Int. J. Nonlinear Anal. Appl. 14 (2023) 1, 2439–2453
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.7300

A three-parameter new lifetime distribution with various failure rates: Characteristics, estimation and application

Adel Abbood Najm*, Mahdi Wahhab Neamah

Department of Statistics, College of Administration and Economics, University of Sumer, Iraq

(Communicated by Javad Vahidi)

Abstract

In this article, we proposed a new lifetime model the so-called Slash Lindley- Rayleigh model. The new model is yielded as a ratio of 2-independent random variables, namely, a Lindley-Rayleigh model (numerator) divided by a special case of Beta distribution (denominator), specifically, an exponent to the uniform model. This proposed distribution may be theorized as stretching for a Lindley Rayleigh model, which is more flexible as concerns the kurtosis of the model. Certain important probabilistic and statistical characteristics have been founded, and the parameters of the model were estimated depending on the maximum likelihood approach. The simulation study technique has been implemented to assess the performance of the ML method by using the Monte Carlo technique. Also, real sample data of lifetimes were applied to state the usefulness and flexibility of the new model to model a positive date with surplus kurtosis.

Keywords: Kurtosis and Skewness, Failure Rate, Lindley-G Model, Rayleigh Distribution, ML Estimation
2020 MSC: 60E05, 62E10, 62E17

1 Introduction

Lindley [20] raised the Lindley ("Lin" for short) probability density function (pdf) for dealing with observations of lifetime which relates to the exponential family. The model of Lindley can be used instead of an exponential positive data model in case of the hazard rate pattern is unimodal or bathtub (Bakouch et al.) [6], see also Mohammed et. al. [3], Rawaa and Mahdi [5]. The mathematical and probabilistic characteristics and the MLE problem for the parameter of the Lin- model were derived by Ghitany et al [13]. Mazucheli and Achcar [21] concluded that the Lin- model was more flexible in comparison with the exponential model beside it can be considered a good choice to use it instead of exponential or Weibull models. The Ray- model was proposed by Rayleigh ("Ray" for short) in (1880) as a particular case of the Weibull model of cdf given by

$$G(x) = \begin{cases} 1 - e^{\frac{-x^2}{2\alpha}} & x > 0, \\ 0, & \text{elsewhere} \end{cases}$$

Several formulas for the Ray-distribution have been devoted to supplying flexibility to model a positive sample. Dyer and Whisenand [11] used the Ray model in the engineering field. Fernandez [12] discussed the parameter

*Corresponding author
Email addresses: adel.abood@uos.edu.iq (Adel Abbood Najm), mehdi.wahab@uokerbala.edu.iq (Mahdi Wahhab Neamah)

Received: November 2022 *Accepted:* January 2022

estimation of the Ray- model. The transmuted Ray-model was developed by Merovci [22], Afaq et.al. [2] studied the transmuted inverse Ray- model and its properties, Alaa and Iqbal derived the reliability and hazard rate functions estimates of the Rayleigh-Logarithmic model [17]. The model of Slashed generalized Rayleigh was discussed by Yuri et al.[18]. Lately, Cakmakyapan et al. [8] have proposed models of Lin-G, that can be yielded through taking the integration for the pdf corresponding to the Lin- model. To be more accurate and clear, an r.v. X follows a Lin –G (LG) pdf If its cdf is given by

$$\begin{aligned} F_{LG}(x;\theta,\omega) &= \int_0^{-\ln(1-G(x,\omega))} \frac{\theta^2}{1+\theta}(1+t)\exp(-\theta t)dt \\ &= 1-\left(1-\frac{\theta}{1+\theta}\ln[1-G(x,\boldsymbol{\omega})]\right)[1-G(x,\boldsymbol{\omega})]^{\theta} \end{aligned} \tag{1.1}$$

And so, the pdf is given by

$$f_{LG}(x;\theta,\boldsymbol{\omega}) = \frac{\theta^2}{1+\theta}g(x,\boldsymbol{\omega})[1-\ln(1-G(x,\boldsymbol{\omega}))][1-G(x,\boldsymbol{\omega})]^{\theta-1} \text{ where } x>0$$

where $G(x,\boldsymbol{\omega})$ is the cdf of the baseline probability density function, $g(x,\boldsymbol{\omega}))$ represents its pdf and $\boldsymbol{\omega}$ denotes to vector of the parameters in G.

The particular state assumed by eq. (1.1) is the family of the Lindley-Rayleigh (LR) model which is obtained when G denotes the distribution function of the one-parameter Rayleigh model, namely, an r.v. X is distributed a LR model, represented by $LR(\alpha,\theta)$), if its pdf is given by

$$f_X(x;\theta,\alpha) = \frac{\theta^2}{\alpha(1+\theta)}\, x\left[1+\frac{x^2}{2\alpha}\right]e^{-\left(\frac{x^2}{2\alpha}\right)} \qquad ,x>0 \tag{1.2}$$

where $\alpha>0$ is a scale parameter,and $\theta>0$ is the shape parameter.

The LR model has been considered to increase the flexibility of the Lin-model via combining a single extra parameter, which it means that Ray- model is assumed to be the baseline pdf. Despite that, the right tail of the LR distribution is not heavy enough to conciliate data with surplus kurtosis and data with outliers. It is worth mentioning that this kind of data appears repeatedly in applied research in various fields like reliability and survival analysis, biometry, demography, environmental sciences, engineering fields, actuarial and management sciences, and some other fields. To vanquish this restriction, in this article we proposed an extension of the LR model, the so-called Slash Lindley-Rayleigh (SLR) model. And to be more precise, the SLR model can be given by the ratio of 2- i.r.vs., the first one is the LR model in the numerator, and the second one is the exponent of a rectangular model (denominator). The new model will be shown later that it can be used instead of the LR law to model observations that show a surplus kurtosis. It is important to mention that the SLR is yielded by using the same concept as that applied in Torres et. al. [15], and Venegas et.al [16], where the slash-elliptical model was obtained as a ratio of 2-independent r.vs., an elliptical model (numerator) divided by an exponent of a uniform pdf (denominator), yielding a distribution with more heaviness tail than the elliptical or normal distributions.

A principal benefit of that technique is that a certain symmetric models can be simply developed to uphold the increased kurtosis. An analogous concept also has been used to lifetime r.vs. in Gomez et.al. [14] , Monasterio et. al. [23] , Bolfarine et.al [25, 24], and Hugo et. at. [28] ,and others [7, 27, 10, 26, 1, 19].

Our article is organized as follows: More than one statistical representation of the SLR distribution are proposed in closed forms and figures of the probability density function with various parameters values are presented. Some statistical characteristics have been concluded, like the non central moments, geometric mean, the reliability and failure rate functions which also are given in closed forms, all those are discussed in section 2. In section 3 , the MLE method is used to estimate parameters of the distribution by solving non-linear system of equations by using R- program . In section 4, A simulation data is applied to show the usefulness and flexibility of the proposed model Finally, in section 5, the conclusions are summarized.

2 The New Lifetime SLR Model

In this section, firstly we introduce more than one probabilistic form of the SLR model together with analytical formulas for both pdf and cdf. Then, some mathematical and statistical characteristics of the model are derived. Specifically, closed forms are derived for the moments including mean, and variance, and besides the geometric mean, also the skewness and kurtosis coefficients have been derived which vary with respect to variation of the parameter values. Also, the reliability and hazard functions are studied and show different shapes.

Definition 2.1. Suppose that X denotes a LR(α, θ) model given in eq. 1.2 with U follows the rectangular model with the interval $(0,1)$. A r.v. Y follows a SLR model, denoted by:

$$Y = \frac{X}{U^{\frac{1}{q}}}, \quad q > 0 \tag{2.1}$$

where X and U are independent.

