Journal of Heat and Mass Transfer Research 9 (2022) 189 - 196

Semnan University

Journal of Heat and Mass Transfer Research

Journal homepage: http://jhmtr.semnan.ac.ir

Fluid Analysis of Double-Layered Blood Flow through a Tapered Overlapping Stenosed Artery with a Porous Wall

Moses Sunday Dada[a], Abiodun Joseph Babatunde[*,a], Mohammed Muritala Tunde[b]

[a] *Department of Mathematics, University of Ilorin, Ilorin, Nigeria*

[b] *School of Early Childhood Care and Primary Education, Federal College of Education (T) Potiskum, Yobe State, Nigeria*

PAPER INFO

Paper history:

Received: 2021-10-04

Received: 2023-02-02

Accepted: 2023-02-03

Keywords:

Porous wall;
Resistance to flow;
Wall Shear Stress;
Slip parameter;
Darcy number.

ABSTRACT

In this present work, we examine the fluid of double-layered blood flow through a tapered overlapping stenosed artery with a porous wall. This two-layered blood flow problem comprises the peripheral layer as Newtonian fluid flows and the central core layer of suspension of the erythrocytes as another Newtonian fluid flows and was analytically solved which the numerical results are shown graphically and discussed. It was found that resistance to flow accelerates with rising slip parameter, blood viscosity, and artery length while a rise in Darcy number and radius of the centre core to the tube radius in the unobstructed region decreases the resistance to flow.

Also, the resistance to flow rises with increasing stenosis height whereas it increases with a rise in values of artery shape. The wall shear stress drops as the Darcy number accelerates and rises with rising viscosity of the blood and slip parameter. Furthermore, fluctuation of wall shear stress at the neck of the stenosis drops as the Darcy number increases. Moreover, it is observed that the shear stress increases with rising viscosity of the blood and slip parameter. This work is able to forecast the major attribute of the physiological flows which have played an important role in biomedical researches.

DOI: 10.22075/jhmtr.2023.24296.1352

1. Introduction

The vascular system is an organ system that allows blood to transport nutrients and circulate oxygen, hormones, blood cells, carbon-II-oxide to and fro the cells in the system of the body to provide nourishment and preventing stabilization of temperature, diseases and sustain homeostasis. Atherosclerosis is a process of continuous thickening and hardening of the walls of medium-sized and large blood vessels due to the deposition of fat on their inner lining. Stenosis growth is responsible for many coronary artery diseases such as strokes, heart attack, peripheral vascular diseases, and death in the world at large. Because of the dangers involved, it is very vital to look forward for the symptoms that cause blockage of arteries, like the Transient Ischemic Attack and stroke, so that adequate measures may be observe before the situation gets worst. The importance of the hemodynamic factors play a vital role in the beginning and the development of atherosclerosis which drew the attention of Mann et al. [1]. Stenosis proliferation conditions under various flow situations have been addressed by a number of researchers such as Asha et al. [2] that studied the geometry of stenosis and its effects on the blood flow through an artery and Srivastava [3] said that the significance of the peripheral layer accelerates as blood vessel diameter decelerating., Arun, [4] examined the mathematical modelling on blood flow under atherosclerotic condition. Two-layered model of blood flow through composite stenosed artery was investigated by Padma et al [5] they observed that existence of peripheral layer is useful in representation of diseased arterial system.

Medhavi [6-8] examined two-phase arterial blood flow through a composite stenosis. A macroscopic two-phase blood flow through a bell-shaped stenosis in an artery with a permeable wall was studied by Srivastav et al., [9], Babatunde and Dada [10] investigated the effects of hematocrit level on wall shear stress and flow resistance in a tapered and overlapping stenosed

[*] *Corresponding Author: Babatunde Abiodun Joseph*
Email: babatundeabiodunjoseph@gmail.com

artery with porosity. they observed that the resistance to flow increases with increase in either stenosis height or artery shapes while the influence of hematocrit level has slight decrease. The two-layered (K.L-Newtonian) model of blood flow in an artery with six types of mild stenoses was examined by Ponalagusamy et al. [11]. Puskar R. Pokhrel, [12] studied the analysis of two-layered blood flow through artery with mild stenosis and observed that the pressure gradient of the blood flow increases in the ratio of thickness of stenosis with radius of artery.

Eldesoky et al. [13] studied the numerical study of unsteady MHD pulsatile flow through a porous medium in an artery using Generalized Differential Quadrature Method and several scholars. The flowing of the blood has been taken as Newtonian fluid, non-Newtonian fluid, single or two-layered fluid flows by different researchers while studying the flows through atherosclerosis. It's obvious that blood flow can be taken as a one-layered model in a big vessel. However, the flow through the small artery is double-layered. Bugliarello and Sevilla [14] examined the velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes and Titiloye et al. [15] studied the mathematical modelling of two-layered blood flow through a tapered artery with an overlapping stenotic condition and observed that there is a cell-free plasma layer, for blood flowing through small arteries and a core region of suspension of all the erythrocytes.

