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On triple θ-centralizers

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Abstract

In this paper, we introduce triple θ-centralizers and weak triple θ-centralizers on an algebra A, where $\theta : A \to A$ is a triple homomorphism. Some observations concerning triple θ-centralizers, weak triple θ-centralizers and approximate weak triple θ-centralizers are given.

Keywords: Triple θ-centralizer, Factorizable, Without order, Semisimple
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1 Introduction

A left (right) centralizer on an algebra A is a mapping T of A into A such that

$$T(ab) = T(a)b \quad (T(ab) = aT(b))$$

for all $a, b \in A$. A centralizer is a mapping $T : A \to A$ such that

$$T(a)b = aT(b)$$

for all $a, b \in A$. The notion of left centralizers was introduced by Wendel [24] who used it to investigate group algebras. The general notion of centralizers on commutative Banach algebras was studied by Helgason [10] and Wang [23]. Helgason used the term *multiplier* instead of *centralizer*. In the non-commutative setting, the notions of left (right) centralizers and centralizers were introduced by Johnson [13] on semigroups, rings, algebras, Banach algebras and topological algebras.

Albas [1] generalized the notion of centralizers and introduced θ-centralizers. For a ring R, if $\theta : R \to R$ is a homomorphism, then a mapping $T : R \to R$ is said to be a left (right) θ-centralizer if

$$T(ab) = T(a)\theta(b) \quad (T(ab) = \theta(a)T(b))$$

for all $a, b \in R$. Jordan left (right) θ-centralizers are obtained if $b = a$. In special case that $\theta = id_A$, we may see that a left (right) id_A-centralizer is a left (right) centralizer. T is said to be a (Jordan) θ-centralizer if it is both (Jordan) left and (Jordan) right θ-centralizer. For a 2-torsion free semiprime ring R (i.e., for $a \in R$, $2a = 0$ implies $a = 0$, and

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$aRa = \{0\}$ implies $a = 0$), Albas [1] proved that every Jordan θ-centralizer of R is a θ-centralizer provided that θ is surjective and $\theta(Z) = Z$, where Z is the center of R. For more properties of θ-centralizers, one can see [1]-[6] and [11, 21].

Let us mention that a Banach algebra A is not without order if there exist nonzero elements a_0 and b_0 in A such that $a_0A = Ab_0 = \{0\}$; for example, semisimple Banach algebras are without order. Wang [23] (see also [13]) showed that every centralizer on a without order Banach algebra is necessarily continuous and linear. Also, Johnson [15] proved that every left (right) centralizer on a Banach algebra with a bounded left (right) approximate identity is continuous and linear. The same result for θ-centralizers has been obtained in [21].

Miura et al. [20] showed that every approximate centralizer (multiplier) on a Banach algebra can be approximated by a centralizer. They also proved that every approximate centralizer on a without order Banach algebra is an exact centralizer. The same results for approximate θ-centralizers have been obtained in [21].

Let A and B be two algebras. A linear mapping $\theta : A \to B$ is called a triple homomorphism if

$$\theta(abc) = \theta(a)\theta(b)\theta(c)$$

for all $a, b, c \in A$. It is evident that if $\theta : A \to B$ is a homomorphism, then θ is a triple homomorphism, but the converse is not true. To see, let $\phi : A \to A$ be a homomorphism, then one can see that $\theta := -\phi$ is a triple homomorphism which is not a homomorphism; for more details, see [7, 9, 17, 18, 22].

In this paper, we introduce triple θ-centralizers and weak triple θ-centralizers on an algebra A, where $\theta : A \to A$ is a triple homomorphism. We will see that the notions of triple θ-centralizers, weak triple θ-centralizers and θ-centralizers are different. We generalize the results of [23, 15, 21] on the linearity and continuity of weak triple θ-centralizers on Banach algebras. We present some observations concerning approximate weak triple θ-centralizers, which improve and extend the same results in [20, 21].

2 Triple and weak triple θ-centralizers

Let us start with the following.