Proposition 2.2. Suppose that Y denotes to a $SLR(\alpha, \theta, q)$ model. Then the pdf of Y is given by:

$$f_Y(y; \alpha, \theta, q) = \frac{\theta^2}{\alpha(1+\theta)} y \int_0^1 t^{\frac{2}{q}} \left[1 + \frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right] exp\left[-\theta\left(\frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right)\right] dt \tag{2.2}$$

where $\alpha > 0$ denotes the scale parameter, $\theta > 0$ is the shape parameter, and $q > 0$ is a kurtosis parameter.

Proof . Using a probabilistic formula given in eq. (2.1) the transformation of the variables technique, getting the Jacobian, and calculate the pdf of Y as follows: From eq. (1.2) we have $X \sim LR(\alpha, \theta)$ and $U \sim u \in (0,1)$, then by using the transformation $Y = \frac{X}{U^{\frac{1}{q}}}$ and $T = U$, we have $X = YT^{\frac{1}{q}}$, where value of the *Jacobian* is calculated by:

$$\|J\| = \left\| \begin{matrix} \frac{\partial x}{\partial y} & \frac{\partial x}{\partial t} \\ \frac{\partial u}{\partial y} & \frac{\partial u}{\partial t} \end{matrix} \right\| = \left\| \begin{matrix} t^{1/q} & y\frac{1}{q}t^{\frac{1}{q}-1} \\ 0 & 1 \end{matrix} \right\| = t^{1/q}.$$

So, the j.p.d.f. of Y and T are given by

$$f(y,t) = \frac{\theta^2}{(1+\theta)\alpha} y t^{\frac{1}{q}} \left[1 + \frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right] e^{-\theta\left(\frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right)} . t^{\frac{1}{q}}, \quad y > 0, \quad 0 < t < 1. \tag{2.3}$$

Now by integrating eq.2.3 with respect to the variable T, we get the marginal probability function of the $r.v.Y$ which represent the required result. □

3 Results

Theorem 3.1. Suppose that Y denotes a SLR(α, θ, q) model. Then,

$$f_Y(y; \alpha, \theta, q) = \frac{q(2\alpha)^{\frac{q}{2}}}{(1+\theta)\theta^{\frac{2}{q}} y^{q+1}} \left[\theta\gamma\left(\frac{q}{2}+1, \frac{\theta y^2}{2\alpha}\right) + \gamma\left(\frac{q}{2}+2, \frac{\theta y^2}{2\alpha}\right)\right], \; y > 0 \tag{3.1}$$

where γ denotes to lower incomplete gamma function which is written as [9]

$$\gamma(s,z) = \int_0^z t^{s-1} e^{-t}\, dt \tag{3.2}$$

furthermore, the upper incomplete gamma function is given as [9]

$$\Gamma(s,z) = \int_z^\infty t^{s-1} e^{-t} dt \tag{3.3}$$

Proof . Using the substitution $w = \frac{\theta y^2 t^{\frac{2}{q}}}{2\alpha}$ in eq. (2.2) and solving the integration:

$$f_Y(y; \alpha, \theta, q) = \frac{\theta^2}{(1+\theta)\alpha} y \int_0^1 t^{\frac{2}{q}} \left[1 + \frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right] e^{-\theta\left(\frac{y^2 t^{\frac{2}{q}}}{2\alpha}\right)} dt.$$

Let $w = \frac{\theta y^2 t^{\frac{2}{q}}}{2\alpha}$, then $t^{\frac{2}{q}} = \frac{2\alpha}{\theta y^2} w$. So, we have $t = \left(\frac{2\alpha}{\theta y^2}\right)^{\frac{q}{2}} w^{\frac{q}{2}}$. This implies that

$$dt = \left(\frac{2\alpha}{\theta y^2}\right)^{\frac{q}{2}} \frac{q}{2} w^{\frac{q}{2}-1}\, dw.$$

Thus,

$$\begin{aligned} f_Y(y;\alpha,\theta,q) =& \frac{\theta^2}{(1+\theta)\,\alpha} y \int_0^{\frac{\theta y^2}{2\alpha}} \left(\frac{2\alpha}{\theta y^2}\right) w \left[1+\frac{w}{\theta}\right] e^{-w} \left(\frac{2\alpha}{\theta y^2}\right)^{\frac{q}{2}} \frac{q}{2} w^{\frac{q}{2}-1}\, dw \\ =& \frac{q\,(2\alpha)^{\frac{q}{2}}}{(1+\theta)\,\theta^{\frac{q}{2}-1} y^{q+1}} \left[\int_0^{\frac{2\alpha}{\theta y^2}} w^{\frac{q}{2}} e^{-w} dw + \frac{1}{\theta}\int_0^{\frac{2\alpha}{\theta y^2}} w^{\frac{q}{2}+1} e^{-w} dw\right]. \end{aligned}$$

Using definition of the lower incomplete gamma function, we get

$$f_Y(y;\alpha,\theta,q) = \frac{q\,(2\alpha)^{\frac{q}{2}}}{(1+\theta)\,\theta^{\frac{q}{2}} y^{q+1}} \left[\theta\gamma\left(\frac{q}{2}+1\,,\frac{\theta y^2}{2\alpha}\right) + \gamma\left(\frac{q}{2}+2,\frac{\theta y^2}{2\alpha}\right)\right].$$

□

Theorem 3.2. Suppose that Y is a SLR(α,θ,q) model. Then,

$$\begin{aligned} f_Y(y;\alpha,\theta,q) =& \frac{q^2(2\alpha)^{q/2}}{2(1+\theta)\theta^{q/2}y^{q+1}} \left(1+\theta+\frac{q}{2}\right)\left[\Gamma\left(\frac{q}{2}\right) - \Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right] \\ & - \frac{q}{(1+\theta)y}\left[1+\theta+\frac{q}{2}+\frac{\theta y^2}{2\alpha}\right]\exp\left[-\frac{\theta y^2}{2\alpha}\right], y>0 \end{aligned} \tag{3.4}$$

Proof . To prove that we can use the following characteristics concern with gamma, lower, and upper incomplete gamma functions [9]

1. $\Gamma(a+1) = a\Gamma(a)$
2. $\gamma(a,z) = \Gamma(a) - \Gamma(a,z), a \neq 0,-1,-2,\ldots$
3. $\Gamma(a+1,z) = a\Gamma(a,z) + z^a e^{-z}$

By returning to eq. 3.1, we have

$$\gamma\left(\frac{q}{2}+1,\frac{\theta y^2}{2\alpha}\right) = \frac{q}{2}\left(\Gamma\left(\frac{q}{2}\right) - \Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right) - \frac{\theta^{\frac{q}{2}} y^q}{(2\alpha)^{\frac{q}{2}}} e^{-\frac{\theta y^2}{2\alpha}} \tag{A}$$

where we use here $z = \frac{\theta y^2}{2\alpha}$ and $a = \frac{q}{2}$. Also, we have

$$\gamma\left(\frac{q}{2}+2,\frac{\theta y^2}{2\alpha}\right) = \Gamma\left(\frac{q}{2}+2\right) - \Gamma\left(\frac{q}{2}+2,\frac{\theta y^2}{2\alpha}\right)$$

and here if we use $a = \frac{q}{2}+1 \quad z = \frac{\theta y^2}{2\alpha}$, we have the left side

$$\frac{q}{2}\left(\frac{q}{2}+1\right)\left(\Gamma\left(\frac{q}{2}\right) - \Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right) - \left(\frac{q}{2}+1\right)\frac{\theta^{\frac{q}{2}} y}{(2\alpha)^{\frac{q}{2}}} e^{\frac{-\theta y^2}{2\alpha}} - \frac{\theta^{\frac{q}{2}+1} y^{q+2}}{(2\alpha)^{\frac{q}{2}+1}} \cdot e^{\frac{-\theta y^2}{2\alpha}}. \tag{B}$$

Now, by putting the relations (A) and (B) in eq. (3.1), we will get the result. □

The following result is very important, because it will be used in the parameters estimation problem of SLR distribution by MLE method.

Corollary 3.3. Suppose that $Y/U = u$ denotes to a r.v. follows LR$(u^{\frac{-2}{q}}\alpha\ ,\theta\ )$ distribution, where U denotes the rectangular distribution on $(0,1)$. Hence, Y has a SLR(α,θ,q) distribution.