In a case of overlapping, there is a suturing of a layer of tissue above or under another in order to add more strength. Chakravarty and Mandal, [16] studied mathematical modelling of blood flow in overlapping arterial stenosis and observed that the flux decreases as the resistive impedance decreasing out of the stenotic flow in vivo, the gravity of the overlapping stenosis affects the resistive impedance seriously, also the wall shear stress accelerates as the amplitude of the pressure gradient drops. Sapna et al., [17] examined mathematical modelling of blood flows in a three-layered stenosed artery, said that the resistance to flow and wall shear stress are significantly very low for the two-fluid non-Newtonian model than those of the two-fluid model.

The endothelial walls are said to be highly porous with ultramicroscopic pores through that filtration occurs and fat is believed to rise the porosity of the blood vessel wall. Such a rise in permeability results from damaged, dilated or inflamed vessel walls. As such Rupesh et al [18], examined double-fluid blood flows in the stenosed artery with a permeable wall.

The study, therefore, considers the combined effects of a tapered and overlapping stenosed artery with a porous wall on the double-layered blood flows. This problem considers the flowing blood as a double-layered Newtonian flow, comprises of a core region of suspension of all the erythrocytes taking to be another Newtonian fluid flows, the viscosity of which can vary depending on the flow situations and a peripheral region of Newtonian fluid flows of constant viscosity, in a vessel that the wall is porous. The problems were solved analytically.

2. Formulation of the problem

The laminar, incompressible, and Newtonian double-layered flow of blood, comprises of a central core layer of red cell suspensions in plasma of radius R_1 and a peripheral plasma layer of a thickness $(R-R_1)$, through axisymmetric one-dimensional tapered and overlapping stenosed artery is being examined. The cylindrical polar coordinate (r, θ, z), is used for any point in the fluid, where z is measured along the axis of the artery and r and θ along the radial and circumferential directions respectively.

The mathematical representation that corresponds to the geometry of this present work is expressed after Titiloye et al., [15] as:

$$\frac{R(z)}{R_0} = \begin{cases} \left(\frac{mz}{R_0}+1\right) - \frac{\delta cos\varphi}{R_0 L_0}(z-d)\left\{11 - \frac{94}{3L_0}(z-d) + \frac{32}{\mathrm{L}_0^2}(z-d)^2 - \frac{32}{3\mathrm{L}_0^3}(z-d)^3\right\}, & d \le z \le d + \frac{3L_0}{2} \\ \left(\frac{z}{R_0}+1\right), & otherwise \end{cases} \tag{1}$$

$$\frac{R_1(z)}{R_0} = \begin{cases} \left(\frac{mz}{R_0}+1\right)\beta - \frac{\delta cos\varphi}{R_0 L_0}(z-d)\left\{11 - \frac{94}{3L_0}(z-d) + \frac{32}{\mathrm{L}_0^2}(z-d)^2 - \frac{32}{3\mathrm{L}_0^3}(z-d)^3\right\}, & d \le z \le d + \frac{3L_0}{2} \\ \left(\frac{mz}{R_0}+1\right)\beta, & otherwise \end{cases} \tag{2}$$

where R(z), R_0 represent the radius of the tapered arterial segment in the constricted region and the constant radius of the normal artery in the non-stenotic region respectively, φ represents the angle of tapering, $\frac{3L_0}{2}$ is the length of overlapping stenosis, d denotes the location of the stenosis, δcosφ is taken to be the critical height of the overlapping stenosis, β is the ratio of the central core radius to the tube radius outside the stenotic region and m=tanφ represents the slope of the tapered vessel. In order to exploring the feasibility of the different shapes of the artery, which classified as diverging tapering (φ>0), non-tapered artery (φ = 0), and converging tapering (φ<0).

