and Dresselhaus spin-orbit interactions for High-energy and low-energy wave-packets have been compared. Evidence showed that the presence of Rashba or Dresselhaus SOIs leads to considerable spin polarization in the wave-packet components. Based on the results, it is found that the wave-packet velocity plays a significant role in the tunneling process of the Gaussian wave-packet through quantum barriers. In addition, by tuning the Rashba and the Dresselhaus coupling strengths, the energy of the wave-packet, and the characteristics of the system, one can control the spin polarization of the wave-packet and its propagation coefficients.

## 1. Introduction

Potential barriers can be the fundamental building blocks of advanced nanoscale nano-electronic and optoelectronic devices. Nowadays, with the advent of modern semiconductor growth technologies such as molecular beam epitaxy, it has become possible to fabricate high-accuracy semiconducting heterostructures based on barriers and wells of arbitrary shapes [1]. Therefore, a thorough theoretical investigation of a wave-packet propagating through such semiconductor nanostructures may facilitate the manufacture of nanoscale devices. Solving the time-dependent nonlinear Schrodinger equation (TDNSE) for an incident wave-packet on a quantum barrier can be a powerful tool to study the time evolution of the wave-packet in low dimensional quantum systems. Since analytical solutions of the TDNSE for an arbitrary-shaped potential profile are not available, one has to use approximate methods to evaluate its numerical solutions.

For this purpose, various numerical approaches, including the split-step finite difference method [2-4], split-step Fourier method [5], quadratic b-spline finite element method [6], finite element method [7-8], discontinuous Galerkin method [9], 4th order Runge-Kutta method [10], etc. have been developed.

Various studies were conducted in this research area in order to investigate the propagation properties of wave packets traveling through semiconducting heterostructures. The first attempt to study wave-packet tunneling through potential barriers was MacColl's work, in which the first exact solution of the time-dependent Schrodinger equation for a wave-packet impinging on a square potential barrier has been presented [11]. A few decades later, Jauho and Nieto [12] studied the time-dependent tunneling of wave-packets through heterostructures and analyzed the effect of the initial wave-packet form on the scattering properties of the system. Afterward, the influence of a transverse magnetic field on the tunneling properties of a spin-less

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Gaussian wave-packet through a square barrier has been investigated [13]. The results indicated that the transmission and reflection coefficients of the potential barrier are field-dependent. Later, Luis et al. [14,15] numerically solved the effective-mass nonlinear Schrodinger equation for an electron wave packet in a coupled quantum-well system. The results have shown that the wave packet has dynamically been trapped in both wells. They have also studied how electron-electron and exchange interactions can affect tunneling processes. So far, the problems of wave-packet tunneling through rectangular and trapezoidal-shaped potential profiles [16], through the time-modulated barrier, [17-19] through a triple-barrier system, [20] and a reflection-less potential barrier [21] have also been examined in detail. However, the time-dependent tunneling of wave-packets through asymmetric heterostructures has attracted considerably less attention, and hence more work is needed in this field of research.

Since in the nonmagnetic multilayered heterostructures, large spin polarization can induce due to the Spin-Orbit interaction (SOI) [22], the effect of SOI has to be taken into account in the calculation of scattering properties. Two different mechanisms in the mesoscopic semiconductors can lead to SOI in the absence of external magnetic fields. The structural inversion asymmetry (SIA) of the system causes the Rashba SOI [23], and the bulk inversion asymmetry (BIA) of the system causes the Dresselhaus SOI [24]. Cruz and Luis [25] have studied the time-dependent evolution of an electron wave-packet in a triple quantum well system. The Rashba SOI, as well as the electron-electron interaction in their calculations, is considered. Their investigation demonstrated that after tunneling through a spin-dependent potential barrier, an initial un-polarized wave packet splits into two spin-up and spin-down wave packets. Dakhlaoui et al. [26] have investigated spin-dependent transmission and spin polarization of heterostructures based on accelerating triple barriers using the transfer matrix method while considering the Rashba SOI. Their findings showed that the wave-packet energy and the system size have an essential effect on the transmission coefficient and the spin polarization of the heterostructure. They have also studied the impact of applied magnetic field on spin polarization and showed that it could polarize either spin-up or down electrons [27]. Solaimani and Izadifard have studied the spin polarization efficiency and spin-dependent transmission of electrons tunneling through a semiconductor built of multiple quantum wells using the quantum transmitting boundary method while taking into account the Dresselhaus SOI [28]. They have found that the number of wells in the structure plays the role of a tuning tool for spin filtering. However, the effect of the Dresselhaus SOI on the scattering properties of asymmetric heterostructures received little attention.