Proof . If $(\mathrm{Y/U=u}) \sim \mathrm{LR}\left(\mathrm{u}^{\frac{-2}{q}}\alpha,\theta\right)$, then

$$\begin{aligned} f_{Y/U}\left(y;\mathrm{u}^{\frac{-2}{q}}\alpha,\theta\right) =&\frac{\theta^2}{\mathrm{u}^{\frac{-2}{q}}\alpha(1+\theta)}y\left[1+\frac{y^2}{2\mathrm{u}^{\frac{-2}{q}}\alpha}\right]\exp\left[-\theta\left(\frac{y^2}{2\mathrm{u}^{\frac{-2}{q}}\alpha}\right)\right]\\ =&\frac{\theta^2}{(1+\theta)}\left(\frac{yu^{\frac{2}{q}}}{\alpha}\right)\left[1+\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right]\exp\left[-\theta\left(\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right)\right] \end{aligned}$$

and

$$\begin{aligned} f_Y(y;\alpha,\theta,\mathrm{q}) =&\int_0^1 f_{Y/U}\left(y;\mathrm{u}^{\frac{-2}{q}}\alpha,\theta\right)f_U(u)du\\ =&\frac{\theta^2}{(1+\theta)}\int_0^1\left(\frac{yu^{\frac{2}{q}}}{\alpha}\right)\left[1+\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right]\exp\left[-\theta\left(\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right)\right]du\\ =&\frac{\theta^2}{\alpha(1+\theta)}\mathrm{y}\int_0^1 u^{\frac{2}{q}}\left[1+\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right]\exp\left[-\theta\left(\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right)\right]du. \end{aligned} \tag{3.7}$$

Now, if we follow the same transformation using in the proof of Theorem 3.2, namely, $w=\theta\left(\frac{y^2u^{\frac{2}{q}}}{2\alpha}\right)$, then, find the Jacobian calculate the integral with respect to the variable U over (01), we will find that the formula of the marginal probability density function for the r.v.Y is given by the form:

$$f_Y(y;\alpha,\theta,q)=\frac{q(2\alpha)^{\frac{q}{2}}}{(1+\theta)\theta^{\frac{q}{2}}y^{q+1}}\left[\theta\gamma\left(\frac{q}{2}+1,\frac{\theta y^2}{2\alpha}\right)+\gamma\left(\frac{q}{2}+2,\frac{\theta y^2}{2\alpha}\right)\right].$$

It is to say, $\mathrm{Y}\sim\mathrm{SLR}(\alpha,\theta,\mathrm{q})$. □

Remark 3.4. As particular state for SLR model ,one can observe that the $\mathrm{SLR}(\alpha,\theta,q)$ model converges to the LR model when $q\ \to\infty$, i.e. we have from Proposition 2.2, the following

$$f_Y\ \ (y;\ \alpha\ ,\theta,q)=\frac{\theta^2}{\alpha\ \ (1+\theta)}y\int_0^1 t^{\frac{2}{q}}\ \left[1+\frac{y^2t^{\frac{2}{q}}}{2\alpha}\right]\exp\left[-\theta\ (\frac{y^2t^{\frac{2}{q}}}{2\alpha})\right]dt$$

$$\lim_{q\to\infty} f_Y\ \ (y;\ \alpha\ ,\theta,q)=\frac{\theta^2}{\alpha\ \ (1+\theta)}y\left[1+\frac{y^2}{2\alpha}\right]\ \exp\left[-\theta\ (\frac{y^2}{2\alpha})\right],\ y>0.$$

Theorem 3.5. Assume that Y denotes to a $\mathrm{SLR}(\alpha,\theta,q)$ model .Then the cumulative density function can be obtained

$$F_Y(y;\alpha,\theta,\mathrm{q})=1-\frac{q(2\alpha)^{\frac{q}{2}}}{2(1+\theta)\theta^{\frac{q}{2}}y^q}\left(1+\theta+\frac{q}{2}\right)\left[\Gamma\left(\frac{q}{2}\right)-\Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right]+\frac{q}{2(1+\theta)}e^{-\theta\left(\frac{t^2}{2\alpha}\right)},\quad \mathrm{y}>0.$$

Proof . Recall that $\Gamma(a,z)$ is defined for all real numbers except when $a\neq 0,-1,-2,\ldots$. So

$$\begin{aligned} F_Y(y;\alpha,\theta,\mathrm{q}) =&\int_0^y f(t;\alpha,\theta,\mathrm{q})dt=1-\int_y^\infty f(t,\alpha,\theta,\mathrm{q})dt\\ =&1-\int_y^\infty\frac{q^2(2\alpha)^{\frac{q}{2}}}{2(\theta+1)\theta^{\frac{q}{2}}t^{q+1}}\left[1+\theta+\frac{q}{2}\right]\left[\Gamma\left(\frac{q}{2}\right)-\Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right]dt\\ &+\frac{q}{(1+\theta)}\int_{\mathrm{y}}^\infty\frac{1}{\mathrm{t}}\left(1+\theta+\frac{q}{2}+\theta\frac{\mathrm{t}^2}{2\alpha}\right)e^{-\theta\left(\frac{t^2}{2\alpha}\right)}dt. \end{aligned}$$

Solving the integrals in the above two terms and after some calculations and simplification, we get the required result. □

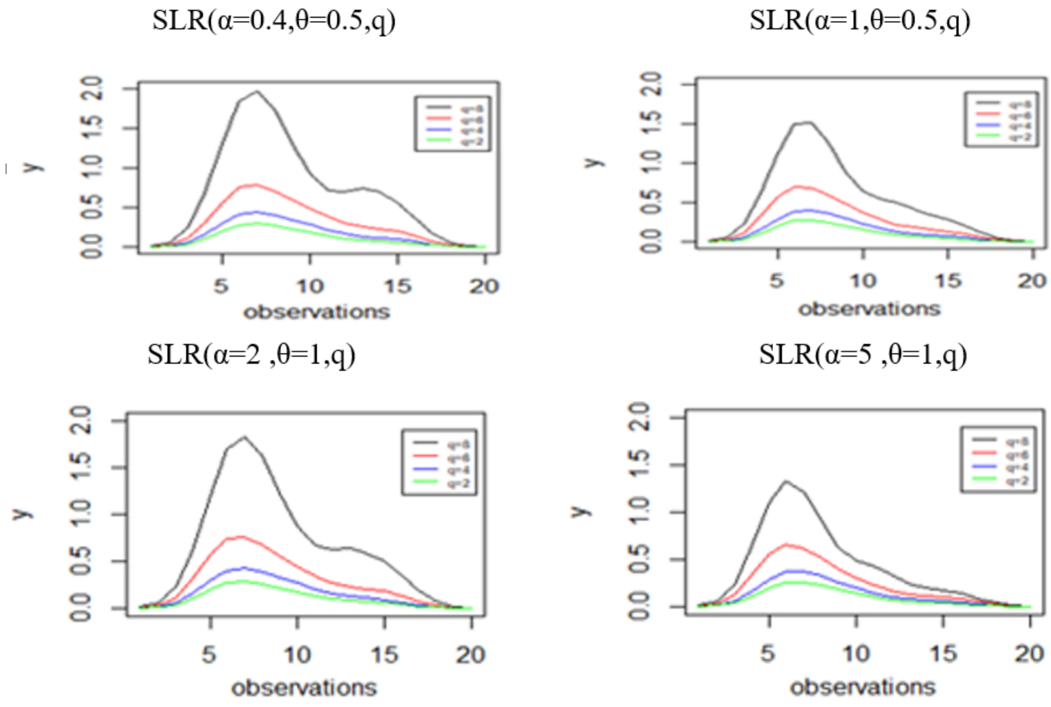

Figure 1: Probability density function of the SLR(α, θ, q) model for various parameters values

3.1 The Moments

The non-central moments of the SLR model could be formulated in closed expression via the gamma function, and for doing that, in the beginning, we provide a general explicit formula for the moments of LR distribution in the following lemma.

