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## Research Article

# Natural Convection and Entropy Generation of Non-Newtonian Hybrid Cu-Al<sub>2</sub>O<sub>3</sub>/Water Nanofluid in an Inclined Partial Porous Cavity with Different Local Heater Positions in the Presence of Magnetic Field

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## ARTICLE INFO

**Article history:**

Received: 2023-05-14

Revised: 2023-12-25

Accepted: 2023-12-26

**Keywords:**Hybrid Cu-Al<sub>2</sub>O<sub>3</sub> nanofluid;

Partially porous media;

Position of heated wall;

Da number;

Porosity;

Non-Newtonian fluid.

## ABSTRACT

In the present paper, natural convection of non-Newtonian hybrid Cu-Al<sub>2</sub>O<sub>3</sub> nanofluid in an inclined partial porous cavity with changing heated wall position was numerically investigated. At first, the governing equations are rewritten non-dimensionally by utilizing dimensionless parameters. Then, the entropy generation equations are expressed in non-dimensional form. The discretization of governing equations is done by CVFVM (Control Volume Finite Volume Method). The coupling between pressure and velocity is handled by SIMPLE method and the obtained algebraic equations are computed by SIP solver. Nu number and irreversibility of flow and heat transfer are examined by main parameters such as Ha number, Da number, porosity, porous media thickness, position of heated wall, power law index and angle of inclination. Nu number increases almost 39%, 50% and 52% with enhancing porous media thickness from 0.1 to 0.6 at inclination angle of -30°, 0° and 30°, respectively. The Nu number increases by setting the hot surface at the lower part of the cavity. Nu decreases almost 10.5%, 7.6% and 5.2% by augmentation of Ha number from 0.0 to 40.0 at viscosity power law index of 0.6, 1.0 and 1.4, respectively. Nu increases almost 36%, 47% and 53% at inclination angle of 30°, 0° and -30° by enhancing volume fraction from 0.02 to 0.12, respectively. Convection heat transfer and fluid flow will be more dominant by augmentation of permeability and Da number and this will lead to more entropy generation by fluid flow and less entropy generation and irreversibility by magnetic field and convective heat transfer process. Fluid flow and magnetic field entropy generation decrease and heat transfer irreversibility and Be number rises by porous thickness ratio. Be number and thermal entropy generation increases with porosity. However, irreversibility due to magnetic field and flow friction decreases.

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## 1. Introduction

Natural heat convection in porous media finds widespread applications in various industrial and engineering areas such as chemical processing, filtration, power technology, liquid food processing [1,2], petroleum industry [3-4],

surgical implants, technology of drying (for textile materials and processing food grains), biological appliance, solar energy storage by phase change materials [5] and biomedical applications especially with non-Newtonian fluids. The use of nanofluids has attracted many

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## Cite this article as:

Kardgar, A. 2023. Natural Convection and Entropy Generation of Non-Newtonian Hybrid Cu-Al<sub>2</sub>O<sub>3</sub>/Water Nanofluid in an Inclined Partial Porous Cavity with Different Local Heater Positions in the Presence of Magnetic Field, *Journal of Heat and Mass Transfer Research*, 10(2), pp. 279-300.

<https://doi.org/10.22075/JHMTR.2023.30638.1442>

attentions because the presence of nanofluids can increase thermal conductivity and heat transfer in the medium [6,7,8]. Comprehensive researches on the nanofluids are observable in the literature. Sheremet and Pop investigated nanofluid heat transfer in the tilted cavity with cold vertical wall [9]. A numerical study was conducted by Jamaludin et al. to elucidate the magnetohydrodynamic mixed convection of hybrid nanofluid [10]. Alsabery et al. studied the finite wall thickness effect in the enclosure filled with porous media. They concluded the convective heat transfer can be impeded by the solid wall [11]. Khan et al. demonstrated that porous medium permeability, thermal buoyancy, the magnetic field and the heating element length has an essential effect in the improvement of Nu number [12]. Geridonmez and Oztop considered partial magnetic field in natural convection in the nanofluids [13]. They demonstrated the heat transfer can be more suppressed by the applying the magnetic field at the middle of the cavity. Porous media can weaken convective heat transfer and strengthen conduction heat transfer. So, the application of porous media in cavity can have both positive and negative effect on the Nu number [14]. Rodríguez-Núñez et al. proposed a numerical model to investigate the unsteady natural convection in a cylinder in order to predict potential of the temperature distribution evolution in the aqueous silica [15]. Sheikholeslami and Vajravelu studied the effect of the Lorentz force on heat transfer with the variable magnetic field [16].