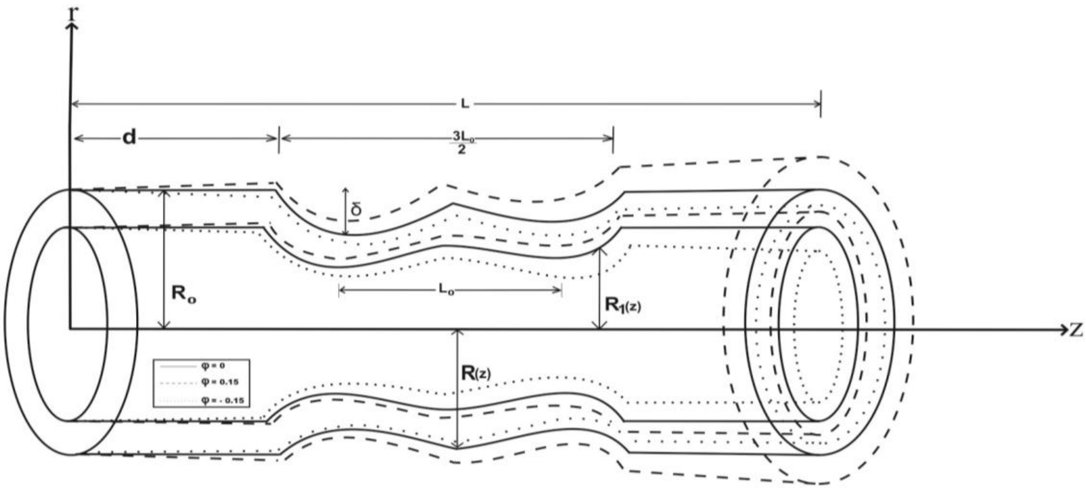

Figure 1. Geometry of a two-layered overlapping stenosed artery [15]

3. Mathematical Formulation

The equations of motion for one-dimensional fluid flow, steady, laminar in the case of stenosed artery ($\delta \ll R_0$) are described by Sharanet al. [19] as

$$\frac{dp}{dz} = \frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)w_p, \quad R_1(z) \leq r \leq R(z), \tag{3}$$

$$\frac{dp}{dz} = \frac{\mu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)w_c, \qquad 0 \leq r \leq R_1(z), \tag{4}$$

where (r, z) are the radial and axial coordinates in the two-dimensional cylindrical polar coordinate system and $\frac{dp}{dz}$ denotes the pressure gradient (μ_p, w_p) and (μ_c, w_c) represents the viscosity and velocity of the fluid flows in the peripheral layer $(R_1(z) \leq r \leq R(z))$ and central layer $(0 \leq r \leq R_1(z))$, respectively.

The corresponding boundary conditions given by (Beavers, [20]) for the present problem can be expressed by [18] as

$$\frac{\partial w_c}{\partial r} = 0 \quad at \;\; r = 0 \tag{5a}$$

$$w_p = w_c \;\; and \;\; \mu_p\frac{\partial w_p}{\partial r} = \mu_c\frac{\partial w_c}{\partial r} \;\; at \;\; r = R_1(z) \tag{5b}$$

$$w_p = w_B \;\; and \;\; \frac{\partial w_p}{\partial r} = \frac{\alpha}{\sqrt{k}}\left(w_B - w_f\right) at \; r = R(z) \tag{5c}$$

where $\boldsymbol{w_f} = -\frac{\boldsymbol{k}}{\boldsymbol{\mu_p}}\frac{\boldsymbol{dp}}{\boldsymbol{dz}}$ represents the velocity in the porosity boundary, $\boldsymbol{\mu_p}$ denotes the plasma viscosity of the fluid in a peripheral layer, α is the slip parameter, k is the Darcy number and $\boldsymbol{w_B}$ is the slip velocity, are dimensionless quantities depending on the material within the boundary region.

4. Analysis

Equation (3) is integrating with respect to r, applying the boundary condition (5a), we have

$$\frac{\partial w_p}{\partial r} = \frac{r}{2\mu_p}\frac{dp}{dz} \tag{6}$$

Also, Equation (6) is integrating with the aid of boundary condition (5b), we have

$$w_p = \frac{r^2}{4\mu_p}\frac{dp}{dz} - \frac{R^2}{4\mu_p}\frac{dp}{dz} + w_B \tag{7}$$

Using boundary condition (5c) on Equation (6), we have

$$w_B = \frac{\sqrt{k}}{2\alpha\mu_p}\frac{dp}{dz}\left[R - 2\alpha\sqrt{k}\right] \tag{8}$$

Substituting Equation (8) into Equation (7), we have

$$w_p = \frac{r^2}{4\mu_p}\frac{dp}{dz} - \frac{R^2}{4\mu_p}\frac{dp}{dz} + \frac{\sqrt{k}}{2\alpha\mu_p}\frac{dp}{dz}\left[R - 2\alpha\sqrt{k}\right]$$

Hence,

$$w_p = -\frac{R_0^2}{4\mu_p}\frac{dp}{dz} \times\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 - 2\left(\frac{R}{R_0}\right)\left(\frac{\sqrt{k}}{\alpha R_0}\right) + \frac{4k}{R_0^2}\right\} \tag{9}$$

Similarly, from Equation (4) following the same processes, we have

$$w_c = -\frac{R_0^2}{4\mu_p}\frac{dp}{dz} \times\left\{\mu\left[\left(\frac{R_1}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2\right] - 2\left(\frac{R}{R_0}\right)\left(\frac{\sqrt{k}}{\alpha R_0}\right) + \frac{4k}{R_0^2}\right\} \tag{10}$$

where $\mu = \frac{\mu_p}{\mu_c}$ and μ_c is the viscosity of the blood flows in the central core layer.