Lemma 3.6. Assume that X denotes a LR(α, θ) model. Then

$$E\left(X^{r}\right)=\frac{r(2 \alpha)^{\frac{r}{2}}}{4 \theta^{\frac{r}{2}}}\left(2+\frac{r}{\theta+1}\right) \Gamma\left(\frac{r}{2}\right), r=1,2, \ldots \tag{3.8}$$

Proof . We have LR(α, θ) distribution given in (1.2), and using the substitution $w=\theta \frac{x^{2}}{2 \alpha}$, and using concepts of the gamma function, we will get

$$\begin{aligned}
\mathrm{E}\left(\mathrm{X}^{\mathrm{r}}\right) &=\frac{\theta^{2}}{\alpha(\theta+1)} \int_{0}^{\infty}\left[\left(\frac{2 \alpha}{\theta}\right)^{\frac{1}{2}} \mathrm{w}^{\frac{1}{2}}\right]^{\mathrm{r}+1}\left(1+\frac{\mathrm{w}}{\theta}\right) e^{-\mathrm{w}} \cdot \frac{1}{2}\left(\frac{2 \alpha}{\theta}\right)^{\frac{1}{2}} w^{-\frac{1}{2}} d w \\
&=\frac{\theta^{2}}{2 \alpha(\theta+1)}\left(\frac{2 \alpha}{\theta}\right)^{\frac{r}{2}+1} \cdot \int_{0}^{\infty} w^{\frac{r}{2}}\left(1+\frac{\mathrm{w}}{\theta}\right) \mathrm{e}^{-\mathrm{w}} \mathrm{dw} \\
&=\frac{r \theta(2 \alpha)^{\frac{r}{2}}}{2(\theta+1) \theta^{\frac{r}{2}}}\left[\frac{2(\theta+1)+\mathrm{r}}{2 \theta}\right] \Gamma\left(\frac{\mathrm{r}}{2}\right)
\end{aligned}$$

Therefore

$$E\left(X^{r}\right)=\frac{r\,(2 \alpha\,)^{\frac{r}{2}}}{4\;\;\theta^{\frac{r}{2}}}\left(2+\frac{r}{\theta+1}\right)\; \Gamma\left(\frac{r}{2}\right).$$

□

Proposition 3.7. Assume that Y denotes a SLR(α, θ, q) model. Then

$$E\left(Y^{r}\right)=\frac{r\,(2 \alpha\,)^{\frac{r}{2}}}{4\;\;\theta^{\frac{r}{2}}}\left(2+\frac{r}{\theta+1}\right)\; \frac{q}{q-r} \Gamma\left(\frac{r}{2}\right), \forall r<q, r=1,2, \ldots \tag{3.9}$$

Proof . By virtue of eq. (2.1), together with an assumption that X and U were. i.r.vs., that will yields

$$E\left(Y^{r}\right)=E\left(X^{r}\right).E\left(U^{\frac{-r}{q}}\right) \tag{A}$$

where $U \sim \; u \in (0,1)$ and we have

$$E\left(U^{\frac{-r}{q}}\right)=\int_0^1 u^{\frac{-r}{q}}\,du=\left.\frac{u^{\frac{-r}{q}+1}}{\frac{-r}{q}+1}\right]_0^1 \tag{3.11}$$

$$=E\left(U^{\frac{-r}{q}}\right)=\frac{q}{q-r},\; r<q. \tag{3.12}$$

Then put (B) in (A), and keep in mind $E\left(X^{r}\right)$ given in eq. (3.8), hence the result is obtained clearly. As a consequence of eq. (3.9), we can find the corresponding formulas of the mean, variance, skewness and kurtosis coefficients as will be seen in the following corollaries. □

Theorem 3.8. Assume that Y denote to a SLR(α,θ,q) model, for all $q>1$, then the mathematical expectation (average) of Y is yielded by

$$\mu_Y=E\left(Y\right)=\frac{1}{2}\sqrt{\frac{\alpha\pi}{2\theta}}\;\left(2+\frac{1}{\theta+1}\right)\;\frac{q}{q-1},$$

for all $q>1$ and for all $q>2$ the variance of Y is given by

$$Var\left(Y\right)=\;\frac{q\alpha}{2\theta}\left[\frac{2}{q-2}\;\left(2+\frac{2}{\theta+1}\right)-\frac{q\pi}{4\left(q-1\right)^{2}}\;\left(2+\frac{1}{\theta+1}\right)^{2}\right],$$

for all $q>2$.

Proof . By putting $r=1$ in eq. (3.9), we get the mean of Y and when $r=2$, we get the second moment. Using the facts $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, and $Var\left(Y\right)=E(Y^{2})-\mu_Y^{2},$ we get the variance of Y. □

Theorem 3.9. Assume that Y denotes a SLR(α,θ,q) model. Then for all $q>3$, the coefficient of skewness is obtained by

$$\sqrt{\beta_1}=\frac{\frac{3\sqrt{\pi}}{q-3}\left(2+\frac{3}{\theta+1}\right)-\frac{3q\sqrt{\pi}}{(q-1)(q-2)}\left(2+\frac{1}{\theta+1}\right)\left(2+\frac{2}{\theta+1}\right)+\frac{q^{2}\pi^{\frac{3}{2}}}{4(q-1)^{3}}\left(2+\frac{1}{\theta+1}\right)^{3}}{\sqrt{q}\left[\frac{2}{q-2}\left(2+\frac{2}{\theta+1}\right)-\frac{q\pi}{4(q-1)^{2}}\left(2+\frac{1}{\theta+1}\right)^{2}\right]^{\frac{3}{2}}}.$$

Proof . When we put $r=3$ in eq. (3.9), we get the third moment, and by using definition of skewness coefficient using the central moments, we get the required result

$$\beta_1=\frac{\left[\mathrm{E}\left(\mathrm{Y}-\mu_{\mathrm{Y}}\right)^{3}\right]^{2}}{[\mathrm{Var}(Y)]^{3}},$$

where $\mathrm{E}\left(\mathrm{Y}-\mu_{\mathrm{Y}}\right)^{3}=E\left(Y^{3}\right)-3\mu_Y E\left(Y^{2}\right)+2\mu_Y^{3}$. □

Theorem 3.10. Assume that Y denotes a SLR(α,θ,q) model, $\forall\, q>4$, then the kurtosis coefficient of Y is calculated as

$$\beta_2=\frac{\frac{4}{q-4}\left(2+\frac{4}{\theta+1}\right)\cdot\frac{6q\pi}{(q-1)(q-3)}\left(2+\frac{1}{\theta+1}\right)\left(2+\frac{3}{\theta+1}\right)+\frac{3q^{2}\pi}{(q-1)^{2}(q-2)}\left(2+\frac{1}{\theta+1}\right)^{2}\left(2+\frac{2}{\theta+1}\right)-\frac{3q^{3}\pi}{16(q-1)^{4}}\left(2+\frac{1}{\theta+1}\right)^{4}}{q\left[\frac{2}{q-2}\left(2+\frac{2}{\theta+1}\right)-\frac{q\pi}{4(q-1)^{2}}\left(2+\frac{1}{\theta+1}\right)^{2}\right]^{2}}.$$

Proof . When we put $r=4$ in eq. (3.9), we get the fourth moment, and by using definition of kurtosis coefficient using the central moments, we get the required result

$$\beta_2=\frac{\mathrm{E}\left(\mathrm{Y}-\mu_{\mathrm{Y}}\right)^{4}}{[\mathrm{Var}(Y)]^{2}},$$

where $\mathrm{E}\left(\mathrm{Y}-\mu_{\mathrm{Y}}\right)^{4}=E\left(Y^{4}\right)-4\mu_{Y}E\left(Y^{3}\right)+6\mu_{Y}^{2}E\left(Y^{2}\right)-3\mu_{Y}^{4}$. □

Note: For various values of the kurtosis parameter q, it can be observed that the coefficient of kurtosis decreases when q increases, and obvious that the SLR model has a broad interval of values for the skewness and kurtosis measurements on the values of the parameters, it is to say that the proposed model SLR is substantial enough to deal with the real lifetime data sets.

