Int. J. Nonlinear Anal. Appl. 14 (2023) 11, 271–286
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.26925.3452

Presenting a multi-objective model to develop depot location through particle swarm optimization algorithm in Artawheel Tire Company

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(Communicated by Mohammad Bagher Ghaemi)

Abstract

This study aimed to present a multi-objective model for the location of depots through a particle swarm optimization algorithm in Artawheel Tire Company. It is applied in terms of purpose and survey and descriptive in terms of the nature of research and data collection. Data collection tools are documents and interviews with experts. Also, the research is of the predictive type in proportion to the fact that the research seeks to locate the depot using the particle swarm optimization algorithm. Given that this problem falls into the category of Hard-PN problems, a meta-heuristic method based on the particle swarm optimization algorithm is used to solve it. Two particle group optimization algorithms and genetics have been used as benchmark algorithms to evaluate the performance of the proposed algorithm. The proposed particle cluster optimization algorithms are implemented in the Matlab 7.5 programming software and the genetic algorithm is implemented using the Matlab 7.5 software toolbox. According to the results of this study, it was found that the use of a particle swarm algorithm to solve the problem of vehicle routing can improve the objective function as well as the total number of routes travelled by vehicles.

Keywords: Optimization, Particle Swarm Algorithm, Location, Artawheel
2020 MSC: 68V30, 90B50

1 Introduction

The location of distribution systems is one of the most important and common issues in logistics management and supply chain as a result of integrating location and transportation decisions in the distribution system of a supply chain. Location is generally an NP-Hard (NP-Hard Non- deterministic polynomial- time hard) problem. The problem of the location of distribution systems is a part of the NP-Hard problem, which requires accurate, heuristic or meta-heuristic algorithms to solve it [18]. The problem of location routing is a research field of positioning studies that have significant features. These features pay special attention to the underlying issues related to vehicle routing. Many studies have been conducted on various aspects of location theory, but the issue of location routing has not been given the attention it deserves. According to Bruns' definition, we consider location-routing by taking into account aspects of tour planning, consistent with Balakrishnan's view considers location-routing issues in terms of fundamentally

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Received: May 2022 *Accepted:* July 2022

strategic location decisions [1]. In this research, the ideal planning model is used in modeling and optimizing the location of depots of Artawheel Tire Company in conditions of discrete uncertainty.

When the location-routing problem is modeled from a mathematical view as a hybrid optimization problem, it forms a part of distribution management from a practical view. The facility location and the routing of vehicles are interrelated. Maran Zana has indicated that factory location, depots, and product offerings are often affected by transportation costs. When location problems do not have the features of routing, many practical situations arise in which the location-routing process is clearly not a good one to solve. Conceptually, the location-routing problem is much more difficult than the problem of classic positioning. Berman showed that in the location-routing problem, the centrality of communication between executive groups and demand points are facility, thus the movement between all of them is still unknown. In classic location problems, they must be located at the desired distances from the unique demand points, which makes the problem more controllable and helps to advance the core of the location-routing problem. When there are a number of different types of problems, we cannot recreate the entire formulation. The following table summarizes the variety of types of location-routing problems [18].

Table 1: Linear programming formulation or modeling for different types of location-routing problems [18]

dynamic location-routing	[14]
Middle P Hamilton	[10]
Rail routing	[23]
Vehicle routing-allocation problem (VRAP)	[7]
Many-to-many location-routing problem	[32]
Euclidean location	[21]
Combined-step location-routing problem	[39]
investment location-routing problem	[25]
Rotational location of vehicle	[21]
Many-to-many location-routing problem	[37]
Multi location-routing investment	[3]
Definite location-routing problem	[2]
Vehicle routing-allocation problem (VRAP)	[22]
non-linear cost location-routing problem	[28]
single-depot location-routing problems	[36]
VRAP Bound	[16]
multi-depot location-routing problem	[32]

Location-Routing Applications

Practical and applied researches prefer to reach the software structure; It is very important to determine the practical applications of location routing. The following table summarizes the main features of the practical applications of this research, which is a reference for the sections discussed in detail, as well as the size of the largest solution and indicates the number of potential facilities and the number of customers.

Table 2: Summary of applications of location-routing problem

papers	**application**	**Country/region**	**facility**	**customers**
[38]	Distribution of soda and food	United Kingdom	40	300
	Distribution of customers' goods	Australia	3	50
	blood bank location	the US	3	117
[19]	Distribution of newspaper	Denmark	42	4510
[31]	rubber machine location	Malaysia	15	300
[34]	Distribution of goods	the US	4	318
[20]	mailbox location	Belgium	Not available	Not available
[31]	rubber machine location	Malaysia	10	47
	Distribution of beans	Switzerland	9	90

	Waste selection	Belgium	13	260
	military equipment location	the US	29	331
[12]	parcel delivery	Switzerland	200	3200
[24]	Design of optical networks	Korea	50	50
[37]	parcel delivery	Australia	10	2042
[9]	Design of communication networks	France	6	70
	Transportation Industry	Europe	24	300

2 Literature Review

The first precise algorithm for the general location-routing problem was proposed by Laporte and Nobert. In this paper, a single depot is selected and a constant number of vehicles are used. The branch and cut algorithms are also used. The authors note that the optimal location of the depot corresponds to the nodes close to the center of importance and attention [22].

Laporte identified the location of several depots with or without fixed parts of the depot with a high limit or without a high limit on the number of depots. He found that for a particular case, such as a vehicle, it would be much more efficient at first to introduce the diminutive constraint of the sub-trip (There may be no exceptional chains). Then the Gomory cut method is used to do the job properly. The authors first suggest the use of Gomory cut and then the constraint of the sub-trip and exception chains. In other words, Laporte's method proposes the application of a branching method where the elimination of sub-trip and exception chain constraints are reintroduced (A similar process is expressed by Laporte for the stochastic location-routing problem). In a study, it was used the graph conversion into a travelling salesman problem to reformulate the location-routing problem. They used a branch and boundary algorithm to search for a tree. Each subproblem is a constrained assignment problem that can be solved efficiently. This process has been generalized to the dynamic location-routing problem by Laporte and Dejax [5].