In the present work, we have compared the scattering properties of high-speed and low-speed spin-dependent wave packets tunneling through one-dimensional double rectangular potential barriers in the presence of the Rashba and the Dresselhaus SOIs, as well as the electron-electron interactions. The results we gained in our previous work have indicated that low-energy wave packets cannot transmit through a double-barrier. By contrast, high-energy wave packets are almost entirely transmitted. As a result,

comparing the behavior of transport properties of high-speed and low-speed wave packets can help better understand the tunneling phenomenon in semiconducting materials. We have solved the resulting coupled nonlinear Schrodinger equations using a split-step finite difference method. This work aims to compare how the magnitude of the Rashba coefficient, the magnitude of the Dresselhaus coefficient, and the height and the width of the potential barriers, can affect the scattering properties of high-energy and low-energy wave packets in a potential double-barrier. The effect of Dresselhaus and Rashba SOIs in the asymmetric heterostructures can be favorably engineered for fabricating spintronic devices based on double-barrier non-magnetic semiconductors [29]. Therefore, this study may be helpful in the design of new devices in the future spintronic industry.

## 2. Formalism

We start with the following three-dimensional time-dependent Schrodinger equation:

$$i\hbar \frac{\partial}{\partial t} \phi(x, y, z, t) = -\frac{\hbar^2}{2m} \nabla^2 \phi(x, y, z, t) + V(x)\phi(x, y, z, t) + Q|\phi|^2\phi(x, y, z, t) + H_{R/D}\phi(x, y, z, t) \quad (1)$$

where  $\hbar$  is the Planck constant divided by  $2\pi$ ,  $V(x)$  is the potential profile,  $Q|\phi|^2$  represents the potential given by the electron-electron interaction in the heterostructure region (a nonlinear interaction) [30-32], and  $H_{R/D}$  indicates the Rashba or the Dresselhaus Spin-Orbit interaction. The effect of each Spin-Orbit interaction on the behavior of the high-speed and low-speed wave-packets is studied independently.

### 2.1 Rashba Spin-Orbit interaction

In the presence of the Rashba Spin-Orbit interaction the Schrodinger equation is rewritten as

$$i\hbar \frac{\partial}{\partial t} \phi(x, y, z, t) = -\frac{\hbar^2}{2m} \nabla^2 \phi(x, y, z, t) + V(x)\phi(x, y, z, t) + Q|\phi|^2\phi(x, y, z, t) + \frac{\alpha}{\hbar} (\sigma_x P_y - \sigma_y P_x) \phi(x, y, z, t) \quad (2)$$

where  $\frac{\alpha}{\hbar} (\sigma_x P_y - \sigma_y P_x)$  indicates the Rashba Spin-Orbit interaction term with strength  $\alpha$  [33,34], and  $\sigma_x$  and  $\sigma_y$  are the Pauli matrices along  $x$  and  $y$  directions, respectively. Also  $P_x$  and  $P_y$  are the momentum operator components along  $x$  and  $y$  directions. Eq. (2) is a three-dimensional partial differential equation, the numerical solution of which is quite demanding indeed. We assume  $\phi(x, y, z, t)$  as the separable function [35]

$$\phi(x, y, z, t) = \psi(x, t)e^{-ik_y y}e^{-ik_z z} \quad (3)$$

Where  $k_y$  and  $k_z$  are the  $y$  and  $z$  direction wave-vector components. Now, we can reduce equation (2) to a one-

dimensional ordinary differential equation by substituting (3) into (2).

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + \frac{\hbar^2 (k_x^2 + k_y^2)}{2m} \psi(x, t) + V(x)\psi(x, t) + Q|\psi(x, t)|^2\psi(x, t) + \alpha\sigma_x k_y \psi(x, t) - i\alpha\sigma_y \frac{\partial \psi(x, t)}{\partial x} \quad (4)$$

Here we have defined the in-plane wave-vector  $\vec{k}_{\parallel} \equiv k_y \hat{y} + k_z \hat{z}$ , which represents the component of the wavevector in the plane of the layers. In the limit of  $\vec{k}_{\parallel} = 0$  we can rewrite the Eq. (4) as,

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x)\psi(x, t) + Q|\psi(x, t)|^2\psi(x, t) - i\alpha\sigma_y \frac{\partial \psi(x, t)}{\partial x} \quad (5)$$

Then, assuming  $\psi = [\psi_{\uparrow}]$  in which  $\psi_{\uparrow}$  and  $\psi_{\downarrow}$  are the wave-functions for spin-up and spin-down wave-packets, we have,

$$\begin{cases} i\hbar \frac{\partial \psi_{\uparrow}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{\uparrow}}{\partial x^2} + V(x)\psi_{\uparrow} + Q|\psi|^2\psi_{\uparrow} - \alpha \frac{\partial \psi_{\downarrow}}{\partial x} \\ i\hbar \frac{\partial \psi_{\downarrow}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{\downarrow}}{\partial x^2} + V(x)\psi_{\downarrow} + Q|\psi|^2\psi_{\downarrow} + \alpha \frac{\partial \psi_{\uparrow}}{\partial x} \end{cases} \quad (6)$$