The flux flow rate is given as

$$Q = 2\pi\left\{\int_0^{R_1} r w_c dr \int_{R_1}^{R} r w_p dr\right\} \tag{11}$$

After simplifying Equation (11), it becomes

$$Q = -\frac{\pi R_0^4}{8\mu_p}\frac{dp}{dz}\left\{\left(\frac{R_1(z)}{R_0}\right)^4(1+\mu) + \left(\frac{R(z)}{R_0}\right)^4 - 2\left(\frac{R(z)}{R_0}\right)^2\left(\frac{R_1(z)}{R_0}\right)^2 + \frac{8k}{R_0^2}\left(\frac{R(z)}{R_0}\right)^2 - \left(\frac{R(z)}{R_0}\right)^3\frac{\sqrt{k}}{R_0\alpha}\right\} \tag{12}$$

Therefore, the pressure gradient from Equation (12) is

$$\frac{dp}{dz} = -\frac{8\mu_p Q}{\pi R_0^4 F(z)} \tag{13}$$

where

$$F(z) = \left\{\left(\frac{R_1(z)}{R_0}\right)^4(1+\mu) + \left(\frac{R(z)}{R_0}\right)^4 - 2\left(\frac{R(z)}{R_0}\right)^2\left(\frac{R_1(z)}{R_0}\right)^2 + \frac{8k}{R_0^2}\left(\frac{R(z)}{R_0}\right)^2 - \left(\frac{R(z)}{R_0}\right)^3\frac{\sqrt{k}}{R_0\alpha}\right\}$$

Integrating Equation (13) along the length of the artery, we have

$$\int_{p_0}^{p_1} dp = \int_0^L -\frac{8\mu_p Q}{\pi R_0^4 F(z)}dz \tag{14a}$$

where $p_0$and $p_1$are the pressures at z=0 and z=L respectively.

$$p_1 - p_0 = -\frac{8\mu_p Q}{\pi R_0^4}\left\{\int_0^d \sigma(z)\,dz + \int_d^{d+\frac{3L_0}{2}} \sigma(z)\,dz + \int_{d+\frac{3L_0}{2}}^L \sigma(z)\,dz\right\} \tag{14b}$$

where $\sigma(z) = \frac{1}{F(z)}$

The resistance to flow λ given by (Malek et al. [21] and Babatunde and Dada, [10]) as

$$\lambda\lambda = \frac{p_1 - p_0}{QQ}$$

Hence,

$$\lambda = -\frac{8\mu_p}{\pi R_0^4}\left\{\int_0^d \sigma(z)\,dz + \int_d^{d+\frac{3L_0}{2}} \sigma(z)\,dz + \int_{d+\frac{3L_0}{2}}^L \sigma(z)\,dz\right\} \tag{15a}$$

The stenosis is present in the region $d \le z \le d + \frac{3L_0}{2}$. If there is no stenosis $\frac{\mathrm{R(z)}}{\mathrm{R_0}} = (\frac{mz}{R_0} + 1)$ and $\frac{R_1(z)}{R_0} = (\frac{mz}{R_0} + 1)\beta$ from Equations (1) and (2) respectively. Therefore,

$$\lambda = -\frac{8\mu_p}{\pi R_0^4}\left\{\int_0^d \zeta(z)\,dz + \int_d^{d+\frac{3L_0}{2}} \sigma(z)\,dz + \int_{d+\frac{3L_0}{2}}^L \zeta(z)\,dz\right\} \tag{15b}$$

where

$$\zeta(z) = \frac{1}{\left\{\left((\frac{mz}{R_0}+1)\beta\right)^4(1+\mu) + \left(\frac{mz}{R_0}+1\right)^4 - 2\left(\frac{mz}{R_0}+1\right)^2\left((\frac{mz}{R_0}+1)\beta\right)^2 + \frac{8k}{R_0^2}\left(\frac{mz}{R_0}+1\right)^2 - \left(\frac{mz}{R_0}+1\right)^3\frac{\sqrt{k}}{R_0\alpha}\right\}}$$

The resistance to flow for Newtonian fluid flow when there is no stenosis ($\delta = 0$) is given by

$$\lambda_N = -\frac{8\mu_p}{\pi R_0^4}\int_0^L \psi(z)dz \tag{16}$$

where

$$\psi(z) = \left[\left(\frac{mz}{R_0}+1\right)^4 + \frac{8k}{R_0^2}\left(\frac{mz}{R_0}+1\right)^2 - \left(\frac{mz}{R_0}+1\right)^3\frac{\sqrt{k}}{R_0\alpha}\right]$$

Thus, the dimensionless resistance to flow may be expressed as

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} \tag{17}$$

The wall shear stress may be expressed as

$$\tau_s = -\frac{R}{2}\frac{dp}{dz} = \frac{4\mu_p Q\left(\frac{R}{R_0}\right)}{\pi R_0^3 F(z)} \tag{18}$$