Averbakh and Berman introduced the minimization/maximization problem, and the location of the P-traveling salesman. The purpose of the problem was to minimize the length of the most distant vehicle. In the problem, the customers are the vertices of the tree. A single depot must be located on a vertex or an edge of the tree, and also the number of vehicles is a predetermined set. The optimal solution to the problem is found by simplifying the problem to the set of minimization Y problems. The equipment location problem was raised by [9]. He formulated the synchronization of the radio communication station location problem with a field planning that connects a radio antenna to such a station. The only difference between this and the location-routing problem is that instead of vehicle routing in the location-routing problem, communication fields are created and there is a number of antennas corresponding to the capacity of the vehicles in each upper boundary field. Finally, the capacity of stations can be found whose value depends on their size [18].

The above problem is also mentioned by Labbe. In which relational (inverse) and correct constraints are reduced but some allowable inequalities are added. In this problem, an initial solution is considered stochastic and then a branch and cut method are used and the distance can be improved by adding the inequalities obtained from the previous phases. When there are no vehicles, then it is a combination, of network planning and positioning problem. In short, in the face of complex location-routing, precise turns can only be effective at short distances. A general location-routing example with location potential and more than 40 depots or 80 customers has been optimally solved. However, precise methods for solving special cases of location-routing problems can be effective, especially the round-trip location problem.

Table 3: The previous table summarizes the types of location problems, their solution method and the biggest problem solved

Type of problem	Solving method	papers	vehicles	customers
definite-general location-routing problem	Plate cut	Laporte et al.	40	40
	Branch and bound	Laporte et al.	3	80
Travel location	Numerical optimization	Dıaz	1	10.000
Euclidean location	Cut and bound	Laporte and Rodrıguez-Martín	50	200
Minimax Taboo Search Location	Graph theory	Averbakh and Berman	1	Not available
Vehicle cycle location	Cut and bound	Labbe et al.	30	120

2.1 Dynamic and Stochastic Problem

The only thing that is definite in life is that nothing is definite. It is unreasonable to assume that the parameters of the location-routing problem are unchanged and perfectly accurate, although taking this uncertainty into account creates additional problems. There are many papers on the stochastic routing location problem. Most of these papers are devoted to a specific case of a depot and a well-known vehicle to the location of the travelling salesman problem. We also note that all of these papers consider customer demand as input subject to stochastic changes. Before reviewing these papers, we will discuss some of the features of this type of paper; Compared to papers on definite location routing, a large proportion of these papers use a precise solution method. Another feature of this type of paper is that they improve most of the heuristics provided by researchers in the worst-case scenario or in their optimization gap and computational complexity [33].

Table 4: Summary of recent papers for dynamic and stochastic problems [33]

Type of problem	**Solving method**	**papers**	**vehicles**	**customers**
Location of traveling salesman	Exact: Graph theory	[27]	Not available	Not available
Location of possible taboo search	Exact: Graph theory	[4]	Not available	Not available
delivery of goods Location	heuristic: Graph theory	[30]	80	1
Total stochastic location-routing problem	Innate heuristic	[2]	100	10
stochastic location-routing problem with investment	Innate heuristic	[25]	200	20
	branch heuristic	[13]	52	9
Dynamic location-routing problem	Innate heuristic	[32]	400	400

2.2 Traveling Salesperson Location Problem

The traveling salesman location problem considers a group of customers at any given time interval with a subset of what a service is demanding; In fact, this demand is not known, but it can be expressed by the distribution of probabilities. The goal is to position the basis of the seller's home so that the expected journey length is consistent. In the papers, two models are considered:

1. A trip is considered for each subset of customers.
2. Among all customers, a previous trip is considered and in the actual trip, every day is skipped among customers who do not need services.

The travelling salesman location problem was introduced by [11]. They proposed an iterative solution method in which at each phase the facility is redeployed as the middle of the customers at the beginning and end of each trip. The surface location is considered with Euclidean and orthogonal distance. Berman and Simchi Levi proved a basic proposition of the travelling salesman location problem that says there is at least one optimal solution at the vertices of the network. An optimal polynomial algorithm is also provided for tree networks. Simchi Levi and Berman [8] introduced a polynomial time heuristic for public networks that uses the formula for estimating lengths that includes travel. This work was developed by these authors in 1987 for surface location at Euclidean and orthogonal distances. However, the worst-case scenario was improved by Bersimas. The travelling salesman location problem was considered by McDiarmid with fewer assumptions about probability distributions; A linear time algorithm was considered for tree networks [25].

Moshiear introduced the pickup and delivery location problem, which the problem is the location of a travelling salesman with pickups and deliveries. In this case, in addition to the group of customers which is variable, their demands also have a possible distribution. The author developed the heuristic theory of Berman and Simchi Levi for this case and provided a solution based on the customer categorization heuristic.

The potential travelling salesman location problem was first introduced in the papers by Berman and Simchi Levi. This is based on the principle of travelling salesman location problem that involves finding a trip for the minimum distance expected to be travelled by the salesman following the trip and passing customers who do not need any service. The authors present a boundary and branch algorithm for the probabilistic problem of travelling salesman location on the network. Balakrishnan modulates the probabilistic problem of travelling salesman location to solve the n potential travelling salesmen problem and offers the polynomial retention heuristic: The nearest neighbourhood based on a salesman on the network and a special curve based on a salesman for the probabilistic problem of travelling salesman location at Euclidean distances. Quantitative works consider the travelling salesman location problem with multiple and non-standard purposes. Averbakh and Berman presented a potential multi-objective pickup and delivery location problem that is an extension of their previous work. Polynomial algorithms were proposed for the location of the tree. This work continued to attract the attention of Averbakh and his colleagues; He considered a change of purpose for the travelling salesman location problem and minimized the average waiting time or total travel time and showed how their previous heuristics for this case could be updated for this purpose [37].