### 2.2 Dresselhaus Spin-Orbit Interaction

In the case of Dresselhaus Spin-Orbit interaction, the Schrodinger equation changes as

$$i\hbar \frac{\partial}{\partial t} \Phi(x, y, z, t) = -\frac{\hbar^2}{2m} \nabla^2 \Phi(x, y, z, t) + V(x)\Phi(x, y, z, t) + Q|\Phi|^2\Phi(x, y, z, t) + \frac{\beta}{\hbar} (\sigma_x P_x - \sigma_y P_y) \Phi(x, y, z, t) \quad (7)$$

Where  $\frac{\beta}{\hbar} (\sigma_x P_x - \sigma_y P_y)$  indicates the Dresselhaus Spin-Orbit interaction term with strength  $\beta$  [36]. By substituting (3) into (7), in the limit of  $\vec{k}_{\perp} = 0$ , and by assuming  $\psi = [\psi_{\uparrow}]$  similarly to the Rashba case, this equation reduces to  $\psi_{\downarrow}$

$$\begin{cases} i\hbar \frac{\partial \psi_{\uparrow}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{\uparrow}}{\partial x^2} + V(x)\psi_{\uparrow} + Q|\psi|^2\psi_{\uparrow} - i\beta \frac{\partial \psi_{\downarrow}}{\partial x} \\ i\hbar \frac{\partial \psi_{\downarrow}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{\downarrow}}{\partial x^2} + V(x)\psi_{\downarrow} + Q|\psi|^2\psi_{\downarrow} + i\beta \frac{\partial \psi_{\uparrow}}{\partial x} \end{cases} \quad (8)$$

### 2.3 Numerical Solution

Equations (6) and (8) are two coupled nonlinear Schrodinger equations which we want to solve by using the Split-Step Finite Difference Method with the rigid boundary condition,

$$\begin{aligned} \psi_j(x, 0) &= \psi_j^0(x), & j &= \uparrow, \downarrow \\ \psi_j(\frac{a}{2}, t) &= \psi_j(-\frac{a}{2}, t) = 0, & j &= \uparrow, \downarrow \end{aligned} \quad (9)$$

where  $a$  is the total system size.

We assume the following initial condition to solve the resulting coupled equations.

$$\psi(x, t = 0) = \frac{1}{\sqrt{2\sigma\pi}} \exp[-\frac{(x-x_0)^2}{2\sigma^2} + ik_x x] \begin{bmatrix} c_{\uparrow} \\ c_{\downarrow} \end{bmatrix} \quad (10)$$

This is a traveling Gaussian wave-packet at  $t = 0$ , where,  $x_0$  is the center of the wave-packet,  $\sigma$  determines the width of the wave-packet,  $k_x$  shows the wave-packet wave-vector and we have  $|c_{\uparrow}|^2 + |c_{\downarrow}|^2 = 1$ . Case of  $c_{\uparrow} = c_{\downarrow}$  represents an un-polarized state and case of  $c_{\uparrow} = 1, c_{\downarrow} = 0$  represents a fully polarized state. We aim to evaluate  $\psi(x, t > 0)$  and we have computed it by using the split-step finite difference (SSFD) method.

Now, we use three physical observables, reflection coefficient ( $R$ ), trapping coefficient ( $L$ ), and transmission coefficient ( $T$ ) of the wave-packet in order to investigate the behavior of the assumed system. We have calculated them at a time well after the initial wave-packet incidence on the double-barrier through,

$$R = \int_{-\infty}^{-L_{eff}/2} dx |\psi(x, t)|^2 \quad (11-1)$$

$$L = \int_{-L_{eff}/2}^{L_{eff}/2} dx |\psi(x, t)|^2 \quad (11-2)$$

$$T = \int_{L_{eff}/2}^{\infty} dx |\psi(x, t)|^2 \quad (11-3)$$

Where  $L_{eff}$  is the region size in which the barriers are present and  $R + L + T = 1$ . If we use the spin-up (down) components of the wave-packet in equations (11), we will

find the spin-up (down) reflection, trapping, and transmission coefficients. However, the total transmission coefficient can be obtained by using the term  $|\psi|^2 = |\psi_{\uparrow}|^2 + |\psi_{\downarrow}|^2$  in the equations (11).

## 3. Results and discussion

In this research, we have compared the propagation properties of high-speed and low-speed spin-polarized Gaussian wave-packets through a one-dimensional rectangular quantum double-barrier in the presence of the Rashba SOI, the Dresselhaus SOI as well as the electron-electron interaction. For this purpose, we have calculated the spin-dependent tunneling properties of the system using Eqs. (11). We have used a numerical split-step finite difference method to perform the required calculations. Then we studied the effect of various parameters, including the Rashba coupling strength, the Dresselhaus coupling strength, the barrier height, the barrier width, and the incident wave-packet velocity on the scattering properties of the system. For the sake of simplicity, we have used the

system of units in which Planck's constant and the particle's mass have been chosen to equal one. Therefore, the effective parameters have been applied in the calculations in their dimensionless form.