Definition 2.1. Let A be an algebra and $\theta : A \to A$ be a triple homomorphism. A mapping $T : A \to A$ is said to be a triple left (right) θ-centralizer if

$$T(abc) = T(a)\theta(b)\theta(c) \quad (T(abc) = \theta(a)\theta(b)T(c))$$

for all $a, b, c \in A$. T is said to be a triple θ-centralizer if it is both triple left and right θ-centralizer.

For the case that $\theta = id_A$, we may see that a triple id_A-centralizer is a triple centralizer. Also, if we set $c = a$, one can see that a triple centralizer is a Jordan triple centralizer [19, p. 1398].

It is easy to see that every θ-centralizer is also a triple θ-centralizer, but the converse is not true. For illustration, see Example 2.3 (ii).

Definition 2.2. Let A be an algebra and $\theta : A \to A$ be a triple homomorphism. A mapping $T : A \to A$ is said to be a weak triple θ-centralizer if

$$T(a)\theta(b)\theta(c) = \theta(a)\theta(b)T(c)$$

for all $a, b, c \in A$.

We notice that if T is a triple θ-centralizer, then it is a weak triple θ-centralizer, but the converse is not true (see Example 2.3 (i)). However, there exist some conditions under which a weak triple θ-centralizer is a triple θ-centralizer (see Theorem 2.4).

Example 2.3. Let

$$\mathcal{A} = \left\{ \begin{bmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix} : r_1, \ldots, r_6 \in \mathbb{R} \right\}.$$

(i) Define mappings $T, \theta \colon \mathcal{A} \longrightarrow \mathcal{A}$ via

$$T\left(\begin{bmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & r_1 & r_2 & r_5 \\ 0 & 0 & r_4 & r_3 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \theta\left(\begin{bmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & r_1 & r_5 & r_3 \\ 0 & 0 & r_4 & r_2 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then θ is a triple homomorphism on $\mathcal{A}$, but it is not a homomorphism and

$$\theta(\mathbf{a})\theta(\mathbf{b})T(\mathbf{c}) = \begin{bmatrix} 0 & 0 & 0 & r_1 s_4 t_6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = T(\mathbf{a})\theta(\mathbf{b})\theta(\mathbf{c}),$$

$$T(\mathbf{abc}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_1 s_4 t_6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where

$$\mathbf{a} = \begin{bmatrix} 0 & r_1 & r_2 & r_3 \\ 0 & 0 & r_4 & r_5 \\ 0 & 0 & 0 & r_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 & s_1 & s_2 & s_3 \\ 0 & 0 & s_4 & s_5 \\ 0 & 0 & 0 & s_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 0 & t_1 & t_2 & t_3 \\ 0 & 0 & t_4 & t_5 \\ 0 & 0 & 0 & t_6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

are arbitrary elements of $\mathcal{A}$. Thus T is a weak triple θ-centralizer, but it is not a triple left (right) θ-centralizer. On the other hand, we have

$$T(\mathbf{ab}) \neq T(\mathbf{a})\theta(\mathbf{b}), \quad T(\mathbf{ab}) \neq \theta(\mathbf{a})T(\mathbf{b}), \quad \theta(\mathbf{a})T(\mathbf{b}) \neq T(\mathbf{a})\theta(\mathbf{b}),$$

whence T is not a left (right) θ-centralizer.

(ii) Taking θ as the above, we may see that if $S = id_{\mathcal{A}}$, then

$$S(\mathbf{abc}) = S(\mathbf{a})\theta(\mathbf{b})\theta(\mathbf{c}) = \theta(\mathbf{a})\theta(\mathbf{b})S(\mathbf{c})$$

for all $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathcal{A}$. Hence S is a triple θ-centralizer. On the other hand, we have

$$S(\mathbf{ab}) \neq S(\mathbf{a})\theta(\mathbf{b}), \quad S(\mathbf{ab}) \neq \theta(\mathbf{a})S(\mathbf{b}),$$

whence S is not a left (right) θ-centralizer.