2.3 A Multi-Vehicle Stochastic Trip Location Problem

There are relatively few papers on this subject. In the case raised by Laporte, both the location of the depot and previous trips must be known before the actual level of demand can be determined. The trips that increase the capacity of the car may be considered as a travel defect, if this happens, the vehicles will temporarily return to the depot. The service is then recalculated for the remaining customers. The cost of this extra trip may be considered a penalty. The authors' goal is to minimize depot s and the cost of previous trips and is one of the following constraints: (a) a limit on the probability of travel violations, (b) a certain amount of penalty expected for travel. Lowe developed the problem for the seller with different capacities as well as the basic theorem for this case; That is, at least one of the optimal solutions to the problem lies on a network node. He proposed a polynomial time heuristic for this state of networks and then modified a solution for the surface location with Euclidean and orthogonal distance [21].

Liu and Lee [25] considered a stochastic customer demand that included inventory costs in the location-routing problem. An initial answer was found by customer clustering, which was based on increasing the order of marginal inventory costs. For each depot cluster, the closest location to the center is located and a travelling salesman problem is solved. They then applied a heuristic recovery method based on the elimination and interpretation movement for the positioning phase. Both travel and inventory costs were assessed for possible movements. Therefore, this method is much slower than nested methods that were used to estimate the length of the trip and other tools to reduce the computational load. In the case studied by Albareda-Sambola et al. [2], both depots and previous trips were planned prior to recognizing demand. Subsequent trips may eliminate some customers if the capacity of the vehicles is such that the capacity of the vehicles must be increased. There will be a penalty for customers who did not see the service. The objective function includes the sum of warehousing costs, expected costs for subsequent trips and the expected penalty costs. The authors also consider an estimate for the latter two costs. An initial answer was found using the heuristic (travel-allocation-location); This solution was then improved by using a local search, including adding and moving movements for location, and 2-opt. For a great extraction and more details, see stochastic travel location, including the Berman travelling salesman location problem [26].

2.4 Dynamic Location-Routing

Dynamic problems divide the planning horizon into several stages. Usually, when planning a horizon, some parameters are uncertain (especially customer demand). Therefore, the dynamic problems are related to the uncertain problems discussed above. The dynamic location-routing problem is a major part of location- routing problem. This is because the static location-routing problem (single-step) is important for critique. The planning horizons below do not address travel, location, and allocation problems. Considering a planning horizon for the facility location that includes the shortest planning intervals for travel planning, the dynamic location-routing problem is a much better model than the real-life location problem with travel perspectives and is an important tool for critics alike. We may distinguish between two types of the planning problem. In one, depots are relocated sequentially, in the other, depots are relocated at the beginning of the planning horizon, and vehicle s trips change with changes in customer demand. The first case is more common if demand increases and the latter one in cases where demand is fluctuating. In the problems studied by Nambian, a rapid increase in demand is predicted, thus the authors provide a sequence of depot locations that should be open at various times. An interesting consideration is that they are allowed to close a factory temporarily while the other is open and later open, although the routes are largely forgotten, thus their method falls into the category of location routing problem [18].

2.5 Problems with Non-Standard Hierarchical Structure

In the routing-location problems discussed so far, they have considered the structure of the facility that serves the customers, which is connected to their depot using a vehicle trip tool. No distance connects different possibilities. Some papers address the facility location-routing problem in which all facilities must be connected for location. This connection is made using directional connections. The cost can include the cost of facility and no-trip planning; Thus, these changes can be ignored. This section considers four types of problems with different hierarchical structures, some simpler and some more complex than the location-routing problem modes discussed so far. Some of them may be out of the definition of the location-routing problem, but some of them are completely consistent with a location-routing problem. They are each different and differ in the standard location-routing problem according to whether or not the trip planning includes different layers below [29].

1. The transportation location problem does not include travel planning.
2. The various location-routing problem includes the planning of travel between the facility and customers, but also the complete routes of the internal facility.
3. The vehicle travel allocation problem includes travel planning between facilities but not between facilities and customers.
4. A multi-layered trip location problem that involves travel planning in both layers and may even be considered as a level of possibilities.

2.6 The Transportation Location Problem

The transportation location problem consists of the facility location problem along with the transfer of goods between supply and demand. Each route from origin to destination (not necessarily a simple route) is a route through the facility location. This is also known as the transportation location problem or the location routing problem. This also happens frequently when transporting hazardous materials, because not having to stop is very sensitive at these times. In the papers, all models of location-routing problems consisting of hazardous materials are in fact location-routing problems [33].

2.7 Many-to-Many Location Routing Problem

Nagy introduced the many-to-many location routing problem. In this case, many customers want to send their goods to each other. In a general case, it is assumed that each customer sends a different product to another, and this corresponds to sending flow among customers. A network of centers includes deployed routing costs (internal center routes are assumed to be direct while customer-to-centre routes have several stops). So far, the location-routing problem has been an approach to establishing the facilities and the many-to-many location-routing problem has been an approach to establishing the centers. It should be noted that when customers can both send and receive goods, the pickup delivery routing method needs to find the routing costs. This is more difficult than the vehicle routing problem, which fluctuates frequently and makes it more difficult for vehicles to test. The step-by-step heuristic solution template is based on the proposed nested methods. A number of logical problems are a particular case of many-to-many location routing problems; Such as the center location problem (if the routes are assumed to be full) and the transportation problem (if the centers are fixed) and of course the location routing problem (if there is no centre-internal flow or all pickups and deliveries are zero). This type of problem has the application of letter design or closed delivery systems. In fact, all other papers in this field consider parcel delivery applications. All authors consider hierarchical solution methods without computational comparisons with other papers [6].