Zhang et al. numerically modeled the fluid flow and energy equation in a porous enclosure with moving wall by lattice Boltzmann method. They studied the internal heat source position on the Nu number [17]. Sheremet and Pop analysed the problem by Tiwari and Das' nanofluid model [18]. Li et al. used LB method to investigate the effect of Lorentz force on heat transfer [19]. Rajarathinam et al. investigated the mixed convection in an inclined cavity with upper and bottom wall moving oppositely. They concluded that the direction of the moving wall has significant role on the flow and heat transfer [20]. The MHD flow around a permeable triangle body in square cavity filled with Alumina nanofluid was examined by Lattice Boltzmann Method [21]. It was observed that the enhancement of permeability augments the fluid momentum, whereas the magnetic force declines the fluid kinetic energy. Gibanov et al. worked on ferrofluid heat transfer characteristics in cavity by performing a numerical modeling by considering the uniform horizontal magnetic field [22]. Rahmat Ellahi et al. investigated nanofluid flow in vertical wavy cone and pipe with analytic solution [23,24].

The phenomenon of buoyancy-driven natural convection in a porous media is extensively under investigation. Aneja et al. carried out an investigation to investigate heat transfer in Casson fluid numerically [25]. Astanina et al. analyzed the heat transfer in an open trapezoidal enclosure. They found that an Hartmann number enhancement can lead to an increment of amplitude of oscillations for average Nu and irreversibility [26]. Selimefendigil and Öztop calculated the influence of corrugation on heat transfer in U-shaped vented cavity. Their research demonstrated that the bottom wall corrugation which was a wave with triangular shape is an effective tool for fluid flow and heat transfer features [27]. Khan et al. [28] and Zeeshan et al. [29] studied darcy- Forchheimer flow. They concluded an increase in thermophoresis parameter and Schmidt number results in a decrease in concentration of nanoparticles.

Most researches on fluid flow in porous media have applied and still utilize the Darcy model to calculate the pressure drop with the superficial velocity [30, 31]. The boundary and inertia effects of the flow must be captured if the fluid velocity is relatively high. Previous investigations elucidated that the steady state Darcy model is adequate to simulate flow in saturated porous media, especially for low porosity media and low velocity flow. However, in order to accommodate different conditions the extended Darcy model is utilized in many studies. For example, Forchheimer and Brinkman extended model is extensively applied to capture non-linearity of porous matrix drag at high velocity [32].

It is apparent that there has been a significant oversight in the investigation of entropy generation within a partial porous enclosure, based on the existing literature review. The identification of key factors affecting entropy generation holds the potential to augment the sustainability, efficiency, and overall performance of thermodynamic processes and systems. Furthermore, a more in-depth analysis of entropy generation concerning parametric effects in natural convection can suggest valuable solutions to different engineering, renewable systems, and environmental challenges. Therefore, this research initiatively targets to explore the fundamental factors contributing to entropy generation in a parametric study of natural convection within a partial porous cavity subjected to a uniformly acting magnetic field. Additionally, a mathematical model based on the Darcy-Forchheimer-Brinkmann framework has been numerically simulated using the finite volume approach. CVFVM was employed to discretize partial differential equations of Navier-Stokes and energy equations. This investigation

will extensively discuss the essential components responsible for volumetric entropy generation, including irreversibilities arising from heat transfer, magnetic field and fluid friction effects. Furthermore, thoroughly exploring the Bejan number associated with each entropy generation component will provide valuable insights into the effect of individual irreversibility factors.

## 2. Theory and physics of the problem

The Physics of the problem and the computational domain is illustrated in Fig. 1. This is an inclined rectangular cavity partially filled with porous matrix with thickness of  $\delta$ . The hybrid Cu/Al<sub>2</sub>O<sub>3</sub> water nanofluid is influenced by the external magnetic field. The Rayleigh number range selected in the present work keeps the nanofluid flow two-dimensional and laminar. The right vertical wall of the cavity is kept at constant temperature TH, while the left vertical wall is maintained at constant cold temperature TC. The top and bottom horizontal wall is kept adiabatic. The following assumptions are considered in the study:

- The flow is considered incompressible, laminar and steady.
- Thermal non-equilibrium condition is between solid matrix and nanofluid.
- The Darcy model and the Boussinesq approximation are applied.
- The properties of the nanofluid and the porous medium are assumed homogeneous and isotropic,
- In comparison with the external magnetic field, the induced magnetic field can be ignored.
- The walls of cavity are considered impermeable.