Theorem 3.11. Assume that Y denotes to a SLR(α,θ,q) model, then the geometric mean of Y (strictly positive) is obtained by

$$\text{G.M.=EXP}\ \left[\frac{1}{2}\left[\ln\left(\frac{2\alpha}{\theta}\right)-\gamma+\frac{1}{\theta+1}\right]+\frac{1}{q}\right]$$

where $\gamma=0.5772156649\ldots$ is the Euler-Mascheroni constant.

Proof . From definition of the geometric mean for strictly positive continuous random variable, we have

$$\text{G.M.=EXP}\ \left[E\left[\ln\left(Y\right)\right]\right]$$

where Y is given in eq. (2.1).

$$E\left[\ln\left(Y\right)\right]=E\ \left(\ln\frac{X}{U^{\frac{1}{q}}}\right)=E\ \left(\ln X\right)-\frac{1}{q}\ E\left(\ln\left(U\right)\right),$$

and after evaluation the expectations in both terms using definition of di (and tri) gamma functions, we get the required result. □

3.2 Reliability and Failure Rate Functions

The reliability and hazard functions are very important in lifetime studies. In the next two corollaries, we provide the expressions of these two measurements which appeared in closed forms according to the upper incomplete gamma function.

Theorem 3.12. Assume that Y denotes to a SLR(α,θ,q) model ,then the reliability function of Y is obtained as

$$R_Y(y)=\frac{q(2\alpha)^{q/2}}{2(1+\theta)\theta^{q/2}y^{q}}\left(1+\theta+\frac{q}{2}\right)\left[\Gamma\left(\frac{q}{2}\right)-\Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right]-\frac{q}{2(1+\theta)}\exp\left[-\theta\left(\frac{y^2}{2\alpha}\right)\right],\ y>0.$$

Proof . From definition of the reliability function $R_Y\left(y\right)=1-F_Y\left(y\right)$, where F represents the cdf of a SLR distribution given in Corollary 3.3, then the required result is obtained. □

Theorem 3.13. Assume that Y denotes to a SLR(α,θ,q) model, then the hazard rate function to Y can be obtained as

$$h_Y(y)=\frac{\frac{q(2\alpha)^{q/2}\left(1+\theta+\frac{q}{2}\right)}{\theta^{q}/2y^{q+1}}\left[\Gamma\left(\frac{q}{2}\right)-\Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right]-\frac{2}{y}\left(1+\theta+\frac{q}{2}+\frac{\theta y^2}{2\alpha}\right)e^{-\frac{\theta y^2}{2\alpha}}}{\frac{(2\alpha)^{q/2}\left(1+\theta+\frac{q}{2}\right)}{\theta^{q}/2y^{q}}\left[\Gamma\left(\frac{q}{2}\right)-\Gamma\left(\frac{q}{2},\frac{\theta y^2}{2\alpha}\right)\right]-e^{-\frac{\theta y^2}{2\alpha}}}.$$

Proof . From definition of the hazard rate function

$$h_Y\left(y\right)=\frac{f_Y\left(y\right)}{1-F_Y\left(y\right)}=\frac{f_Y\left(y\right)}{R_Y\left(y\right)}$$

where F represents the cdf of a SLR distribution given in Theorem 3.2, and f denotes to pdf given in Corollary 3.3, then the required result is obtained. □

Note: If we try to take various values of the parameters, we can be observed obviously that both functions R(y)and h(y) show a wide variety of ranges and hence, shapes. As a result. we can say that our new distribution SLR is useful and flexible enough to model real lifetime data sets.

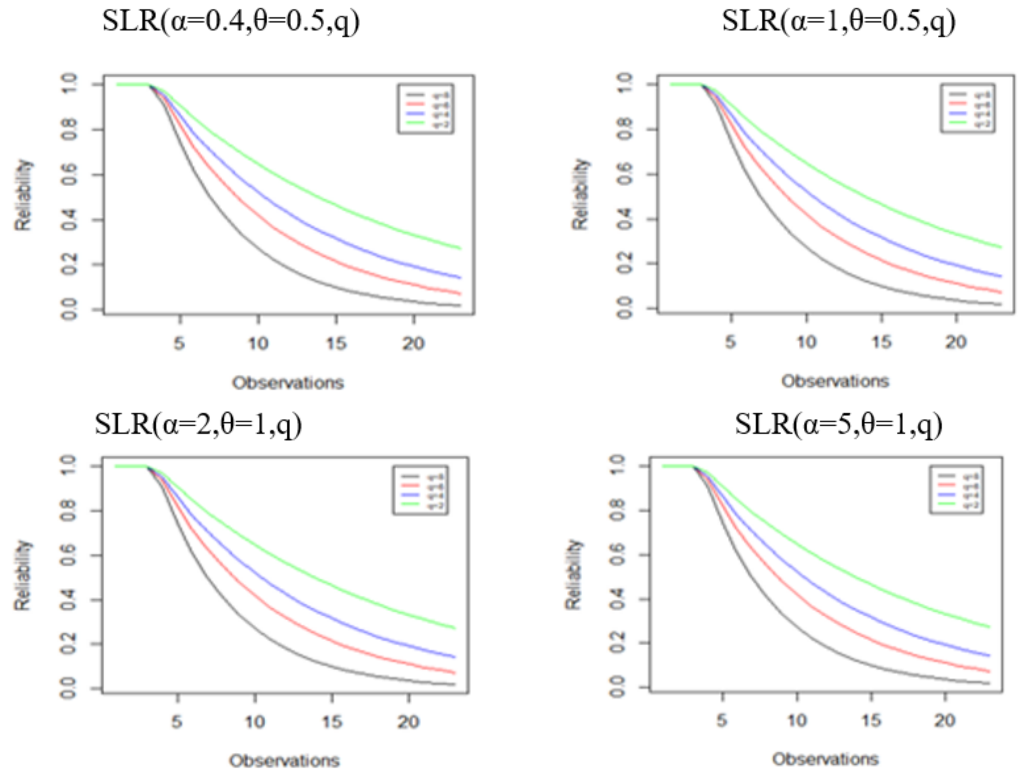

Figure 2: Reliability function for the SLR(α, θ, q) distribution for various parameters values

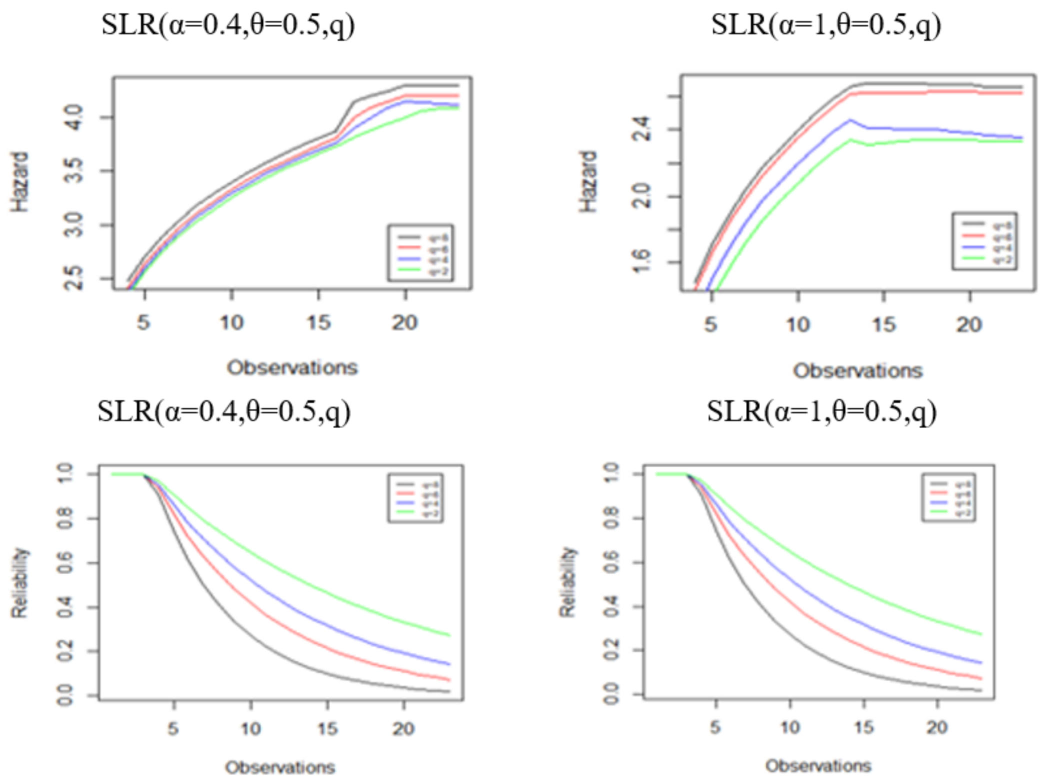

Figure 3: The failure rate function for the SLR(α, θ, q) distribution with various parameters values