2.8 Vehicle Routing Allocation Problem

In the above papers, the centre-internal distances are straight, while the customers' distances to the center is a trip. We can change it and consider travel including centers and customers to direct centers. It is desirable that such problems should be considered as a part of the location routing problem because there is no travel planning at the customer's level. Beasley and Nascimento [7] called this problem as the vehicle routing allocation problem and provided an overview of the work done. However, different authors used different names and created different modes of the problem. Nambiar [31] considered the tire-including allocation to the stations of the tire assembly along with the planning of the route of vehicle assembly, but the algorithm for solving them ignored the allocation problem. Labbe and Laporte [20] solved the mailboxes' location problem. They minimized the linear combination of vehicles' travel costs which was a set of postal items and customer dissatisfaction costs which depended on the total distance between customers and the allocated mailbox. A recurring heuristic was presented. A complex military logic problem centered on the vehicle routing allocation problem had been discussed as well. Trained mobile simulators surveyed the sites. A sequential solution based on the site coverage set of the sites was presented with a series of heuristics to route the simulators. Labbe et al. [22] used the branch and bound method to solve the vehicle routing allocation problem (known as the middle run problem), which aimed to minimize routing costs provided there is an upper bound on allocation costs. Routes from resources to centers include stations (in resources or centers), but routes from centers to customers are always straightforward. The authors solved the problem by considering only a small subset of all possible multi-station distances and assigning a variable to each simple distance. Therefore, the solution algorithm does not include the planning of any distance. The discounted problem is solved by using a business linear programming solver. In addition, three heuristics based on binary and then the re-introduction of some variables and constraints were presented.

2.9 Multilevel Location Routing problem

Readers' minds may think that routing takes place at both the customer center and level. Others may well think that this happens in the structure of theory, not in practice. In the two-level location routing problem, introduced by Jacobs and Madsen, newspapers were transferred from the factory to the points of delivery and from there to the customers [35].

2.10 PSO Algorithm

The idea of Particle Swarm Optimization (PSO) was first mooted by Kennedy and Eberhart in [15]. The particle swarm algorithm is an evolutionary computational algorithm inspired by nature and based on iteration. The source of inspiration for this algorithm was the social behavior of animals, such as the mass movement of birds and fish. Since the particle swarm algorithm starts with an initial stochastic population matrix, it is similar to many other evolutionary algorithms such as the continuous genetic algorithm and the colonial competition algorithm. Unlike the genetic algorithm, the particle swarm algorithm has no evolutionary operator such as mutation and coupling. Therefore, it can be said that the colonial competition algorithm is more similar to a particle swarm. Each element of the population is called a particle which is the equivalent of a chromosome in a genetic algorithm or a country in a colonial competition algorithm. In fact, the particle swarm algorithm consists of a specific number of papers that are randomly assigned the initial value. For each particle, two values of position and velocity are defined, which are modeled with a location vector and a velocity vector, respectively. These papers move repetitively into the next n-space of the problem to look for possible new options by calculating the optimum value as a measurement criterion. The dimension of the problem space is equal to the number of parameters in the desired function for optimization. One memory is dedicated to storing the best position of each particle in the past and one memory is dedicated to storing the best position occurring among all papers. From the experience of these memories, the papers decide how to move next time. In each iteration, all the papers move in the next n-space of the problem to finally find the general optimal point. Papers update their velocities and position according to the best global and local answers.

$$P_{m,n}^{new} = P_{m,n}^{old} + V_{m,n}^{new}$$

which in: $V_{m,n}$ is the particle velocity, $P_{m,n}$ is particle variables, r_2 and r_1 are independent random numbers with uniform distribution, $\Gamma_1\Gamma_2$ are learning factors, $P_{m,n}^{\text{local best}}$ are the best local answer, $P_{m,n}^{\text{global best}}$ are the best global answer.

The particle swarm algorithm optimizes the velocity vector of each particle and then adds the new velocity value to the position or particle value. Repetition in speed updating is affected by both the value of the best local answer and the best global answer. The best local and global answers are the best answers that have been obtained by one particle and the whole population, respectively, as the algorithm is implemented. Constants $\Gamma_1\Gamma_2$ are respectively, perceptual parameters and social parameters. The main advantage of the particle swarm algorithm is that the implementation of this algorithm is simple and requires the determination of quantitative parameters. This algorithm is also able to optimize complex cost functions with a large number of local minimums.

3 Method

The present study is applied in terms of purpose because Artawheel Tire Company is looking to solve the problem regarding the selection of the depot. The research method is a descriptive survey and the data collection tool is documents and interviews with experts. Also, considering that the research seeks to locate the depot using the particle swarm optimization algorithm, the research is of a predictive type.

3.1 Community and Statistical Sample of Research

The statistical population of the present study consists of managers, deputies, employees, and experts of Artawheel Tire Company.

3.2 Data Collection Method

The library method is used to study the research literature review, the documents (documents of Artawheel Tire Company), the student dissertations and the search in electronic databases were used as the main tools of data collection in this research.

3.3 Data Analysis Method

In this research, a meta-heuristic method based on the optimization method of the particle swarm algorithm is used to solve the problem. This algorithm is a minimization technique that is used to deal with problems in which the best answer is multi-dimensional. First, the set of papers is placed in the response space and begins to move at the initial velocity. There are also communication channels between them. Then, these members move in the response space and the results obtained in each stage are evaluated according to the cost. Over time, these members accelerate toward other members of their communication group who have a higher value for the evaluation function. In the particle swarm optimization algorithm, group members exchange information with each other about the best location found to date. In other words, the best or optimal response found is known to all members of the group.

The PSO algorithm is based on the following two equations, which calculate the velocity and position of the i^{th} particle at $t+1$, respectively:

$$V_i(t+1) = W * V_i(t) + c_1 r_1 (X_{p_i}(t) - X_i(t)) + c_2 r_2 (X_{g_i}(t) - X_i(t)) \tag{3.1}$$

$$X_i(t+1) = X_i(t) + V_i(t+1). \tag{3.2}$$

The parameters r_2 and r_1 of random variables from the uniform function between 0 and 1 are P^{best} and g^{best}, c_1 and c_2 represent the acceleration coefficient of P^{best} and g^{best}, and W represents the coefficient of inertia.