The wall shear stress for Newtonian fluid flow when there is no stenosis ($\delta = 0$) is given as

$$\tau_N = \frac{4\mu_p Q\left(\frac{mz}{R_0}+1\right)}{\pi R_0^3\psi(z)} \tag{19}$$

Thus, the wall shear stress in non-dimension form is expressed as

$$\bar{\tau} = \frac{\tau_s}{\tau_N} \tag{20}$$

The wall shear stress at the neck of the stenosis is define as

$$\tau_{wm} = \left[\frac{4\mu\mu_c Q\left[\left(\frac{mz}{R_0}+1\right) - \frac{\delta}{R_0}\right]}{\pi R_0^3\eta}\right] \tag{21}$$

where

$$\eta = \left[\left(\left(\frac{mz}{R_0}+1\right) - \frac{\delta}{R_0}\right)^4 + \frac{8k}{R_0^2}\left(\left(\frac{mz}{R_0}+1\right) - \frac{\delta}{R_0}\right)^2 - \left(\left(\frac{mz}{R_0}+1\right) - \frac{\delta}{R_0}\right)^3\frac{\sqrt{k}}{R_0\alpha}\right]$$

Thus, the dimensionless wall shear stress at the neck of the stenosis is given as

$$\bar{\tau}_m = \frac{\tau_{wm}}{\tau_N} \tag{22}$$

The analytical solution of the second integral on the right-hand side of Equation (15b) is a tedious work which can be solved by numerical method, while the solution of the first and third integrals is easier. Equations (15b) - (22) can be use to determine the resistance to flow and the wall shear stress in the stenosed artery.

5. Numerical results and discussion

In order to explode this present work, the results are displayed graphically. In this section, fluid analysis of two-layered blood flow through a tapered overlapping stenosed artery with porosity had been shown. The values of the parameters are considered with its range (Titiloye et al. [15] and Babatunde and Dada, [10]) as

Q=0.1, L=2.5, $L_0 = 1.0$, d=0.5, $R_0 = 1$,

$\frac{\delta}{R_0} = 0.1 - 0.5$, $k = 0.1 \; to \; 0.2$, $\alpha = 0.1 \; to \; 0.2$,

β=0.90−0.96 and μ= 0.1−0.3, φ= − 0.15, 0.00 and 0.15.

Figure 2 shows the resistance to flow against stenosis height with artery shape for several values of slip parameter. Revealed that resistance to flow accelerates with a rise in the slip parameter.

Figure 3 represents the resistance to flow against stenosis height with artery shape for several values of Darcy number. It was observed that resistance to flow decreases with increasing the Darcy number.

Figure 4 displays the resistance to flow against stenosis height with artery shape for several values of blood viscosity. It depicts that resistance to flow rises with increasing the blood viscosity.

Figure 5 displays the resistance to flow against stenosis height for several values of artery shapes. It revealed that resistance to flow increases with a rise in the slip parameter and Its increases with a rise in values of artery shape φ.

Figure 6 represents the resistance to flow against stenosis height with artery shape for several values of β. Depicts that resistance to flow reduces with a rise in the value of β.

Figure 7 shows the resistance to flow against stenosis height with artery shape for several values of artery length. Reveals that resistance to flow increases with increases in the artery length.

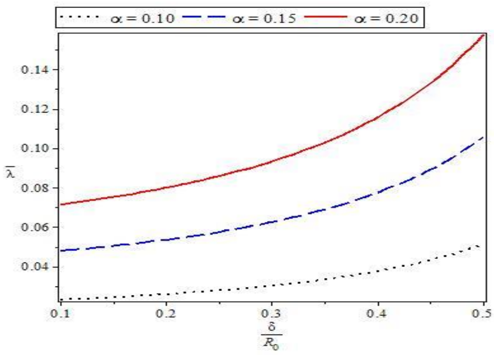

Figure 2. Resistance to flow for several values of slip parameter at an angle φ=0.15.

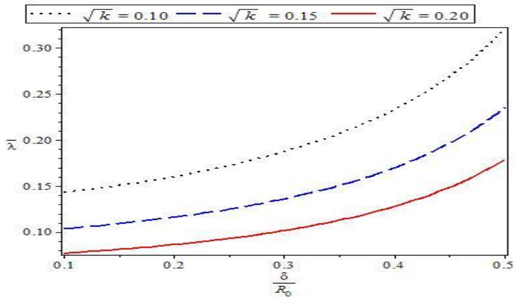

Figure 3. Resistance to flow for several values of Darcy number at an angle φ=0.1

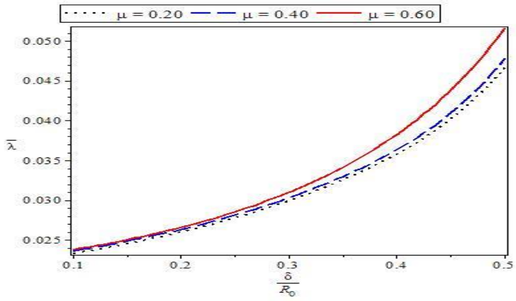

Figure 4. Resistance to flow for several values of viscosity of the blood at an angle $\varphi = 0.15$