At first, we conducted some simulations in which the wave-packet's velocity was set differently. Their results which we gained in our previous work, have indicated that wave-packets with a velocity up to  $v = 0.6$  cannot transmit through a double-barrier. By contrast, wave-packets with a velocity higher than  $v = 1.2$  are almost entirely transmitted. As a result, we chose a low-energy wave-packet with a velocity of  $v = 0.6$  and a high-energy wave-packet with a velocity of  $v = 1.2$  and compared their behavior. Figures 1 to 4 show the variation of the transmission, reflection, and trapping coefficients of the aforementioned Gaussian wave-packets traveling through the double-barrier as a function of the barrier height and width. In these figures, panels A to C are associated with the case in which  $\alpha = \beta = 0$ , panels D to F are related to the case of heterostructure with the dimensionless Rashba SOI ( $\alpha = 0.009$ ), and panels E to G are related to the case in which the dimensionless Dresselhaus SOI ( $\beta = 0.009$ ) affects the tunneling process.

To study the effect of the barrier height on the tunneling process, we have assumed a fully spin-up polarized Gaussian wave-packet impinging rectangular double-barrier in which the width of each barrier is 6, and their height varies from 0.3 to 0.8. The variation of scattering properties for two different assumed wave-packets versus the barrier height has been shown in Figures 1 and 2.

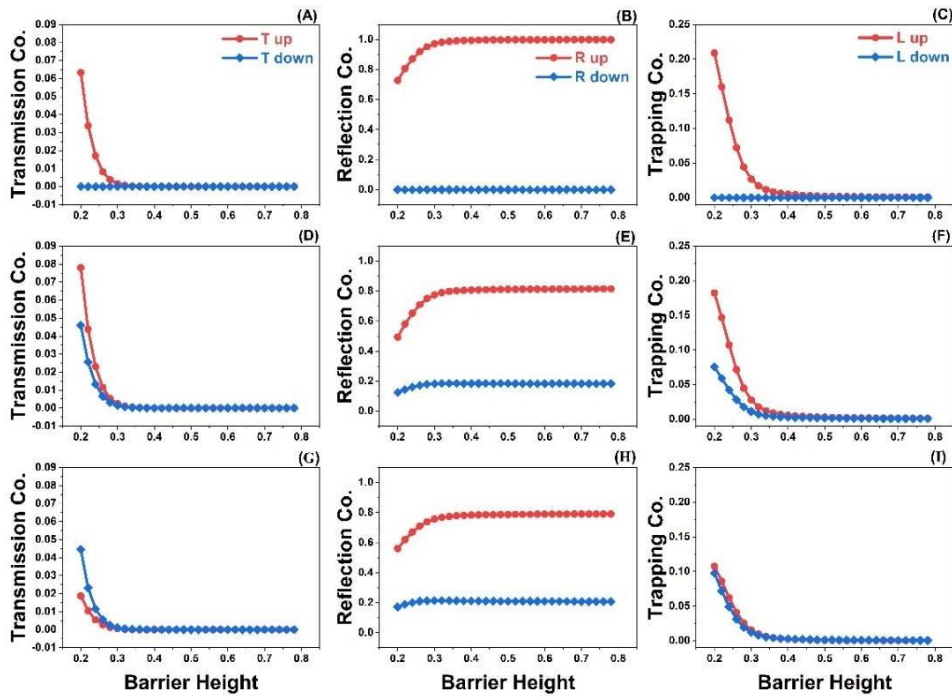
As we expected, panels A to C in Figures 1 and 2 show that in the case of  $\alpha = \beta = 0$ , the scattering properties for the spin-down wave-packet component are zero; which means in the absence of Rashba or Dresselhaus SOI, spin-flip does not occur. Furthermore, panels A, D, and G in Figure 1 show that when the barrier height increases, the transmission coefficient of a low-energy wave-packet sharply reduces, and for barriers higher than 0.3 vanishes. Moreover, panels A, D, and G in Figure 2 indicate that increasing the barrier height makes a smooth reduction in the transmission coefficient of a high-energy wave-packet. However, it should be noted that the maximum value of the transmission coefficient for low-energy wave-packets is only about 0.08 while, for high-energy wave-packets in the absence of SO interaction, and in the case of Dresselhaus SOI is approximately 0.9 and in the case of Rashba SOI is about 0.5. This means that, in general, low-energy wave-packets do not transmit through double-barriers. In addition, the high-speed wave-packet behavior in the

presence of Dresselhaus SOI is similar to the case in which there is no SO interaction in the system. The only difference is that in the case of Dresselhaus SOI, the spin-polarization of the wave-packet completely changes. Whereas, under the influence of Rashba SOI, the contribution of spin-up and spin-down components of the transmission coefficient are almost equal. Hence, it can be said that by tunneling through a heterostructure by structural inversion asymmetry, an initially spin-up polarized wave-packet converts to a fully spin un-polarized wave-packet [22,25].