The mathematical model of the problem is presented using equations (3.3) to (3.10):

$$\min Z = \sum_t f_t . Z_t + \sum_{t,j|j\neq 0} c_j . X_{jt} + \sum_{p,j,t} h_{pj} . I_{pjt} \tag{3.3}$$

$$I_{pj(t-1)} + w_{pjt} - I_{pjt} = d_{pjt} \quad \forall p, j | j \neq 0, t \tag{3.4}$$

$$I_{p0(t-1)} + P_{pt} - I_{p0t} = \sum_j w_{pjt} \quad \forall p, t \tag{3.5}$$

$$\sum_p k_p . P_{pt} \leq P_{\max} . Z_t \quad \forall t \tag{3.6}$$

$$\sum_p a_p . w_{pjt} \leq Q . X_{jt} \quad \forall j | j \neq 0, t \tag{3.7}$$

$$\sum_p a_p . I_{pjt} \leq I \max_j \quad \forall j, t \tag{3.8}$$

$$X_{jt}, Z_t \in \{0,1\} \quad \forall j, t \tag{3.9}$$

$$P_{pt}, w_{pjt}, I_{pjt} \geq 0 \quad \forall p, j, t \tag{3.10}$$

Equation (3.3) represents the objective function of the proposed model, which includes fixed start-up costs, the cost of sending the product to parts, and the cost of maintaining the product by the manufacturer and parts. Equation (3.4) shows the inventory balance between the demand for the p-type product in retail j in period t and the sum of type p product sent from the manufacturer to retail l in period t. The balance between the total p-type product sent from the manufacturer to all retailers in period t and the total p-type product produced in period t is expressed using Equation (3.5). Equation (3.6) guarantees the limit of production capacity. The capacity limit of the means of transport is presented using Equation (3.7). Equations (3.6) and (3.7) implicitly show whether, in period t, the product was made by the manufacturer or the product was shipped to retailer j. Equation (3.8) guarantees constraint of storage capacity for the manufacturer and retailers. The technical constraints on the decision variables are established by Equations (3.9) and (3.10).

The PSO algorithm starts with a group of random responses (papers) and then seeks the optimal solution by updating the papers in each iteration [17]. If the decision variables, and in turn the position of the papers, are of the type zero and binary; The velocity and position vectors of each particle in each iteration of the algorithm are calculated according to Equations (3.11) to [18]:

$$V_i(t) = W * V_i(t-1) + c_1 r_1 (P^{best}(i) - X_i(t)) + c_2 r_2 (n^{best}(i) - X_i(t)) \tag{3.11}$$

$$-V \max < V_i(t) < V \max \tag{3.12}$$

$$S_i = \frac{1}{1 + e^{V_i(t)}} \tag{3.13}$$

$$X_i(t) = \begin{cases} 1 & \rho < S_i \\ 0 & O.W \end{cases} \tag{3.14}$$

According to Equation (3.11), the new velocity vector of each particle based on the previous velocity of the particle $V_i(t-1)$ is the best position the particle has ever reached $(i)P^{best}$ and the position of the best particle in the particle neighborhood $(i)n^{best}$ is calculated. If the neighborhood of each particle includes all the papers in the group, then $(i)n^{best}$ indicates the position of the best particle in the group, which is denoted by $(i)g^{best}$.

In Equation (3.14), ρ is a random number with a uniform distribution between zero and one. According to equations (3.3) to (3.10) of the routing-inventory problem, is a mixed integer programming problem in which the decision variables can be divided into two categories of continuous variables (w_{pjt}, I_{pjt} and P_{pt}) and zero and binary variables (Z_t and X_{jt}). Since the difficulty of solving this problem is due to the variables zero and binary, therefore, a two-part solution to the problem can be developed by separating the decision variables. In the proposed solution, first, the values of zero and binary variables (Z_t and X_{jt}) are determined using the optimized particle group optimization algorithm, then due to the specific values of the variables zero and binary, the routing-inventory problem becomes a linear programming problem to determine the values of continuous variables (w_{pjt}, I_{pjt} and P_{pt}) with the aim of minimizing the total cost. The resulting linear programming problem can be easily solved with the existing methods for solving linear programming problems, and finally, the final solution to the main problem can be achieved. The general structure of the proposed algorithm is presented in the following diagram. As presented in the figure, the proposed structure is designed based on the particle group optimization algorithm.

Typically, in meta-heuristic algorithms, each response at each phase is evaluated using the fitness function and its fitness value is calculated. In this paper, like other single-objective problems, the objective function of the problem, expressed by Equation (3.1), is used as a particle fitness function. In the structure of the proposed algorithm, the position of each particle only represents the variables Z_t and X_{jt}. Therefore, to calculate the objective function, it is necessary to calculate w_{pjt}, I_{pjt} and P_{pt}, followed by the value of the objective function based on Z_t and X_{jt}. For constant values of Z_t and X_{jt} variables, different values can be obtained for w_{pjt}, I_{pjt} and P_{pt} variables; Among these values, w_{pjt}, I_{pjt} and P_{pt} should be determined in such a way that while setting the constraints of the problem, it also has the lowest costs. In the proposed algorithm, a linear programming model is used to find the mentioned optimal values. For this purpose, according to relations (3.4) and (3.5) and considering $I_{p00} = I_{pj0} = 0$

$$I_{pjt} = \sum_{l=1}^{t} (w_{pjl} - d_{pjl}) \quad \forall p, t, j | j \neq 0 \tag{3.15}$$

$$I_{p0t} = \sum_{l=1}^{t} (P_{pl} - \sum_{j} (w_{pjl})) \quad \forall\ p, t. \tag{3.16}$$

Placing relations (3.15) and (3.16) in relation (3.3) and by specifying the production and transmission times, after simplification we will have:

$$\min Z = \text{constant value } + \sum_{p,j|j\neq 0,t} (T-t+1)(h_{pj} - h_{p0}) w_{pjt} + \sum_{p,t} (T-t+1) h_{p0}.P_{pt}. \tag{3.17}$$