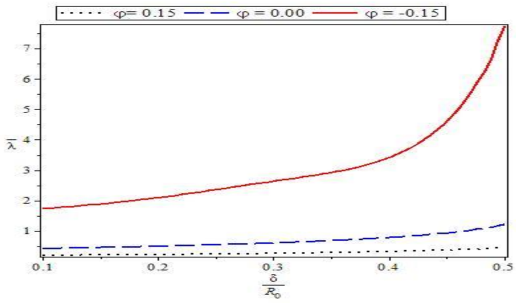

Figure 5. Resistance to flow for several values of artery shape

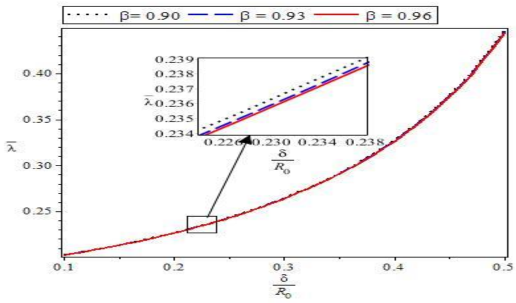

Figure 6. Resistance to flow for several values of β at an angle φ=0.15.

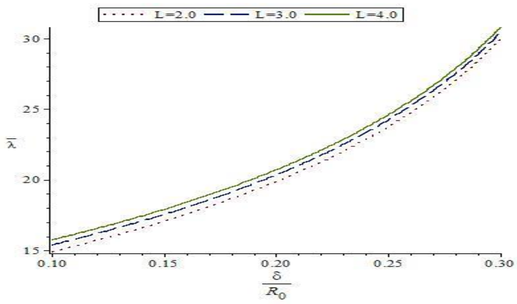

Figure 7. Resistance to flow for several values of artery length at an angle φ=0.15.

Figure 8 represents the wall shear stress for several values of slip parameters. It depicts that wall shear stress rises with a rise in the slip parameter.

Figure 9 displays the wall shear stress for several values of Darcy number. It is found that shear stress reduces with a rise in the value of Darcy number.

Figure 10 shows the wall shear stress for several values of stenosis height. It depicts that wall shear stress rises with increasing the value of stenosis height at the stenosis region while the wall shear stress drops with a rise in the value of stenosis height where there is no stenosis.

Figure 11 displays the wall shear stress for several values of blood viscosity. It depicts that shear stress rises as the value of viscosity of blood increases.

Figure 12 represents wall shear stress at the neck of the stenosis for different values of Darcy number. It depicts that increasing Darcy number reduces the fluctuation of the wall shear stress at the throat of stenosis.

Figure 13 represents the fluctuation of wall shear stress at the neck of the stenosis for several values of slip parameter. It displays that as slip parameter increases, the fluctuation of the shear stress at the neck of stenosis rises.

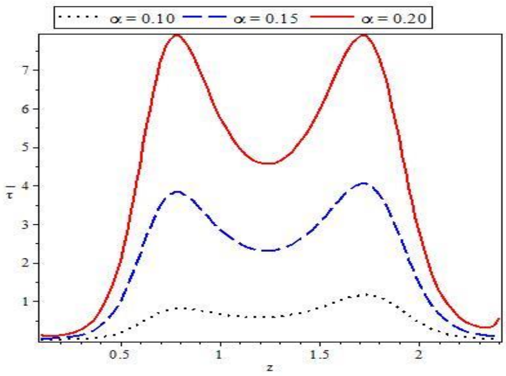

Figure 8. Wall shear stress for several values of slip parameter at an angle φ=0.15.

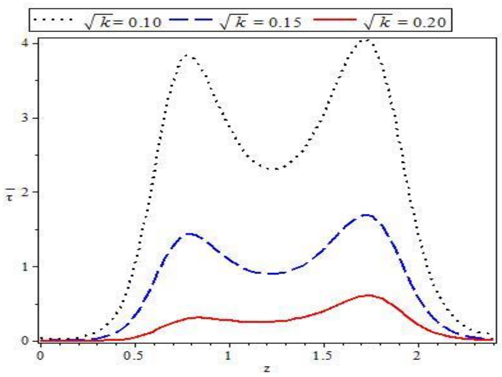

Figure 9. Wall shear stress for several values of Darcy number angle φ=0.15.