Let us mention that an algebra A is factorizable if for each a in A, there exist a_1 and a_2 in A such that $a = a_1 a_2$. By a classical theorem due to Cohen [4], Banach algebras with a bounded approximate identity are factorizable.

Theorem 2.4. Let A be a without order factorizable algebra and $\theta : A \to A$ be a surjective triple homomorphism. If $T : A \to A$ is a weak triple θ-centralizer, then T is a linear triple θ-centralizer.

Proof . Let a, b, c be arbitrary elements of A. Take x in A, since A is facotizable there exist x_1 and x_2 in A such that $x = x_1 x_2$. On the other hand, θ is surjective, so there exist y_1 and y_2 in A such that $\theta(y_1) = x_1$ and $\theta(y_2) = x_2$. We have

$$\begin{aligned} xT(abc) &= \theta(y_1)\theta(y_2)T(abc) \\ &= T(y_1)\theta(y_2)\theta(abc) \\ &= (T(y_1)\theta(y_2)\theta(a))\theta(b)\theta(c) \\ &= (\theta(y_1)\theta(y_2)T(a))\theta(b)\theta(c)) \\ &= \theta(y_1)\theta(y_2)((T(a)\theta(b)\theta(c)) \\ &= x(\theta(a)\theta(b)T(c)). \end{aligned}$$

Hence,

$$x(T(abc) - \theta(a)\theta(b)T(c)) = 0,$$

and this is true for each $x \in A$. Since A is without order, so

$$T(abc) - \theta(a)\theta(b)T(c) = 0.$$

Thus, T is a triple right θ-centralizer. Similarly, it is proved that T is a triple left θ-centralizer. Thus T is a triple θ-centralizer.

To see that T is linear, let $\lambda \in \mathbb{C}$, $a, b \in A$ and

$$\Lambda := T(\lambda a + b) - \lambda T(a) - T(b).$$

Let $x \in A$ be arbitrary, since again A is factorizable and θ is surjective, there exists $x_1, x_2, y_1, y_2 \in A$ such that $x = x_1x_2$ and $x_1 = \theta(y_1), x_2 = \theta(y_2)$. Now we can see that

$$\begin{aligned} x\Lambda &= \theta(y_1)\theta(y_2)[T(\lambda a + b) - \lambda T(a) - T(b)] \\ &= \theta(y_1)\theta(y_2)T(\lambda a + b) - \theta(y_1)\theta(y_2)\lambda T(a) - \theta(y_1)\theta(y_2)T(b) \\ &= T(y_1)\theta(y_2)\theta(\lambda a + b) - \lambda T(y_1)\theta(y_2)\theta(a) - T(y_1)\theta(y_2)\theta(b) \\ &= T(y_1)\theta(y_2)[\theta(\lambda a + b) - \lambda\theta(a) - \theta(b)] \\ &= 0. \end{aligned}$$

This is true for each $x \in A$. Since A is without order, so $\Lambda = 0$ for all $a, b \in A$ and $\lambda \in \mathbb{C}$; that is, T is linear. □

Remark 2.5. Let A be a factorizable algebra and $\theta : A \to A$ be a homomorphism. If $T : A \to A$ is a triple θ-centralizer, then T is a θ-centralizer.

To see, let a, b be arbitrary elements of A. Since A is factorizable, there exist $b_1, b_2 \in A$ such that $b = b_1b_2$. Since T is a triple left θ-centralizer, so

$$T(ab) = T(ab_1b_2) = T(a)\theta(b_1)\theta(b_2) = T(a)\theta(b_1b_2) = T(a)\theta(b).$$

Thus T is a left θ- centralizer. Similarly, one can see that T is a right θ-centralizer and so T is a θ-centralizer.

Corollary 2.6. Let A be a without order factorizable algebra and $\theta : A \to A$ be a surjective homomorphism. If $T : A \to A$ is a weak triple θ-centralizer, then T is a linear θ-centralizer.