Also, with placing relations (3.15) and (3.16) in relation (3.8), we will have:

$$\sum_{p} \sum_{l=1}^{t} a_p (P_{pl} - \sum_{j} w_{pjl}) \leq I \max_{0} \quad \forall t \tag{3.18}$$

$$\sum_p \sum_{l=1}^{t} a_p.w_{pjl} \leq I \max_j + \sum_p \sum_{l=1}^{t} a_p.d_{pjl} \quad \forall t, j|j \neq 0. \tag{3.19}$$

Also, considering the relations (3.15), (3.16) and as $I_{pjt} \geq 0$, we can replace constraints (3.4) and (3.5) with constraints (3.20) and (3.21):

$$\sum_{l=1}^{t} w_{pjl} \geq \sum_{l=1}^{t} d_{pjl} \quad \forall p, t, j|j \neq 0 \tag{3.20}$$

$$\sum_{l=1}^{t} (P_{pl} - \sum_j w_{pjl}) \geq 0 \quad \forall p, t. \tag{3.21}$$

Therefore, considering the above relations and according to equations (3.3) to (3.10), the simplified model is obtained as follows:

$$\min Z = \text{constant value } + \sum_{p,j|j\neq 0,t} (T-t+1)(h_{pj} - h_{p0})w_{pjt} + \sum_{p,t} (T-t+1)h_{p0}.P_{pt} \tag{3.22}$$

$$\sum_p k_p.P_{pt} \leq P_{\max} \quad \forall t \in \{t|Z_t = 1\} \tag{3.23}$$

$$\sum_p a_p.w_{pj1} \leq K_j \quad \forall j \in \{j|X_{j1} = 1\} \tag{3.24}$$

$$\sum_p a_p.w_{pjt} \leq Q \quad \forall (j,t) \in \{(j,t)|X_{jt} = 1, t > 1\} \tag{3.25}$$

$$\sum_{l=1}^{t} w_{pjl} \geq \sum_{l=1}^{t} d_{pjl} \quad \forall p, j|j \neq 0, t|t > 1 \tag{3.26}$$

$$\sum_p \sum_{l=1}^{t} a_p.w_{pjl} \leq I \max_j + \sum_p \sum_{l=1}^{t} a_p.d_{pjl} \quad \forall j|j \neq 0, t|t > 1 \tag{3.27}$$

$$w_{pj1} \geq d_{pj1}, w_{pjt} \geq 0, P_{pt} \geq 0 \forall p, \forall (j,t) \in \{(j,t)|X_{jt} = 1\}, \forall t \in \{t|Z_t = 1\} \tag{3.28}$$

$$w_{pjt} = P_{pt} = 0 \quad \forall p, \forall (j,t) \in \{(j,t)|X_{jt} = 0\}, \forall t \in \{t|Z_t = 0\} \tag{3.29}$$

which in relation (3.22), $K_j = \min(Q, I\max_j + \sum_p a_p.d_{pj1})$. The above-simplified model has a variable number of $P(N+1)T$ and a maximum limit of $N(P+1)$ less than the original model. Now in each stage of the algorithm, by specifying the producing and transmission times (X_{jt} and Z_t) and using the model, the variables W_{pjt} and P_{pt} as well as the fitness values of each particle can be calculated and according to the values obtained from solving the linear programming model for variables W_{pjt} and P_{pt}, if necessary, make necessary corrections in production and transmission times. Because by solving the linear programming model, the optimal value of some production or transmission values may be zero, in which case the corresponding transmission time will also be zero. Calculating the particle fitness using a linear programming model solution requires some high computational time. Therefore, to increase the speed of the algorithm, this method is used only in the second stage of the algorithm and to calculate the fitness value in the first step of the algorithm, an approximate method has been developed which is described below:

1. Using Equation (3.28), we calculate the initial value of W_{pjt}:

$$W_{pjt} = \sum_{k=1}^{r-1} d_{pjk} \quad \forall p, j|j \neq 0 \tag{3.30}$$

In this relation, r indicates the next period in which the next transmission takes place.

2. Using Equations (3.29) to (3.31), the value obtained for W_{pjt} is modified so that all W_{pjt} values are possible ($W_{pjt} \leq Q$):

$$\Delta_{jt} = X_{jt}.Q - \sum_{p} a_p.w_{pjt} \quad \forall t, j|j \neq 0 \tag{3.31}$$

$$\Delta_{j(t-1)} = \Delta_{j(t-1)} + \min(0, \Delta_{jt}) \quad \forall t|t \neq 1, j|j \neq 0 \tag{3.32}$$

$$W_{pjt} = W_{pjt}.(Q - \max(0, \Delta_{jt}))/\sum_{p} a_p.w_{pjt} \quad \forall p, t, j|j \neq 0 \tag{3.33}$$

3. The initial value of P_{pt} is calculated using equation (3.32):

$$P_{pt} = \sum_{k=1}^{r-1} w_{pjk} \quad \forall p, j|j \neq 0 \tag{3.34}$$

   In this relation, r indicates the next period in which the next production takes place.
4. Using Equations (3.33) to (3.35), the value obtained for P_{pt} is modified so that all P_{pt} values are possible ($P_{pt} \leq P_{\max}$):

$$\Delta_{0t} = Z_t.P_{\max} - \sum_{p} k_p.P_{pt} \quad \forall t \tag{3.35}$$

$$\Delta_{0(t-1)} = \Delta_{0(t-1)} + \min(0, \Delta_{0t}) \quad \forall t|t > 1 \tag{3.36}$$

$$P_{pt} = P_{pt}.(P_{\max} - \max(0, \Delta_{0t}))/\sum_{p} k_p.P_{pt} \quad \forall p, t \tag{3.37}$$