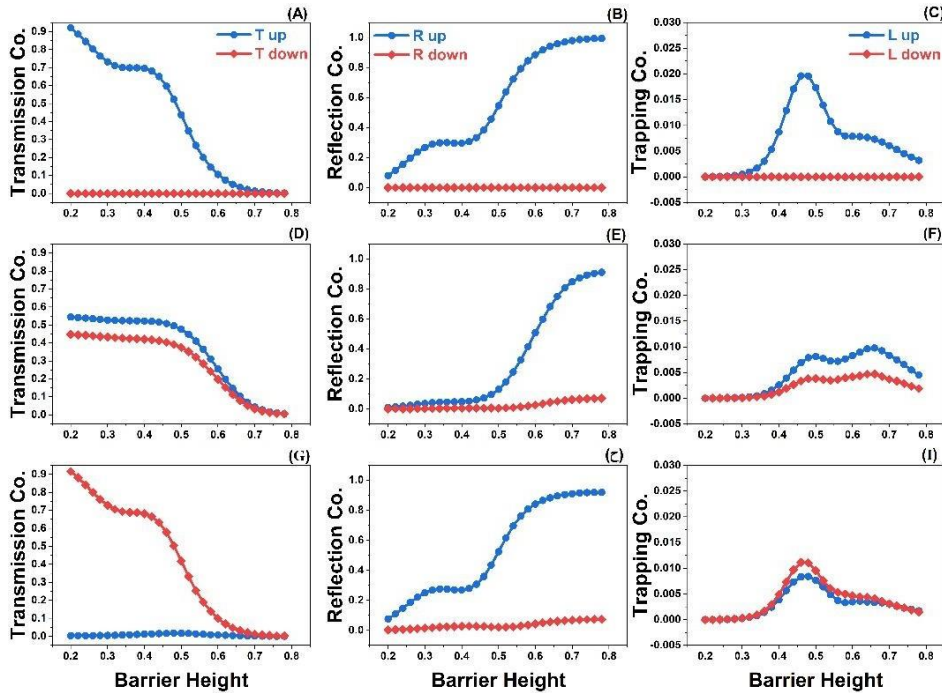
According to panels B, E, and H of Figure 1 for low-energy wave-packets, the reflection coefficient grows as the barrier height increases to 0.3 and then remains constant. Furthermore, panels B, E, and H of Figure 2 demonstrate that the reflection coefficient increases by increasing the barrier height for high-energy wave-packets. This behavior can easily be explained by the fact that the potential energy of the potential barrier increases by increasing the barrier height and can dominate the kinetic energy of the wave-packet. Consequently, the wave-packet does not transmit through high potential barriers and experiences almost complete reflection.

After that we studied the effect of the barrier width on the propagation coefficients of the system. In this case, we assumed barrier height  $V_0 = 0.3$ , and barrier width increased from 6 to 24. Then in Figures 3 and 4, we plotted the variation of propagation coefficients versus the barrier width for the two assumed wave-packets.

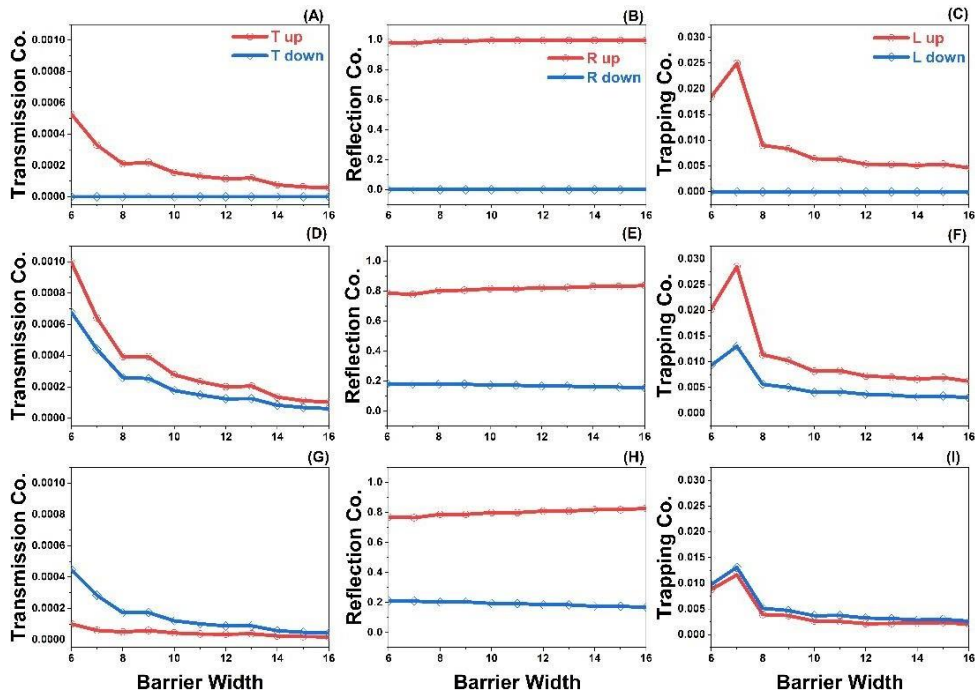
According to panels A, D, and G of Figure 3, when the barrier width grows to 10, the wave-packet's transmission coefficient experiences a rapid reduction and then vanishes. The energy dependence of the transmission coefficient can also explain this behavior. By increasing the barrier width, the double barrier becomes broader and thus leads to reducing the confinement effects. Therefore, the energy eigenvalues of the system, and as a result, the transmission coefficient of the system decreases. Besides, as can be seen from Figure 3 (panels E and H), the reflection coefficient shows the same variation trend in the presence of both Rashba and Dresselhaus SOIs. In both cases, when the barrier width increases, the spin-up reflection coefficient shows a linear rise, and the spin-down reflection coefficient shows a linear fall. Panels (3-F) and (3-I) demonstrate that both the spin-up and the spin-down trapping coefficients increase and then reduce smoothly by growing the barrier width. The results indicate that the spin-up trapping coefficient in the systems lacking structural inversion symmetry has more value than in the systems with bulk inversion asymmetry.