### 2.1. Mathematical Modeling

By taking into account the afore-said assumptions, the coupled non-linear PDE's for mass continuity, momentum and energy are [33]:

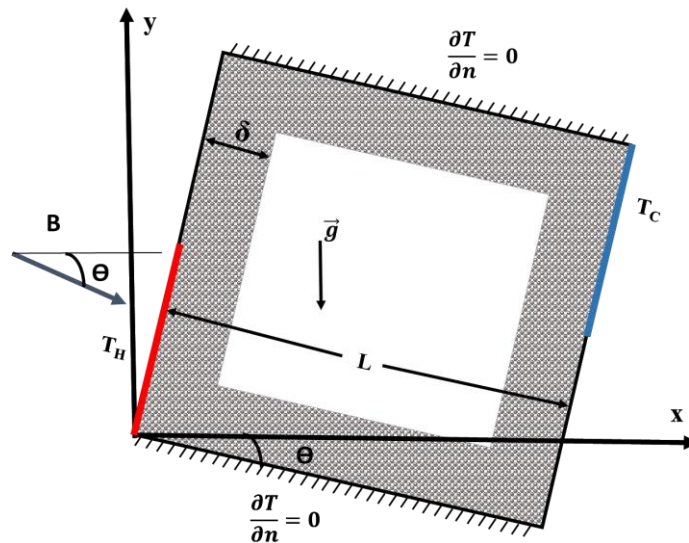


Fig. 1. The geometry of the problem with physical model and boundary conditions

$$\text{Continuity} \quad \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\begin{aligned} \text{Momentum-x} \quad \frac{\rho_{nf}}{\varepsilon^2} \left( \frac{\partial(u^*u^*)}{\partial x^*} + \frac{\partial(u^*v^*)}{\partial y^*} \right) = & -\frac{\partial p^*}{\partial x^*} + \frac{\mu_{nf}}{\varepsilon} \left( \frac{\partial}{\partial x^*} \left( \frac{\partial u^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left( \frac{\partial u^*}{\partial y^*} \right) \right) - \frac{\mu u}{\rho_{hnf} K} \\ & + \sigma_{nf} B^2 (v^* \sin\theta \cos\theta - u^* \sin^2\theta) - (\rho\beta)_{nf} g_x (\bar{T} - T_c) \end{aligned} \tag{2}$$

$$\begin{aligned} \text{Momentum-y} \quad \frac{\rho_{hnf}}{\varepsilon^2} \left( \frac{\partial(u^*v^*)}{\partial x^*} + \frac{\partial(v^*v^*)}{\partial y^*} \right) = & -\frac{\partial p^*}{\partial y^*} + \frac{\mu_{hnf}}{\varepsilon} \left( \frac{\partial}{\partial x^*} \left( \frac{\partial v^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left( \frac{\partial v^*}{\partial y^*} \right) \right) - \frac{\mu v}{\rho_{hnf} K} \\ & + \sigma_{hnf} B^2 (u^* \sin\theta \cos\theta - v^* \cos^2\theta) - (\rho\beta)_{hnf} g_y (\bar{T} - T_c) \end{aligned} \tag{3}$$

$$\text{Fluid Energy} \quad \varepsilon(\rho c_p)_{hnf} \left( \frac{\partial(u^* T_f^*)}{\partial x^*} + \frac{\partial(v^* T_f^*)}{\partial y^*} \right) = \varepsilon k_{hnf} \left( \frac{\partial}{\partial x^*} \left( \frac{\partial T_f^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left( \frac{\partial T_f^*}{\partial y^*} \right) \right) + h_v(T_s^* - T_f^*) \quad (4)$$

$$\text{Solid Energy} \quad 0 = (1 - \varepsilon)k_s \left( \frac{\partial}{\partial x^*} \left( \frac{\partial T_s^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left( \frac{\partial T_s^*}{\partial y^*} \right) \right) - h_v(T_s^* - T_f^*) \quad (5)$$

The viscosity  $\mu_{hnf}$ , the density  $\rho_{hnf}$ , the thermal capacity  $(\rho c_p)_{hnf}$ , and the coefficient of thermal expansion  $(\rho\beta)_{hnf}$  are computed by the following formulas, respectively.