Proof . By Theorem 2.4, T is a linear triple θ-centralizer. Thus, by Remark 2.5, T is a linear θ-centralizer. □

We introduce a useful result that can be easily derived from Eshaghi et al. [7, Theorem 2.4] .

Lemma 2.7. Let A be a semisimple Banach algebra. Then every surjective triple homomorphism $\theta : A \to A$ is continuous.

Theorem 2.8. Let A be a semisimple Banach algebra with a bounded left approximate identity and $\theta : A \to A$ be a surjective triple homomorphism. Then every weak triple θ-centralizer $T : A \to A$ is linear and continuous.

Proof . Let $a_m \to 0$ in A, by Johnson's theorem there exist $c \in A$ and a sequence $(b_m) \in A$ such that $b_m \to 0$ and $a_m = cb_m$ for each m, see [15]. On the other hand, by Cohen's factorization theorem, there exist $c_1, c_2 \in A$ such that $c = c_1c_2$. By assumptions and Theorem 2.4, T is a linear triple left θ-centralizer, so we have

$$T(a_m) = T(cb_m) = T(c_1c_2b_m) = T(c_1)\theta(c_2)\theta(b_m).$$

By Lemma 2.7, θ is a continuous map. It forces the last sentence approaches to zero whenever $m \to \infty$; that is, T is continuous. □

Corollary 2.9. Let A be a C*-algebra and $\theta : A \to A$ be a surjective triple homomorphism. Then every weak triple θ-centralizer $T : A \to A$ is linear and continuous.

3 Approximate weak triple θ-centralizers

Here, we give some sufficient conditions under which every approximate weak triple θ-centralizer is a linear triple θ-centralizer.

Theorem 3.1. Let A be a without order factorizable Banach algebra, $\theta : A \to A$ be a surjective triple homomorphism and $\ell \in \{-1, 1\}$ be fixed. Let $T : A \to A$ be a mapping satisfy

$$\|\theta(a)\theta(b)T(c) - T(a)\theta(b)\theta(c)\| \leq \phi(a, b, c),$$

where $\phi : A^3 \to [0, \infty)$ is a mapping such that

$$\lim_{k\to\infty} \frac{\phi(2^{\ell k}a, b, c)}{2^{\ell k}} = 0 \tag{3.1}$$

for all $a, b, c \in A$. Then T is a linear triple θ-centralizer.

Proof . Let $\mu \in \mathbb{C}$ and $a \in A$ be arbitrary, we show that $T(\mu a) = \mu T(a)$. Take $x \in A$, since A is factorizable and θ is surjective, there exist x_1, x_2 and y_1, y_2 in A such that $x = x_1x_2$, $\theta(y_1) = x_1$, $\theta(y_2) = x_2$. Then

$$\begin{aligned}
\|2^{\ell k}x[T(\mu a) - \mu T(a)]\| &= \|\theta(2^{\ell k}y_1)\theta(y_2)[T(\mu a) - \mu T(a)]\| \\
&\leq \|\theta(2^{\ell k}y_1)\theta(y_2)T(\mu a) - T(2^{\ell k}y_1)\theta(y_2)\theta(\mu a)\| \\
&\quad +\|T(2^{\ell k}y_1)\theta(y_2)\theta(\mu a) - \theta(2^{\ell k}y_1)\theta(y_2)(\mu T(a))\| \\
&\leq \phi(2^{\ell k}y_1, y_2, a) + |\mu|\phi(2^{\ell k}y_1, y_2, a).
\end{aligned}$$

So we have

$$\|x[T(\mu a) - \mu T(a)]\| \leq \frac{1}{2^{\ell k}}[\phi(2^{\ell k}b, c, a) + |\mu|\phi(2^{\ell k}b, c, a)].$$

By letting $k \to \infty$, we get

$$x[T(\mu a) - \mu T(a)] = 0.$$

Since A is without order and this is true for each $x \in A$, $T(\mu a) - \mu T(a) = 0$ for all $a \in A$ and $\lambda \in \mathbb{C}$.