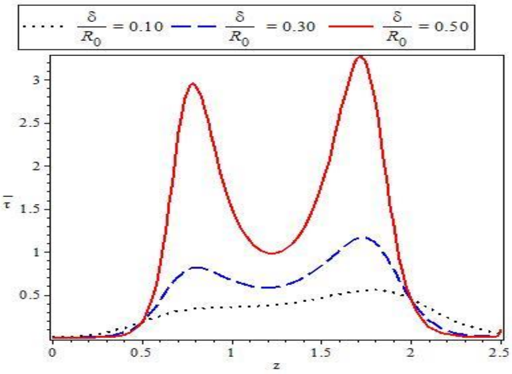

Figure 10. Wall shear stress for several values of stenosis height at an angle φ=0.15.

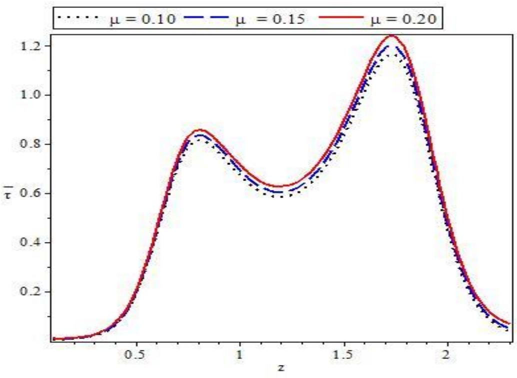

Figure 11. Wall shear stress for several values of viscosity of blood at an angle φ=0.15.

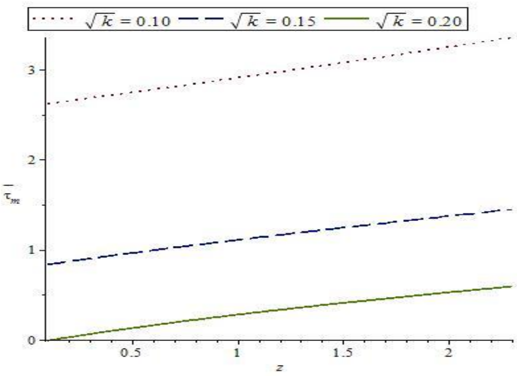

Figure 12. Wall shear stress at the neck of the stenosis for several values of Darcy number at φ=0.15.

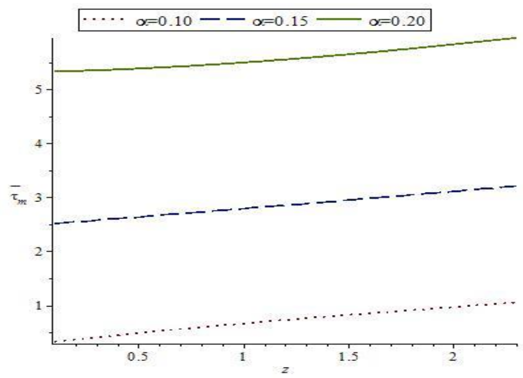

Figure 13. Fluctuation of shear stress at the neck of the stenosis for several values of slip parameter at φ=0.15.

Conclusion

The fluid analysis of two-layered blood flow through a tapered overlapping stenosed artery with permeable wall was determined in this present work. This double-layered blood flow comprises the peripheral layer fluid flows and the central core layer of suspension of the erythrocytes as Newtonian fluid flows and was solved analytically. Different fluid parameters were introduced to analyse the effects of slip parameter, Darcy number, blood viscosity, artery shape and stenosis height on the resistance to the flow and the wall shear stress of the blood. The results have been studied in the case of tapered overlapping stenosed artery are shown as follow:

i. It is found that resistance to flow increases with increasing slip parameter, blood viscosity, artery length. It occurs due to influence of slip parameter on blood flow increases the internal viscosity of the blood flow which causes rise to the Lorentz force.

ii. The influence of β and Darcy number decreases the resistance to flow.

iii. Fluctuation of wall shear stress at the neck of the stenosis decreases as Darcy number increases.

iv. We found that shear stress rises as viscosity of the blood increasing and slip parameter. However, we observed that the presence of peripheral layer in a porous artery aids the functioning of the diseased artery.

Nomenclature

All variables using this manuscript, listed in nomenclature.

(r, θ, z)	Cylindrical polar coordinate system
L	Length of the arterial segment
w_f	velocity in the porosity boundary
w_{B}	Slip velocity
μ_{c}	Viscosity of the blood flows in the central core layer
μ_{p}	Viscosity of the blood flows in the peripheral layer
Q	Volumentric flow flux
w_{c}	Axial velocity of the blood flows in the central core layer
w_{p}	Axial velocity of the blood flows in the peripheral layer
α	Slip parameter
τ	Wall shear stress
τ_s	Wall shear stress at the maximum height of the stenosis
τ_N	Wall shear stress in the absence of stenosis
τ_{wm}	Wall shear stress at the neck of stenosis
$\bar{\tau}_m$	Wall shear stress in dimensionless form
β	Ratio of center core radius to the tube radius outside the stenotic region
m	Slope of the tapered vessel
k	Darcy number
R(z)	Radius of tapered artery in the region of stenosis
R_0	Radius of the non-tapered artery in the non-stenotic region
R_1	Radius of the plasma
L_0	Length between throat of two stenosis
φ	Tapered angle
δ	Maximum height of the stenosis at some location z
λ_N	Resistance to flow with no stenosis
r	Radius of the artery
P	Pressure
d	Location of the stenosis
λ	Resistance to flow
$\frac{dp}{dz}$	Pressure gradient
$\delta\cos\phi$	Critical height of the overlapping stenosis
z	Axial distance

Acknowledgements

The authors gratefully acknowledge the reviewers' comments that improved the quality of the article.