$$\mu_{hnf} = \frac{\mu_f}{(1 - \varphi_1)^{2.5}(1 - \varphi_2)^{2.5}} \quad (6)$$

$$(\rho c_p)_{hnf} = (1 - \varphi_2) \left( (1 - \varphi_1)\rho_f c_{pf} + \varphi_1 \rho_{n1} c_{pn1} + \varphi_2 \rho_{n2} c_{pn2} \right) \quad (7)$$

$$(\rho\beta)_{hnf} = (1 - \varphi_2) \left( (1 - \varphi_1)\rho_f \beta_f + \varphi_1 \rho_{n1} \beta_{n1} + \varphi_2 \rho_{n2} \beta_{n2} \right) \quad (8)$$

$$\rho_{hnf} = (1 - \varphi_2) \left( (1 - \varphi_1)\rho_f c_{pf} + \varphi_1 \rho_{n1} c_{pn1} + \varphi_2 \rho_{n2} c_{pn2} \right) \quad (9)$$

$$\frac{k_{nhf}}{k_{bf}} = \frac{k_{n2} + 2k_{bf} - 2\varphi_2(k_{bf} - k_{n2})}{k_{n2} + 2k_{bf} + \varphi_2(k_{bf} - k_{n2})} \quad (10)$$

$$\frac{k_{bf}}{k_f} = \frac{k_{n1} + 2k_f - 2\varphi_1(k_f - k_{n1})}{k_{n1} + 2k_f + \varphi_1(k_f - k_{n1})} \quad (11)$$

$$\frac{\sigma_{nhf}}{\sigma_{bf}} = \frac{\sigma_{n2} + 2\sigma_{bf} - 2\varphi_2(\sigma_{bf} - \sigma_{n2})}{\sigma_{n2} + 2\sigma_{bf} + \varphi_2(\sigma_{bf} - \sigma_{n2})} \quad (12)$$

$$\frac{\sigma_{bf}}{\sigma_f} = \frac{\sigma_{n1} + 2\sigma_f - 2\varphi_1(\sigma_f - \sigma_{n1})}{\sigma_{n1} + 2\sigma_f + \varphi_1(\sigma_f - \sigma_{n1})} \quad (13)$$

in above the equations, the solid nanoparticle volume fraction is denoted by  $\varphi$  and subscript  $hnf$ ,  $n$  and  $f$  stands for hybrid nanofluid, solid nanoparticle and base fluid, respectively. The fluid viscosity is

$$\mu_f = N \left\{ 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right\}^{\frac{n-1}{2}}$$

where  $N$  is the consistency coefficient and  $n$  is the power-law index.

The convection heat transfer coefficient between solid matrix and fluid is determined by the following relation[34].

$$h_v = \frac{Nuk}{d} \quad (14)$$

$$Nu = (1 + 0.99(RePr)^{0.66})\varepsilon^{1.79} \quad (15)$$

In the above relations, dimensional temperature, pressure, y-velocity and x-velocity are denoted by  $T^*$ ,  $p^*$ ,  $v^*$  and  $u^*$  respectively. The no-slip BC for velocity near wall and zero gradient for insulated wall and fixed temperature for right and left wall can be applied by the following equations:

$$\text{Upper outer wall} \quad y^* = L, 0 \leq x^* \leq L, \frac{\partial T_s^*}{\partial y^*} = 0 \quad (16)$$

$$\text{Lower outer wall} \quad y^* = 0, 0 \leq x^* \leq L, \frac{\partial T_s^*}{\partial y^*} = 0 \quad (17)$$

$$\text{Left outer wall} \quad x^* = 0, L/2 \leq y^* \leq L, T^* = T_H \quad (18)$$

$$\text{Right outer wall} \quad x^* = L, L/2 \leq y^* \leq L, T^* = T_C \quad (19)$$