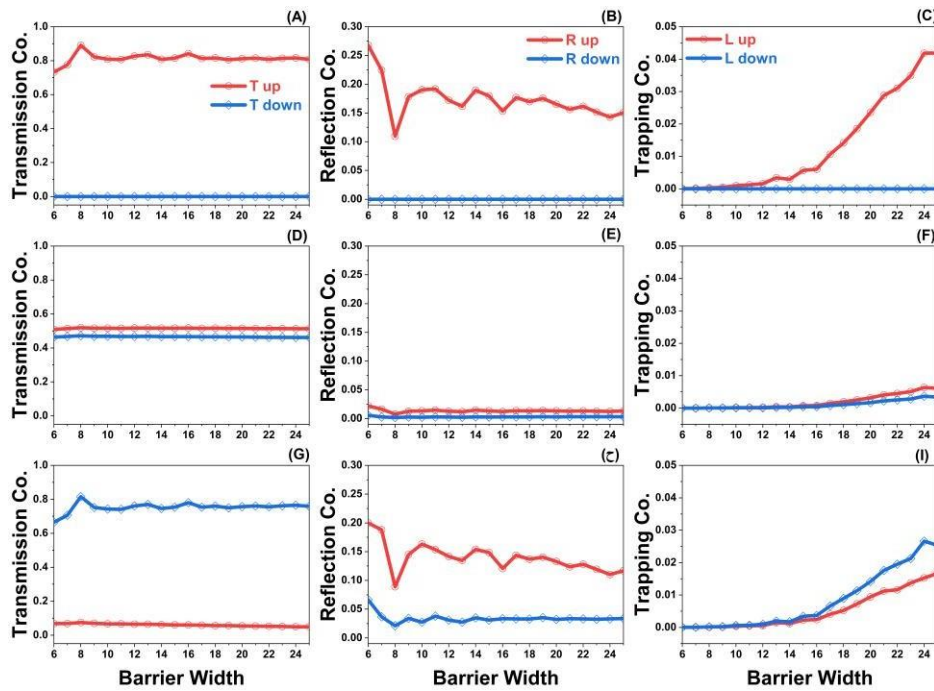
**Fig.1.** Transmission coefficient (Panel A), reflection coefficient (Panel B), and trapping coefficient (Panel C) as a function of the barrier height for an initially spin-up polarized low-energy Gaussian wave-packet with electron-electron interaction strength  $Q = 0.5$  and in the absence of the Rashba and the Dresselhaus SOCs. Panels (D), (E), and (F) are the same as panels (A), (B), and (C), respectively, but in the case of the dimensionless Rashba coefficient  $\alpha = 0.009$ . Panels (G), (H), and (I) are the same as panels (A), (B), and (C), respectively, but in the case of the dimensionless Dresselhaus coefficient  $\beta = 0.009$ . We also assumed, barrier width 6, and wave-packet velocity 0.6.



**Fig.2.** Same as Fig.1 but for an initially spin-up polarized high-energy Gaussian wave-packet with velocity 1.2.



**Fig.3.** Transmission coefficient (Panel A), reflection coefficient (Panel B), and trapping coefficient (Panel C) as a function of the barrier width for an initially spin-up polarized low-energy Gaussian wave-packet with electron-electron interaction strength  $Q = 0.5$  and in the absence of the Rashba and the Dresselhaus SOCs. Panels (D), (E), and (F) are the same as panels (A), (B), and (C), respectively, but in the case of the dimensionless Rashba coefficient  $\alpha = 0.009$ . Panels (G), (H), and (I) are the same as panels (A), (B), and (C), respectively, but in the case of the dimensionless Dresselhaus coefficient  $\beta = 0.009$ . We also assumed, barrier height 0.3, and wave-packet velocity 0.6.