4 Estimation of the Parameters Problem

In this section, we study the estimation of the parameters for the SLR(α, θ, q) distribution. The MLE method is assumed to find MOLEs.

4.1 The MLE Method

Assume that $Y_1, \ldots, \ldots, Y_n$ be a r.s. of size n taken from the SLR$(\alpha, \theta, \mathrm{q})$ model where its parameters are unknown. And let $\mathrm{y}_1, \ldots, \ldots, \mathrm{y}_\mathrm{n}$ represent the corresponding observed values. By taking the L-function for the observations. $L(\alpha, \theta, q) = \prod_{i=1}^{n} f_Y(\mathrm{y}, \alpha, \theta, \mathrm{q})$, it follows that the ln-L function is given by

$$\ln \mathrm{L}(\alpha, \theta, \mathrm{q}) = \frac{\mathrm{n}}{2} \ln 2 - \frac{\mathrm{n}}{2} \ln \alpha + 2\mathrm{n} \ln \theta - \mathrm{n} \ln(1+\theta) + \sum_{\mathrm{i}=1}^{\mathrm{n}} \ln \left[\mathrm{G}\left(\mathrm{y_i}\right)\right] \tag{4.1}$$

$$\mathrm{G}\left(\mathrm{y_i}\right) = \mathrm{G}(\mathrm{y}; \alpha, \theta, \mathrm{q}) = \int_0^1 u^{1/q} t_i \left(1 + {t_i}^2\right) e^{-\theta\left({t_i}^2\right)} du$$

and

$$t_i = \frac{y_i u^{1/q}}{\sqrt{2\alpha}}, \quad i = 1, 2, 3, \ldots, n.$$

To get the maximum likelihood estimators, we must find the partial derivatives of eq. (4.1) with respect to the parameters (α, θ, q) respectively, and trying to find a solution for the following non-linear equations

$$\sum_{i=1}^{n} \frac{G_1(y_i)}{G(y_i)} = \frac{n}{2\alpha} \tag{4.2}$$

$$\sum_{i=1}^{n} \frac{G_2(y_i)}{G(y_i)} = \frac{n}{1+\theta} - \frac{2n}{\theta} \tag{4.3}$$

$$\sum_{i=1}^{n} \frac{G_3(y_i)}{G(y_i)} = 0 \tag{4.4}$$

where

$$G_1\left(y_i\right) = \frac{1}{2\alpha} \int_0^1 u^{\frac{1}{q}} t_i \left[{t_i}^2 \left(2\theta {t_i}^2 + 2\theta - 3\right)\right] e^{-\theta\left({t_i}^2\right)}\, du \tag{4.5}$$

$$G_2\left(y_i\right) = -\int_0^1 u^{\frac{1}{q}} {t_i}^3 \left(1 + {t_i}^2\right) e^{-\theta\left({t_i}^2\right)}\, du \tag{4.6}$$

$$G_3\left(y_i\right) = \frac{2}{q^2} \int_0^1 \ln(u) u^{\frac{1}{q}} t_i \left[{t_i}^2 \left(\theta {t_i}^2 + \theta - 2\right) - 1\right] e^{-\theta\left({t_i}^2\right)}\, du. \tag{4.7}$$

Note that

$$G\left(y_i\right) = \int_0^1 u^{\frac{1}{q}} t_i \left(1 + {t_i}^2\right) e^{-\theta\left({t_i}^2\right)}\, du.$$

4.2 Important Practical Treatment

If we take a look at the system of equations 14-16, it is clear that the maximum likelihood estimators cannot be yielded in closed forms. To get the MLE of the parameters it may either solve the mentioned non-linear system of equations using numerical methods such as the Newton-Raphson method, which looks very difficult to be solved or depending on another technique by solving the optimization problem bellow

$$\begin{aligned} &\mathrm{MAX} \ln \mathrm{L}(\alpha, \theta, \mathrm{q}) \\ &\text{subject to } \alpha > 0 \\ &\theta > 0 \\ &q > 0 \end{aligned} \tag{4.8}$$

Note that ln $\mathrm{L}(\alpha, \theta, q)$ is given in eq. (4.1). So, eq. (4.8) can be solved depending on R-programming language [30] according to a function optimisation which requires convenient and feasible initial values (points) in the parameters domain because it plays an important role and has a high effect in the obtained results, so, our choice was (0.25,0.5,1), besides we need to prepare the gradient function to ln $\mathrm{L}(\alpha, \theta, q)$.

4.3 Simulation Study

Here, we start with an algorithm to generate a positive sample from SLR(α, θ, q) model, where the algorithm was depending on the formula in eq. (2.1), where Lindley distribution(say Z) comes from a mixture distribution between two random variables one of them is Exp(θ) and the other is Gamma distribution with $(2, \theta)$, after this we generate the LR distribution (say V) by computing $V = \sqrt{2\alpha}Z^{1/2}$, finally, we generate the SLR distribution (say Y)by calculating the formula $Y = VU^{-1/q}$ where U $\sim$ Uni$(0, 1)$. Next, we provide the technique of simulation to evaluate effectiveness with performance for ML estimators by taking N = 1000 r of sizes n = $50, 75, 100$ from SLR(α, θ, q) distribution for various values parameters. Some statistical measurements were calculated for the simulated estimators such as the bias (B) and the root mean square error (RMSE), for example, $(i)B = \theta^\wedge - \theta$ and,

(ii) $RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\theta^\wedge - \theta)^2}$, and similar calculations were done for the other two parameters. All results are stated in Table 1, and it can be seen that the maximum likelihood method exhibits reasonable estimates for three parameters of SLR distribution, note that the RMSE decreases when n increases.

5 Real Data Application

Now, in this section, real sample observations are used for stating that the SLR model can be more convenient in comparison to some else models used for dealing with and modelling positive sample set which exhibits high or surplus kurtosis. We use 75 observations representing the lifetime of some machines taken from the general company of textiles in Iraq which collects during three months and a half. Table 2 shows the following.

Some descriptive statistical measurements such as minimum, maximum, mean, median, quartiles, skewness, kurtosis and standard deviation of the observations, where all these statistics are shown in a box plot of the lifetime observations stated in Fig. 4. Then we will see that the SLR give a better fit in comparison with some other lifetime distribution such as Lindley, Rayleigh, and Lindley-Rayleigh (LR) distributions. So, for the mentioned four distributions, Table 3 provides the estimates of ML with its standard deviations (SD), and its confidence intervals together with its length of interval or difference (D). Also, the log-likelihood (lnL) values are stated and it is clear that the SLR model has the greatest one.

To give a comparison among the four distributions, we use the Criterion of AIC [4], and the Criterion of BIC [29]. The least values of these statistics are the better to fit the lifetime data, it is obvious that the SLR model gave a better fit than the other three pdfs. Table 4 provides statistics of Kolmogorov-Smirnov (KSS) together with so-called p-values which give us another certainty that the SLR model is the better.