Exergy is a thermodynamic concept, used for many years within engineering analyses of chemical and mechanical processes and systems. Officially, in thermodynamics, the exergy of a system is construed as the maximum potential of work of a component or system in a specified environment at a given state. The exergy destruction rate is proportional to the production of entropy. Irreversibilities can destroy exergy completely or partially in actual cycles and processes. In the studied problem, the exergy destruction is due to magnetic field, heat transfer and fluid flow. The rate of entropy generation due to fluid flow  $S_F^*$ , heat transfer  $S_T^*$  and magnetic field  $S_G^*$  can be formulated as follows [35]:

$$S_F^* = \frac{\mu_{hnf}}{T_0} \left[ 2 \left( \frac{\partial u^*}{\partial x^*} \right)^2 + 2 \left( \frac{\partial v^*}{\partial y^*} \right)^2 + \left( \frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*} \right)^2 \right] \quad (20)$$

$$S_T^* = \frac{k_{hnf}}{T_0^2} \left[ \left( \frac{\partial T^*}{\partial x^*} \right)^2 + \left( \frac{\partial T^*}{\partial y^*} \right)^2 \right] \quad (21)$$

$$S_G^* = \frac{\sigma_{hnf} B^2}{T_0} (u^* \sin\theta - v^* \cos\theta)^2 \quad (22)$$

The total entropy generation can be achieved by summing up the aforementioned irreversibility as follows:

$$S_S^* = S_F^* + S_T^* + S_G^* \quad (23)$$

One the important non-dimensional number in MHD flow investigation is Bejan number which can be computed as:

$$Be = \frac{S_T^*}{S_S^*} \quad (24)$$

### 2.2. Numerical Procedure

Finite volume method (FVM) are utilized to discretize governing PDE's. The collocated arrangement of mesh is applied and in order to deal with velocity and pressure coupling the entrenched SIMPLE technique is employed with using the Rhie and Chow's momentum interpolation method [36]. X-momentum convective fluxes through right-face(e-face) of control volume demonstrated in Fig. 2 can be calculated as [37]:

$$\begin{aligned} F_e^c &= \dot{m}_e u_e \\ u_e &= (\rho u)_e \Delta y \\ F_e^c &= \begin{cases} \max(\dot{m}_e, 0) u_p + \min(\dot{m}_e, 0) u_E \\ \dot{m}_e (1 - \lambda_e) u_p + \dot{m}_e \lambda_e u_E \end{cases} \quad (25) \\ \lambda_e &= \frac{x_e - x_p}{x_E - x_p} \end{aligned}$$

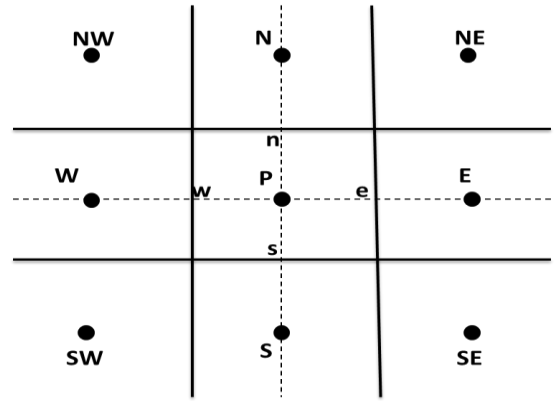


Fig. 2. Control volume and its north, south, west and east neighborhoods in the collocated arrangement of grid [37]

The same equations for other faces is obtained by applying central and upwind discretization scheme. For instance, the resulted equations for upwind scheme is as follow:

$$\begin{aligned} A_E^c &= \min(\dot{m}_e, 0), & A_W^c &= \min(\dot{m}_w, 0) \\ A_N^c &= \min(\dot{m}_n, 0), & A_S^c &= \min(\dot{m}_s, 0) \\ A_P^c &= -(A_E^c + A_W^c + A_N^c + A_S^c) \end{aligned} \quad (26)$$

By applying the central discretization scheme, the coefficient of convective fluxes in algebraic equations are:

$$\begin{aligned} A_E^c &= \dot{m}_e \lambda_e, & A_W^c &= \dot{m}_w \lambda_w \\ A_N^c &= \dot{m}_n \lambda_n, & A_S^c &= \dot{m}_s \lambda_s \\ A_P^c &= -(A_E^c + A_W^c + A_N^c + A_S^c) \end{aligned} \quad (27)$$

The equation for  $A_P^c$  is resulted from the continuity condition:

$$\dot{m}_e + \dot{m}_w + \dot{m}_n + \dot{m}_s = 0 \quad (28)$$

The diffusion terms is discretized by central differencing due to its physics. The coefficients are computed by the following relations:

$$\begin{aligned} F_e^d &= \left( \mu \frac{\partial u}{\partial x} \right)_e \Delta y = \frac{\mu_e S_e}{x_E - x_p} (u_E - u_p) \\ A_E^d &= -\frac{\mu_e S_e}{x_E - x_p}, & A_W^d &= -\frac{\mu_w S_w}{x_p - x_W} \\ A_N^d &= -\frac{\mu_n S_n}{x_N - x_p}, & A_S^d &= -\frac{\mu_s S_s}{x_p - x_S} \end{aligned} \quad (29)$$