References

[1] Mann F. C., Herrick J. F., Essex H. E., and Blades E. J., 1938. Effects on blood flow of decreasing the lumen of blood vessels, Surgery, 4, 249–252.

[2] Asha K. N. and Neetu Srivastava, 2021. Geometry of Stenosis and Its Effects on the Blood Flow Through an artery-A Theoretical Study, AIP Conference Proceedings, 2375, Issue 1: 10.1063/5.0066510.

[3] Srivastava V.P., Rastogi R. and Vishnoi R., 2010. A two-layered suspension blood flow through an overlapping stenosis, Computers and Mathematics with Applications, 60, 432–441.

[4] Arun Kumar Maiti, 2016. Mathematical Modelling on Blood Flow Under-

Atherosclerotic condition. Americal Journal of Applied Mathematics, Vol. 4, No.6, 324-329.

[5] Padma Joshi, Ashutosh Pathak and Joshi B.K, 2009. Two-layered model of blood flow through composite stenosed artery. AAM, vol. 4(2), 343-354.

[6] Medhavi A., 2011. On macroscopic two-phase arterial blood flow through an overlapping stenosis, e-Journal of Science and Technology, 6, 19–31.

[7] Medhavi A., Srivastav R. K., Ahmad Q. S. and Srivastava V. P., 2012. Two-phase arterial blood flow through a composite stenosis, e-Journal of Science and Technology, 7(4), 83–94.

[8] Medhavi A., 2013. A macroscopic two-phase blood flow through a stenosed artery with a permeable wall, Appl Bionics and Biomechanics, 10(1), 11–18.

[9] Srivastava V.P., Tandon M. and Srivastav R. K., 2012. A macroscopic two-phase blood flow through a bell-shaped stenosis in an artery with a permeable wall, Appl. and Appl Math., 7(1), 37–51.

[10] Babatunde A. J. and Dada M. S., 2021. Effects of Hematocrit level on Blood flow through a tapered and overlapping stenosed Artery with Porosity. JHMTR, (1-9), 2007-1293.

[11] Ponalagusamy R and Manchi R., 2020. The two-layered (K.L-Newtonian) model of blood flow in an artery with six types of mild stenosis. Applied Mathematics and Computation, Elsevier, vol. 367.

[12] Puskar R. Pokhrel, 2021, Analysis of Two-layered Blood Flow through Artery with Mild Stenosis, Humanities and Social Sciences Journal, 13(1), 145-154.

[13] Eldesoky I. M., Kernel M. H. & Hussien R. M., Abumandour, 2013. Numerical study of unsteady MHD pulsatile flow through a porous medium in an artery using Generalized Differential Quadrature Method, Intern. Jour. of Materials, Mechanics and Manufacturing, 1(2).

[14] Bugliarello G. and Sevilla, 1970. Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes, Biorheol, 7, 85–107.

[15] Titiloye E.O, Babatunde A.J. and Dada M.S., 2016. Mathematical modelling of two-layered blood flow through a tapered artery with an overlapping stenotic condition, faculty of physical sci., University of Ilorin. ISSN: 2408-4808. Vol.3, 208-223.

[16] Chakravarty. S. and Mandal P. K., 1994. The Mathematical modelling of blood flow through an overlapping Arterial stenosis. Math. & Comp. Modelling, Vol.19, Issue 1, page 59-70.

[17] Sapna Ratan Shah, Anuradha, and Anamika, 2017. Mathematical modelling of blood flow through three-layered stenosed artery". International Journal for Research in Applied Science & Eng. Tech., Vol. 5 Issue VI, ISSN: 2321-9653.

[18] Rupesh K. Srivastav and V.P. Srivastava, 2014. On two-fluid blood flow through the stenosed artery with a permeable wall. Applied Bionics and Biomechanics, 11. 39-45.

[19] Sharan M. and Popel A. S., 2001. A two-phase model for the flow of blood in narrow tubes with increased viscosity near the wall, Birheol, 38, 415–428.

[20] Beavers G. S. and Joseph D. D., 1967. Boundary conditions at a naturally permeable wall. Journal of Fluid Mech, 30(1), 197–207.

[21] Malek A. and Haque A., 2017. Hematocrit level on blood flow through a stenosed artery with a permeable wall. AAM: An international journal, ISSN: 1932-9466.