5. According to Equations (3.38) and (3.39), we calculate the values of I_{p0t} and I_{pjt}:

$$I_{p0t} = I_{p0(t-1)} + P_{pt} - \sum_{j|j \neq 0} w_{pjt} \quad \forall p, t \tag{3.38}$$

$$I_{pjt} = I_{pj(t-1)} + w_{pjt} - d_{pjt} \quad \forall p, j, t \tag{3.39}$$

6. Using the relations (3.40) to (3.41), we get the uncertainty of the obtained responses:

$$P(X) = \sum_{k=1}^{2} \delta_k.P_k(X) \tag{3.40}$$

$$P_1(X) = \sum_{p,j,t} \max(0, I_{pjt}) \tag{3.41}$$

$$P_2(X) = \sum_{j,t} \max(0, \sum_{p} a_p.I_{pjt} - I \max_{j}) \tag{3.42}$$

   In the above relations, δ_k is the penalty coefficient for violating k^{th} limit.
7. Using equation (3.43), we calculate the particle fitness value:

$$fitness(X) = Z(X) + P(X) \tag{3.43}$$

   In the above relation, $Z(X)$ represents the objective function obtained from relation (3.3).

At the end of the approximate procedure, like solving the linear programming model, according to the values obtained for the variables W_{pjt} and P_{pt}, the necessary modifying is applied to the production and transmission times if it is necessary. In the second stage of the algorithm, if the papers are uncertain, it is not possible to solve the linear programming model. Therefore, in the second stage of the algorithm, even if it is an uncertain particle, the above approximate procedure is used to calculate the fitness of that particle.

Table 5: The results obtained from solving sample problems using LP-IPSO and IPSO

problem number	Objective function		uncertainty		Time (second)	
	IPSO	LP-IPSO	IPSO	LP-IPSO	IPSO	LP-IPSO
1	2358	2358	0	0	4.5	160.4
2	3487	3482	0	0	5.8	233
3	3533.6	3496.3	0	0	7.1	278.6
4	5127.6	5048	0	0	8.7	418.4

4 Conclusion

In this part of the research, considering several simulations, the proposed method is evaluated from various aspects, including the time to reach the optimal response and the degree of optimality. Since the proposed method has several different parameters the appropriate value of these parameters may affect the final results of the proposed method, and then the impact of these parameters on the result of the proposed method is investigated. In each simulation, the number of iterations, the number of populations, the nostalgia coefficient operator rate and the inertia coefficient operator rate are different. Therefore, different values are obtained for the desired objective function. In the simulations, the optimal nostalgia coefficient rates, the optimal inertia coefficient rate, the average of the total optimal distance travelled (minimum distance), the optimal population number and as a result, the desired objective function value are obtained for 10 separate repetitions. The values are presented in the following charts.