Kholsa and Rubin [38] is applied to avoid diagonally domination in system of algebraic equations resulted from central differencing approximation. Because, system of algebraic equations with diagonally domination may diverge.

$$F_e^c = \dot{m}_e u_e^{UDS} + \dot{m}_e [u_e^{CDS} - u_e^{UDS}]^{m-1} \quad (30)$$

in which superscripts, CDS and UDS means central and upwind discretization scheme,

respectively. The last term in bracket is computed from former iteration and will vanish at convergence and the precision is analogous to CDS whilst the rate of convergence is like UDS [38].

The pressure term can be dealt like a source term in the momentum equation and is calculated as [39]:

$$Q_p^u = -(p_e S_e - p_w S_w)^{m-1} \tag{31}$$

The pressure treatment in order to avoid the checker board problem in collocated grid is expressed in [38]. The system of algebraic equations is obtained by rearranging the above relations:

$$\begin{aligned} A_p^u u_p + A_E^u u_E + A_W^u u_W + A_N^u u_N + A_S^u u_S &= Q_p^u \\ A_E^u &= A_E^c + A_E^d, & A_W^u &= A_W^c + A_W^d \\ A_N^u &= A_N^c + A_N^d, & A_S^u &= A_S^c + A_S^d \\ A_p^u &= A_p^t - (A_E^u + A_W^u + A_N^u + A_S^u) \end{aligned} \tag{32}$$

$$Q_p^u = Q_u^p + Q_u^c + Q_u^t$$

where  $Q_u^c$  is:

$$Q_u^c = [(F_u^c)^{UDS} - (F_u^c)^{CDS}]^{m-1} \tag{33}$$

The systems of linear algebraic equations are solved by the Stone's SIP solver. Stone's scheme, also is well-known as SIP<sup>1</sup>, is an algorithm developed for systems of equations resulting from the discretization of partial differential equations. The Biggest disadvantage of LU method is ignoring the advantage of coefficient matrix. So, Stone's method accelerate the convergence rate by considering a preconditioner matrix.

### 3. Grid Independency and Validation

The discretization of the governing equations was done by Finite volume method (FVM). A numerical code was developed in FORTRAN compiler to solve the resulted linear equations. An iterative procedure is adopted to obtain the converged solution. Therefore, the convergence criteria is considered to stop the iteration:

$$\left| \frac{\theta_{i,j} - \theta_{i,j}^{old}}{\theta_{i,j}} \right| < 10^{-10} \tag{34}$$

where  $\theta$  may stand for velocity or temperature. Under relaxation number is considered 0.7 and 0.4 for velocity and for pressure, respectively.

Mesh independency is checked by using different grid sizes in the simulation domain. The results are seen in Fig. 3.

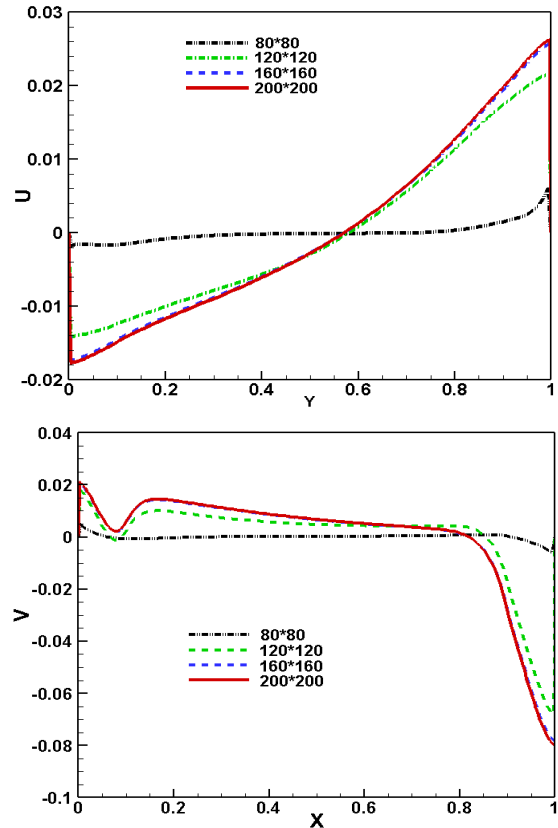


Fig. 3. U-velocity on vertical axis and V-velocity on horizontal axis with different number of meshes

Grid independent solution was achieved by 200\*200 meshes. The domain with the generated grid is demonstrated in Fig. 4.