In spacial case that $\mu = 2^{\ell k}$, we get $T(a) = \dfrac{1}{2^{\ell k}}T(2^{\ell k}a)$. Thus for each $a, b, c \in A$, we have

$$\begin{aligned}
\|\theta(a)\theta(b)T(c) - T(a)\theta(b)\theta(c)\| &= \frac{1}{2^{\ell k}}\|2^{\ell k}\theta(a)\theta(b)T(c) - 2^{\ell k}T(a)\theta(b)\theta(c))\| \\
&= \frac{1}{2^{\ell k}}\|\theta(2^{\ell k}a)\theta(b)T(c) - T(2^{\ell k}a)\theta(b)\theta(c)\| \\
&\leq \frac{1}{2^{\ell k}}\phi(2^{\ell k}a, b\, c).
\end{aligned}$$

By taking limit whenever $k \to \infty$, the last sentence approaches to zero; that is, T is a weak triple θ-centralizer. By Theorem 2.4, T is a linear triple θ-centralizer. □

Corollary 3.2. Let A be a without order factorizable Banach algebra, $\theta : A \to A$ be a surjective triple homomorphism, $\ell \in \{-1, 1\}$ be fixed and ϵ, r be positive real numbers with $\ell r < \ell$. If $T : A \to A$ is a mapping such that

$$\|\theta(a)\theta(b)T(c) - T(a)\theta(b)\theta(c)\| \leq \epsilon\|a\|^r\|b\|^r\|c\|^r$$

for all $a, b, c \in A$, then T is a linear triple θ-centralizer.

Proof . Set $\phi(a, b, c) := \epsilon\|a\|^r\|b\|^r\|c\|^r$ and use Theorem 3.1. □

Theorem 3.3. Let A be a Banach algebra (need not be without order), $\theta : A \to A$ be a triple homomorphism (need not be surjective), $\ell \in \{-1, 1\}$ be fixed and $\varphi : A^2 \to [0, \infty)$ and $\phi : A^3 \to [0, \infty)$ be mappings such that

$$\sigma(a, b) := \sum_{i=\frac{1-\ell}{2}}^{\infty} \frac{\varphi\left(2^{\ell i}a, 2^{\ell i}b\right)}{2^{\ell i}} < \infty, \quad \lim_{k\to\infty} \frac{\phi\left(2^{\ell k}a, b, 2^{\ell k}c\right)}{4^{\ell k}} = 0$$

for all $a, b, c \in A$. If $f : A \to A$ is a mapping satisfying

$$\|f(a + b) - f(a) - f(b)\| \leq \varphi(a, b), \quad \|\theta(a)\theta(b)f(c) - f(a)\theta(b)\theta(c)\| \leq \phi(a, b, c)$$

for all $a, b, c \in A$, then there exists a unique additive weak triple θ-centralizer $T : A \to A$ such that

$$\|f(a) - T(a)\| \leq \frac{\sigma(a, a)}{2} \tag{3.2}$$

for all $a \in A$.

Proof . By [8] and [16, Corollary 2.19], there exists a unique additive mapping $T\colon A \to A$ such that (3.2) holds for all $a \in A$. The mapping T is given by

$$T(a) := \lim_{k\to\infty} \frac{1}{2^{\ell k}} f\left(2^{\ell k}a\right)$$

for all $a \in A$. On the other hand, by the homogeneity of θ, we have

$$\begin{aligned}
&\left\|\theta(a)\theta(b)\frac{1}{2^{\ell k}} f\left(2^{\ell k}c\right) - \frac{1}{2^{\ell k}} f\left(2^{\ell k}a\right)\theta(b)\theta(c)\right\| \\
&= \frac{1}{4^{\ell k}} \left\|\theta(2^{\ell k}a)\theta(b)f\left(2^{\ell k}c\right) - f\left(2^{\ell k}a\right)\theta(b)\theta(2^{\ell k}c)\right\| \\
&\leq \frac{1}{4^{\ell k}} \phi\left(2^{\ell k}a, b, 2^{\ell k}c\right)
\end{aligned}$$