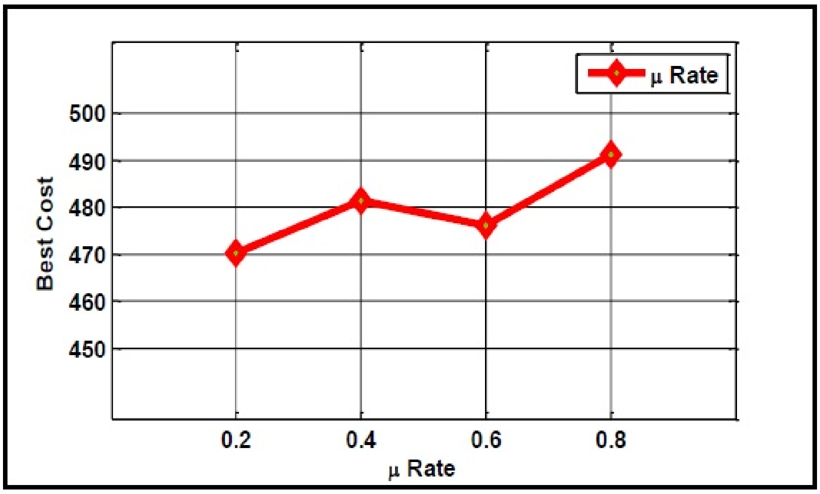

Figure 1: The value of the objective function for different rates of inertia

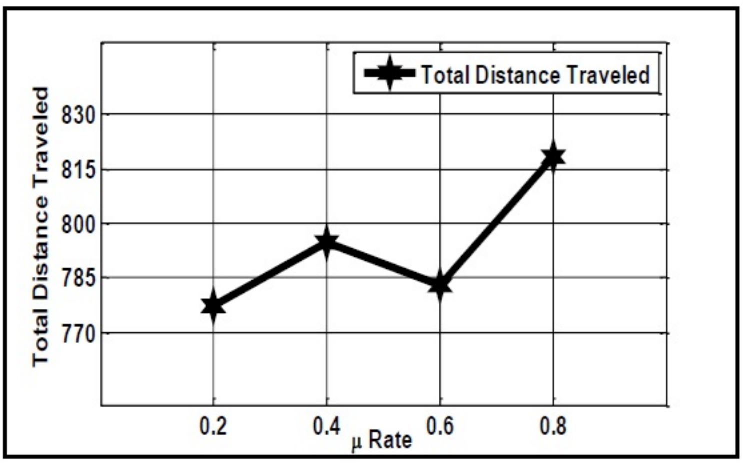

Figure 2: Total distance traveled for different rates of nostalgia

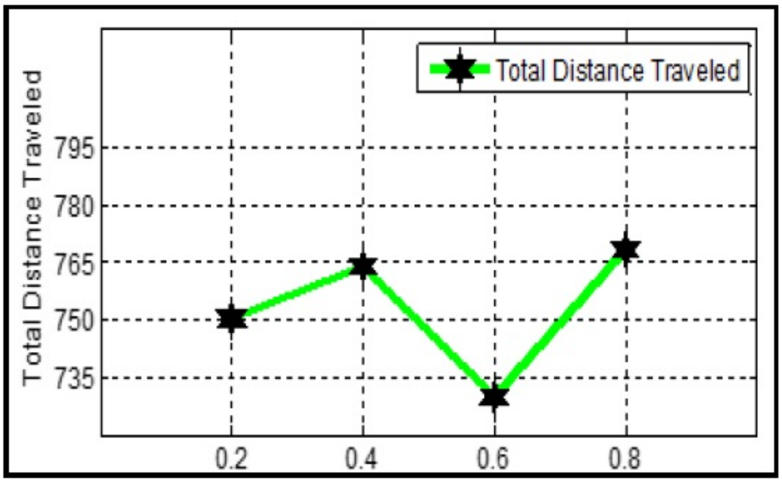

Figure 3: Total distance traveled for different rates of nostalgia

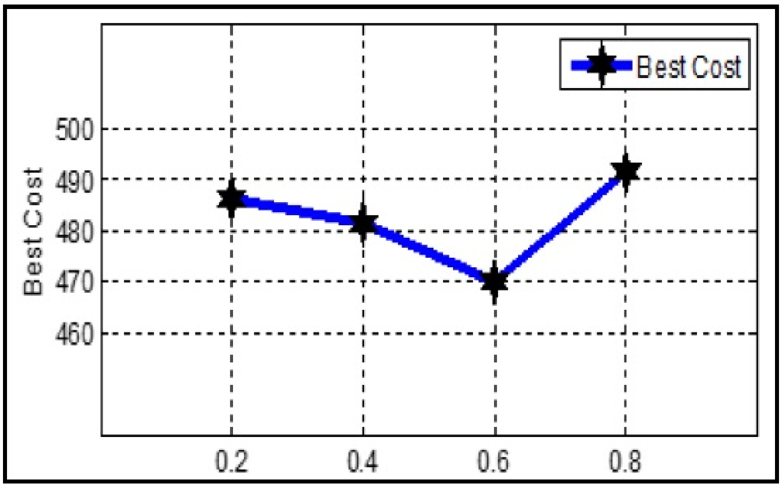

Figure 4: The value of the objective function for different

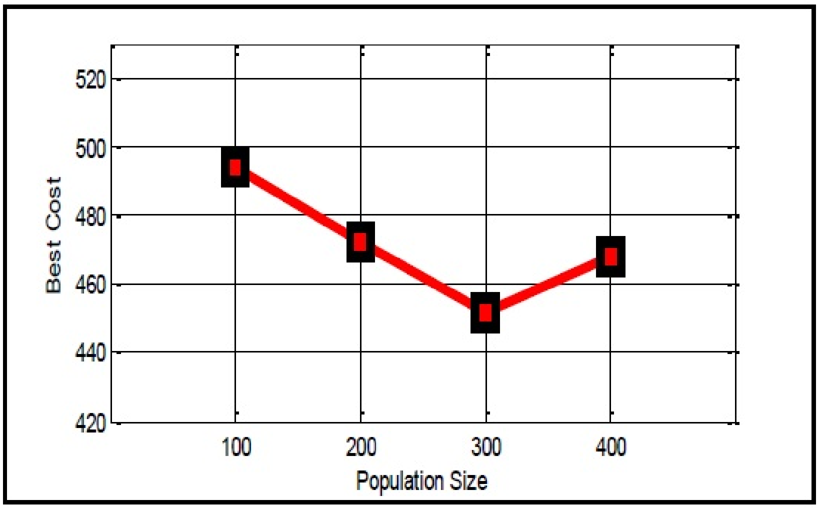

Figure 5: The value of objective function per number of different populations in the proposed method

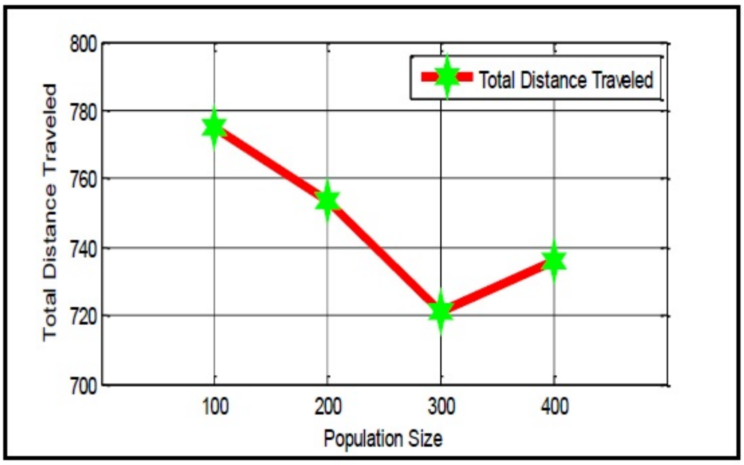

Figure 6: rates of the nostalgia coefficient operator

These comparisons have been performed for a general comparison between the simulations performed for 10 different problems. In the following tables, the results of the objective function and the obtained values have been compared in terms of time.

Table 6: The answers obtained from the proposed methods

best answers from PSO	**optimal answer**	**Number of vehicles**	**Number of customers**	**Type of problem**
825	784	5	32	P_1
710	661	5	33	P_2
786	742	6	33	P_3
965	914	7	46	P_4
1138	1073	7	48	P_5
1135	1073	9	55	P_7
1183	1117	9	65	P_8
1229	1168	9	69	P_9
1908	1764	10	80	P_{10}

In the smaller examples, the difference increases slightly. This difference with the optimal answer is between 5 to 10 percent. The following is a comparison by time:

Table 7: Time obtained in terms of second by the proposed methods

Time obtained from PSO	**Number of vehicles**	**Number of customers**	**Type of problem**
61	5	32	P_1
73	5	33	P_2
74	6	33	P_3
92	7	46	P_4
117	7	48	P_5
175	9	55	P_7
186	9	65	P_8
209	9	69	P_9
344	10	80	P_{10}

According to the results obtained from the simulations in this study, the optimal values of the objective function were obtained for various parameters including cost, time and distance travelled by vehicles. According to the results of these simulations, it was found that using the PSO algorithm to solve the vehicle routing problem can improve the value of the objective function as well as the total number of routes travelled by vehicles. This will reduce fuel consumption and vehicle costs.

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