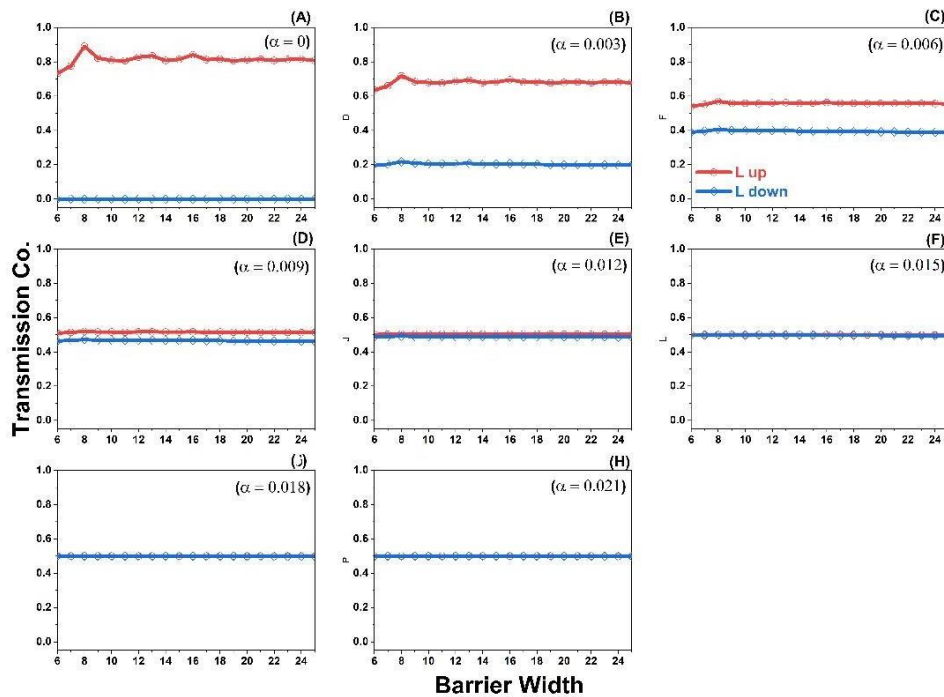


**Fig.4.** Same as Fig.3 but for an initially spin-up polarized high-energy Gaussian wave-packet with velocity 1.2.

According to Figure 4, in comparison to low-energy wave-packets (Fig.3), by increasing the width of the barrier, the transmission coefficient of the high-energy wave-packet increases, and its reflection coefficient decreases. However, the trapping coefficient value of the high-energy wave-packet does not change significantly. It should be mentioned that when an initially spin-up polarized wave-packet tunnels through a system in the presence of Rashba SOI, the wave-packet becomes completely spin-unpolarized. Furthermore, from panels B, E, and H of Figure 4, we can see that in the presence of the Rashba SOI, the reflection probability of the wave-packet is small. In contrast, in the case of Dresselhaus SOI and in the absence of any SO interactions, the reflection probability is approximately 20% and 26%, respectively. Moreover, by increasing the barrier width, the reflection probability gradually drops and becomes 6% and 10%, respectively. Panels C, E, and I of Figure 4 demonstrate the behavior of the trapping coefficient versus the barrier's width. The trapping coefficient is about zero for barriers with a width up to 12. However, as expected, the more the barrier width grows, the more the trapping coefficient increases. It is worth noting that in the presence of Rashba SOI, there is only a slight possibility that the wave-packet traps into a double-barrier. By contrast, in the case of Dresselhaus SOI,

and in the absence of SO interactions, the trapping probability increases up to 4%, which is a remarkable value. For investigating the effect of Rashba and Dresselhaus SOIs on the transmission of wave-packets, we have plotted the variation of transmission coefficient versus the barrier width for some different Rashba and Dresselhaus coupling strengths.

Panels (5-A) to (5-C) indicate that in the absence of Rashba SOI and in the case in which the Rashba SOI is weak, the transmission coefficient of the wave-packet oscillates as a function of the barrier width. In quantum mechanics, particles can be defined using plane waves. In this case, the Schrödinger equation in the absence of SOIs is exactly solvable [37]. Gaussian wave-packets are always built from a linear combination of plane waves. Therefore, the behavior of Gaussian wave-packets tunneling through the potential barriers is expected to be similar to the plane waves, which can be seen in the obtained results. However, when the Rashba coupling strength increases, the dependence of the transmission coefficient on the barrier width disappears, and the transmitted wave-packet becomes completely spin-unpolarized (panels (5-D) to (5-G)).