for all $a, b, c \in A$. The last sentence approaches to zero whenever $k \to \infty$; that is,

$$\theta(a)\theta(b)T(c) = T(a)\theta(b)\theta(c)$$

for all $a, b, c \in A$. So T is an additive weak triple θ-centralizer. □

Corollary 3.4. Let A be a Banach algebra (need not be without order), $\theta : A \to A$ be a triple homomorphism (need not be surjective), $\ell \in \{-1, 1\}$ be fixed and ϵ, p, r be positive real numbers with $\ell p, \ell r < \ell$. If $f : A \to A$ is a mapping such that

$$\|f(a + b) - f(a) - f(b)\| \leq \epsilon\left(\|a\|^p + \|b\|^p\right), \quad \|\theta(a)\theta(b)f(c) - f(a)\theta(b)\theta(c)\| \leq \epsilon\|a\|^r\|b\|^r\|c\|^r$$

for all $a, b, c \in A$, then there exists a unique additive weak triple θ-centralizer $T : A \to A$ such that

$$\|f(a) - T(a)\| \leq \frac{\ell\epsilon}{1 - 2^{p-1}}\|a\|^p$$

for all $a \in A$.

Proof . Set $\varphi(a, b) := \epsilon\left(\|a\|^p + \|b\|^p\right)$, $\phi(a, b, c) := \epsilon\|a\|^r\|b\|^r\|c\|^r$ and use Theorem 3.3. □

Let A and B be Banach algebras. A linear mapping $T : A \to B$ is said to be an almost homomorphism (or almost multiplicative linear mapping) if there exists $\epsilon \geq 0$ such that

$$\|T(ab) - T(a)T(b)\| \leq \epsilon\|a\|\|b\|$$

for all $a, b \in A$ (see, e.g., [12, 14]). Also, a linear mapping $T : A \to B$ is said to be an almost triple homomorphism if there exists $\epsilon \geq 0$ such that

$$\|T(abc) - T(a)T(b)T(c)\| \leq \epsilon\|a\|\|b\|\|c\|$$

for all $a, b, c \in A$ (see, e.g., [17]).

Theorem 3.5. Let A be a semisimple Banach algebra with a bounded left approximate identity and $\theta : A \to A$ be a surjective triple homomorphism. If $T : A \to A$ is a weak triple θ-centralizer, then T is a continuous almost triple homomorphism.

Proof . By Theorems 2.4 and 2.8, T is a continuous linear triple left θ-centralizer. Also θ is a continuous linear map, so

$$\|\theta(a)\| \leq \|\theta\|\|a\|, \qquad \|T(a)\| \leq \|T\|\|a\|$$

for all $a \in A$. Thus

$$\begin{aligned}
\|T(abc) - T(a)T(b)T(c)\| &= \|T(a)\theta(b)\theta(c) - T(a)T(b)T(c)\| \\
&\leq \|T(a)\|\|\theta(b)\theta(c) - T(b)T(c)\| \\
&\leq \|T(a)\|(\|\theta(b)\|\|\theta(c)\| + \|T(b)\|\|T(c)\|) \\
&\leq \|T\|\|a\|(\|\theta\|^2\|b\|\|c\| + \|T\|^2\|b\|\|c\|) \\
&= \|T\|\|\theta\|^2\|a\|\|b\|\|c\| + \|T\|^3\|a\|\|b\|\|c\| \\
&= \|T\|(\|\theta\|^2 + \|T\|^2)\|a\|\|b\|\|c\|
\end{aligned}$$

for all $a, b, c \in A$. By taking $\epsilon = \|T\|(\|\theta\|^2 + \|T\|^2)$, we see that T is almost triple homomorphism. $\square$

Corollary 3.6. Let A be a C*-algebra and $\theta : A \to A$ be a surjective triple homomorphism. If $T : A \to A$ is a weak triple θ-centralizer, then T is a continuous almost triple homomorphism.

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