Table 1: Bias and RMSE to a maximum likelihood estimators of α, θ, and q

$\alpha = 0.5,\ \theta = 0.1$										
q	n	$\hat{\alpha}$	Bias	RMSE	$\hat{\theta}$	Bias	RMSE	$\hat{q}$	Bias	RMSE
1	50	0.4485	0.0515	0.0216	0.086	0.014	0.0047	0.8269	0.1731	0.0255
	75	0.5383	-0.0383	0.0112	0.1123	-0.0123	0.0036	0.8946	0.1054	0.0242
	100	0.4628	0.0372	0.0109	0.1113	-0.0113	0.0032	0.9022	0.0978	0.0141
2	50	0.5655	-0.0119	0.0034	0.105	-0.005	0.0015	2.3424	-0.3424	0.0411
	75	0.5081	-0.0033	0.0034	0.1037	-0.0037	0.0009	2.1926	-0.1926	0.0389
	100	0.4967	0.0033	0.0008	0.1032	-0.0032	0.0007	2.1529	-0.1529	0.0386
3	50	0.4785	0.0215	0.0067	0.0928	0.0072	0.0026	2.2799	0.7015	0.0787
	75	0.5199	-0.0199	0.005	0.0934	0.0066	0.0022	3.1111	-0.1111	0.0508
	100	0.4851	0.0149	0.0049	0.0939	0.0061	0.0019	3.092	-0.092	0.0449
5	50	0.4654	0.0346	0.0102	0.0904	0.0096	0.0032	5.5886	-0.5886	0.2472
	75	0.529	-0.029	0.0084	0.1093	-0.0093	0.003	5.1326	-0.1326	0.2058
	100	0.4767	0.0233	0.0068	0.1077	-0.0077	0.0027	5.0612	-0.0612	0.0809

$\alpha = 0.5,\ \theta = 0.1$										
q	n	$\hat{\alpha}$	Bias	RMSE	$\hat{\theta}$	Bias	RMSE	$\hat{q}$	Bias	RMSE
1	50	0.3505	0.0505	0.0232	0.5103	0.0897	0.0311	0.8269	0.1731	0.0393
	75	0.3413	0.0413	0.0142	0.5159	0.0841	0.0287	0.8946	0.1054	0.0289
	100	0.2691	0.0309	0.0132	0.6676	0.0676	0.0219	0.9022	0.0978	0.0274
2	50	0.3119	0.0119	0.0034	0.6299	0.0299	0.0096	2.3424	0.3424	0.0489
	75	0.3393	0.0071	0.0033	0.6224	0.0224	0.0058	2.1926	0.1926	0.045
	100	0.2980	0.0020	0.0005	0.5978	0.0022	0.0010	2.1529	0.1529	0.0423
3	50	0.3174	0.0174	0.0055	0.6460	0.0460	0.0169	2.2985	0.7015	0.0886
	75	0.2860	0.0140	0.0045	0.5605	0.0395	0.0149	3.1111	0.1111	0.0683
	100	0.2871	0.0129	0.0045	0.5632	0.0368	0.0126	3.0920	0.0920	0.0554
5	50	0.3230	0.0230	0.0074	0.5326	0.0674	0.0212	4.0894	0.9106	0.4189
	75	0.3227	0.0227	0.0067	0.5425	0.0575	0.0179	5.2626	0.2626	0.2245
	100	0.2792	0.0208	0.0059	0.6559	0.0559	0.0175	5.1227	0.1227	0.089

$\alpha = 0.3,\ \theta = 0.6$										
q	n	$\hat{\alpha}$	Bias	RMSE	$\hat{\theta}$	Bias	RMSE	$\hat{q}$	Bias	RMSE
1	50	0.8971	0.1029	1.1232	0.8598	0.1402	1.1127	0.9022	0.0978	0.0255
	75	1.0766	-0.0766	1.1127	1.1232	-0.1232	0.0473	0.8269	0.1731	0.0242
	100	0.9256	0.0744	1.0932	1.1127	-0.1127	0.0361	0.8946	0.1054	0.0141
2	50	0.9307	0.0693	1.0767	1.0932	-0.1127	0.0323	2.1926	-0.1926	0.0411
	75	1.0579	-0.0579	1.0499	0.9042	0.0932	0.0322	2.1529	-0.1529	0.0389
	100	0.9535	0.0465	1.0373	1.0767	-0.0767	0.0301	2.3424	-0.3424	0.0386
3	50	0.9571	0.0429	1.0317	0.9282	0.0718	0.0274	3.0920	-0.0920	0.0787
	75	1.0398	-0.0398	0.9386	0.9342	0.0658	0.0258	2.2985	0.7015	0.0508
	100	0.9702	0.0298	0.9342	0.9386	0.0614	0.0193	3.1111	-0.1111	0.0449
5	50	1.1310	-0.0237	0.9282	1.0499	-0.0499	0.0146	5.5886	-0.5886	0.2472
	75	1.0162	-0.0162	0.9042	1.0373	-0.0373	0.0086	5.1326	-0.1326	0.2058
	100	0.9933	0.0067	0.8598	1.0317	-0.0317	0.0073	5.0612	-0.0612	0.0809

$\alpha = 3,\ \theta = 1.5$										
q	n	$\hat{\alpha}$	Bias	RMSE	$\hat{\theta}$	Bias	RMSE	$\hat{q}$	Bias	RMSE
1	50	3.5049	-0.5049	0.2222	1.2758	0.2242	0.0744	0.8269	0.1731	0.0267
	75	3.4125	-0.4125	0.1359	1.2896	0.2104	0.0687	0.8946	0.1054	0.0376
	100	2.6912	0.3088	0.1265	1.6690	-0.1690	0.0506	0.9022	0.0978	0.0253
2	50	3.1194	-0.1194	0.0313	1.5749	-0.0749	0.0230	2.3424	-0.3424	0.0469
	75	3.3930	-0.0712	0.0309	1.5560	-0.0560	0.0134	2.1926	-0.1926	0.0430
	100	2.9799	0.0201	0.0048	1.4946	0.0054	0.0024	2.1529	-0.1529	0.0405
3	50	3.1738	-0.1738	0.0525	1.6151	-0.1151	0.0404	2.2985	0.7015	0.0822
	75	2.8604	0.1396	0.0428	1.4013	0.0987	0.0345	3.1111	-0.1111	0.0630
	100	2.8712	0.1288	0.0417	1.4079	0.0921	0.0303	3.0920	-0.0920	0.0531
5	50	3.2299	-0.2299	0.0705	1.3315	0.1685	0.0505	4.0894	0.9106	0.4007
	75	3.2265	-0.2265	0.0637	1.3563	0.1437	0.0429	5.2626	-0.2626	0.2151
	100	2.7922	0.2078	0.0544	1.6399	-0.1399	0.0405	5.1227	-0.1227	0.0847

Table 2: Descriptive statistics of data

Summary data

Min	1st Qu	Median	Mean	3rd Qu.	Max.	skewness	kurtosis	SD
1.39	5.25	6.53	7.19	8.67	17.49	0.74	3.29	3.54

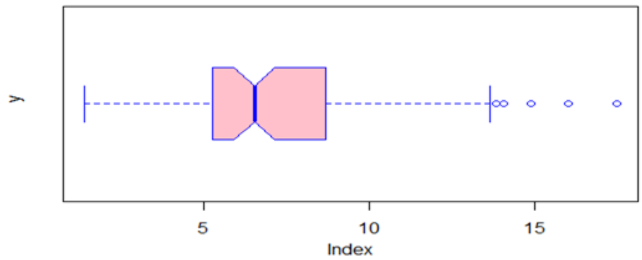

Figure 4: Box plot of data

Table 3: Lifetime data, ML estimates, SD, confidence limits (L,U,D),log L ,AIC, and BIC for the fitted models