**Fig.5.** Transmission coefficient as a function of the barrier width for an initially spin-up polarized high-energy Gaussian wave-packet with electron-electron interaction strength  $Q = 0.5$  and in the presence of the Rashba SOC with different strengths. We also assumed, barrier height 0.3, and wave-packet velocity 1.2.

According to Figure 6, we can see that when the Dresselhaus coefficient increases, the spin polarization of the transmitted wave-packet changes. Therefore, there are cases with specific Dresselhaus coupling strengths that lead to fully un-polarized transmitted wave-packets. By contrast, when the Rashba coupling strength is more than

$\alpha = 0.012$  (panel (5-E)), the transmitted wave-packet becomes almost entirely un-polarized. Another noteworthy point is that by tuning the Dresselhaus coefficient and the barrier width, one may design a system in which the high-energy wave packet's polarization completely changes when it tunnels through.

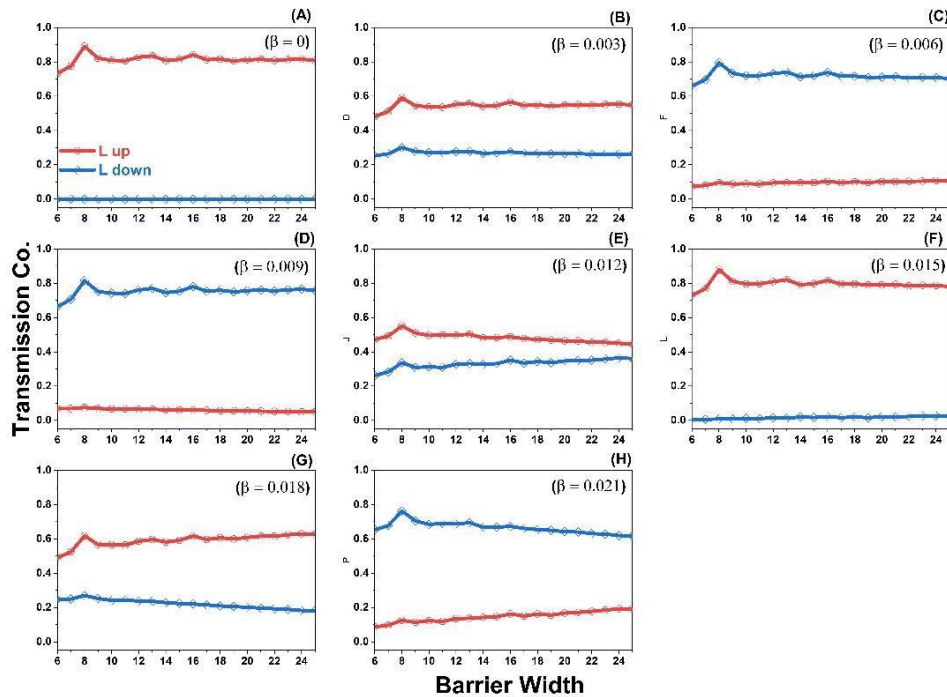


Fig.6. Same as Fig. 5 but in the presence of Dresselhaus SOI with different strengths.

#### 4. Conclusion

In the present investigation, we have compared the scattering properties of high-energy and low-energy spin-polarized Gaussian wave-packets impinging a one-dimensional rectangular quantum double-barrier. The effect of Rashba and Dresselhaus SOIs and the geometrical parameters of the double-barrier on the tunneling properties of the wave-packets have been studied. Evidence showed that the presence of Rashba or Dresselhaus SOIs leads to considerable spin polarization in the wave-packet components. Furthermore, when the double-barrier height increased, the transmission coefficient of the low-energy wave-packet sharply reduced, and for double-barriers higher than 0.3 vanished, whereas for high-energy wave-packets increasing the barrier height made a smooth reduction in the transmission coefficient. However, by contrast to the high-energy wave-packets, the maximum value of the transmission coefficient for low-energy wave-packets was pretty small. Besides, for low-energy wave-packets, the reflection coefficient grew as the barrier height increased to 0.3 and then remained constant. By contrast, for high-energy wave-packets, the reflection coefficient increased by increasing the barrier height. In addition, for low-energy wave-packets, the transmission coefficient experienced a rapid decrease when the barrier width in the double-barrier grew to 10 and then vanished. Besides, the spin-up reflection coefficient increased linearly, and the spin-down reflection coefficient decreased linearly when the barrier width increased. Furthermore,

both components of the trapping coefficient were reduced smoothly by increasing the barrier width. However, for high-energy wave-packets, by increasing the width of the barrier, the transmission coefficient and the reflection coefficient decreased. It should be pointed out that, in the presence of the Rashba SOI, the high-energy wave-packet did not reflect. By contrast, in attending Dresselhaus SOI, as well as in the absence of any SO interactions, the reflection probability was approximately 20% and 26%, respectively. Moreover, by increasing the barrier width, the reflection probability gradually dropped and became 6% and 10%, respectively.

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