Dist.	Parameter	MLE	SD	L	U	D	Log L	AIC	BIC
SLR	$\hat{\theta}$	0.1426	0.0035	0.0851	0.1996	0.1145	284.64	-563.29	-556.33
	$\hat{\alpha}$	2.6571	0.0501	2.5985	2.7161	0.1176			
	$\hat{q}$	4.8223	0.1777	4.7627	4.8853	0.1226			
LR	$\hat{\theta}$	0.1826	0.0435	0.1202	0.2453	0.1251	212.53	-421.07	-416.43
	$\hat{\alpha}$	2.4971	0.1099	2.4341	2.5607	0.1266			
L	$\hat{\alpha}$	0.3140	0.1749	0.2504	0.3782	0.1278	199.25	-396.50	-394.18
R	$\hat{\alpha}$	0.3588	0.2198	0.2940	0.4243	0.1303	203.05	-404.09	-401.77

Table 4: Lifetime data, K-S statistic and p- values

sample Kolmogorov-Smirnov test

Distribution	SLR	LR	R	L
K.S. Statistic	0.085619	0.095212	0.12833	0.10713
p-value	0.6107	0.4759	0.1549	0.3316

6 Conclusions

In this study, we introduce an extension of the LR model for the purpose of getting a model that exhibits flexibility to model lifetime observations with surplus kurtosis. The new model is formulated like a quotient of 2- r.vs., the first variable (numerator) represents the Lindley-Rayleigh (LR) model and the second one (denominator) represents an exponent of the uniform model. The proposed pdf gives us the ability to accommodate positive data exhibiting high kurtosis besides outliers since the measurements of skewness and kurtosis show a large or broad interval of values. We discuss the characteristics of this model (SLR) including moments, skewness and kurtosis coefficients, geometric mean, reliability and hazard rate functions. Parameters of the SLR model have been estimated depending on the following MLE approach The importance, performance, and priority of SLR distribution have been illustrated by using the Monte Carlo technique and a real lifetime sample. The yielded calculations show that the new model gives advantage fits with comparison to the other existing probability density functions.

References

[1] Ş. Acıtaş, T. Arslan, and B. Şenoğlu, *Slash Maxwell Distribution: Definition, Modified Maximum Likelihood Estimation and Applications*, Gazi Univer. J. Sci. (2020).

[2] A. Ahmad, S.P. Ahmad, and A. Ahmed, *Transmuted inverse Rayleigh distribution: A generalization of the inverse Rayleigh distribution*, Math. Theory Model. **4** (2014), no. 7, 90–98.

[3] M.T. Ahmed, M.A. Khaleel, and E.Kh. Khalaf, *The new distribution (Topp Leone Marshall Olkin-Weibull) properties with an application*, Period. Engin. Natural Sci. **8** (2020), no. 2, 684–692.

[4] H. Akaike, *A new look at the statistical model identification*, IEEE Trans. Automatic Control **19** (1974), no. 6, 716–723.

[5] R.S. AL-Saffar and M.W. Neamah, *On a new double weighted exponential-pareto distribution: Properties and estimation*, Period. Engin. Natural Sci. **10** (2022), no. 1, 330–342.

[6] H.S. Bakouch, B.M. Al-Zahrani, A.A. Al-Shomrani, V.A.A. Marchi, and F. Louzada, *An extended Lindley distribution*, J. Korean Statist. Soc. **41** (2012), no. 1, 75–85.

[7] I. Barranco-Chamorro, Y.A. Iriarte, Y.M. Gómez, J.M. Astorga, and H.W. Gómez, *A generalized Rayleigh family of distributions based on the modified slash model*, Symmetry **13** (2021), no. 7, 1226.

[8] S. Cakmakyapan and O. Gamze, *The Lindley family of distributions: properties and applications*, Hacettepe J. Math. Statist. **46** (2016), no. 6, 1113–1137.

[9] M.A. Chaudhry and S.M. Zubair, *Generalized incomplete gamma functions with applications*, J. Comput. Appl. Math. **55** (1994), no. 1, 99–123.

[10] M. Dwiki, S. Nurrohmah, and M. Novita, *Lindley–exponential slash distribution*, vol. 1725, IOP Publish., 2021, p. 012093.

[11] D.D. Dyer and C.W. Whisenand, *Best linear unbiased estimator of the parameter of the Rayleigh distribution-Part I: Small sample theory for censored order statistics*, IEEE Trans. Reliab. **22** (1973), no. 1, 27–34.

[12] A.J. Fernández, *Bayesian estimation and prediction based on Rayleigh sample quantiles*, Qual. Quant. **44** (2010), no. 6, 1239–1248.

[13] M.E. Ghitany, B. Atieh, and S. Nadarajah, *Lindley distribution and its application*, Math. Comput. Simul. **78** (2008), no. 4, 493–506.

[14] H.W. Gómez, J.F. Olivares-Pacheco, and H. Bolfarine, *An extension of the generalized Birnbaum–Saunders distribution*, Statist. Probab. Lett. **79** (2009), no. 3, 331–338.

[15] H.W. Gómez, F.A. Quintana, and F.J. Torres, *A new family of slash-distributions with elliptical contours*, Statist. Probab. Lett. **77** (2007), no. 7, 717–725.

[16] H.W. Gómez and O. Venegas, *Erratum to:"A new family of slash-distributions with elliptical contours"[Statist. Probab. Lett. 77 (2007) 717–725]*, Statist. Probab. Lett. **78** (2008), no. 14, 2273–2274.

[17] A.F. Hameed and I.M. Alwan, *Bayes estimators for reliability and hazard function of Rayleigh-Logarithmic (RL) distribution with application*, Period. Engin. Natural Sci. **8** (2020), no. 4, 1991–1998.

[18] Y.A. Iriarte, H.W. Gómez, H. Varela, and H. Bolfarine, *Slashed rayleigh distribution*, Rev. Colombiana Estadíst. **38** (2015), no. 1, 31–44.

[19] F. Kanfer and S. Millard, *Beta slashed generalised half-normal distribution*, South Afr.Statist. J. **51** (2017), no. 2, 329–343.

[20] D.V. Lindley, *Fiducial distributions and Bayes' theorem*, J. Royal Statist. Soc. Ser. B (1958), 102–107.

[21] J. Mazucheli and J.A. Achcar, *The Lindley distribution applied to competing risks lifetime data*, Comput. Meth. Prog. Biomed. **104** (2011), no. 2, 188–192.

[22] F/ Merovci and I/ Elbatal, *Weibull Rayleigh distribution: Theory and applications*, Appl. Math. Inf. Sci/ **9** (2015), no. 5, 1–11.

[23] J.F. Olivares-Pacheco, H.C. Cornide-Reyes, and M. Monasterio, *An extension of the two-parameter Weibull distribution*, Rev. Colombiana Estadíst. **33** (2010), no. 2, 219–231.

[24] N.M. Olmos, H. Varela, H. Bolfarine, and H.W. Gómez, *An extension of the generalized half-normal distribution*, Statist. Papers **55** (2014), no. 4, 967–981.

[25] N.M. Olmos, H. Varela, H.W. Gómez, and H. Bolfarine, *An extension of the half-normal distribution*, Statist. Papers **53** (2012), no. 4, 875–886.

[26] N.M. Olmos, O. Venegas, Y.M. Gomez, and Y.A. Iriarte, *Confluent hypergeometric slashed-Rayleigh distribution: Properties, estimation and applications*, J. Comput. Appl. Math. **368** (2020), 112548.

[27] J. Reyes, O. Venegas, and H.W. Gómez, *Modified slash Lindley distribution*, J. Probab. Statist. **2017** (2017).

[28] H.S. Salinas, Y.A. Iriarte, and H. Bolfarine, *Slashed exponentiated rayleigh distribution*, Rev. Colombiana Estadíst. **38** (2015), no. 2, 453–466.

[29] G. Schwarz, *Estimating the dimension of a model*, Ann. Statist. **6** (1978), no. 2, 461–464.

[30] R.C. Team, *R: A language and environment for statistical computing*, (2013), Available online: https://www.r-project.org (accessed on 24 January 2020).