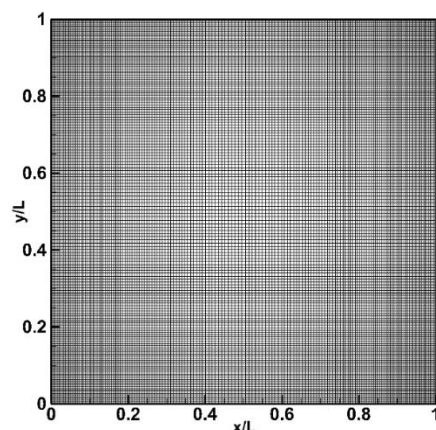


Fig. 4. Sample of the uniform structured mesh used for simulation

The present code has been verified successfully against the works of Khanafer et al. [40]. The present code result in a close agreement with numerical simulation of Khanafer et al. [40].

<sup>1</sup> Strongly Implicit Procedure

The comparison is concerned with u-velocity, v-velocity and temperature along the centerline. For more confident, the prepared numerical code was also verified against the experimental work of Krane and Jessee [41] with natural convection in a cavity with air as fluid in Fig. 6. There is a good agreement between numerical simulation and experimental result.

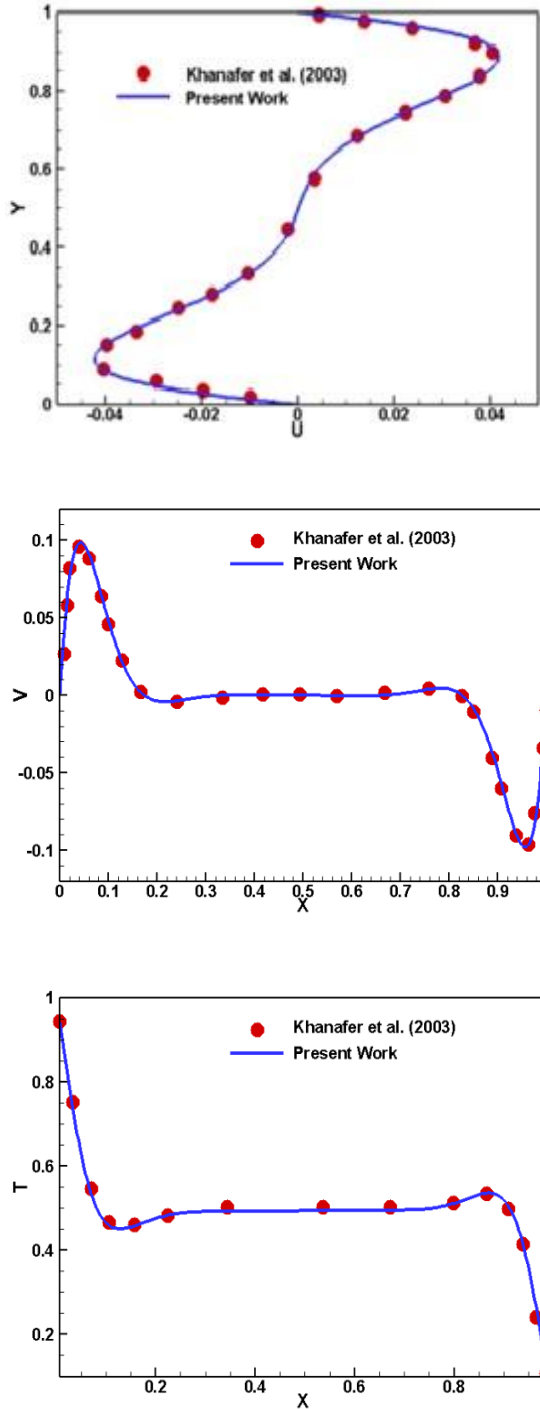


Fig. 5. Comparison between numerical results of Khanafer et al. [40] and present work for the temperature, U-velocity and V-velocity distribution on centerline at  $Gr=10^5$ ,  $Pr=6.2$  and  $\phi=5\%$

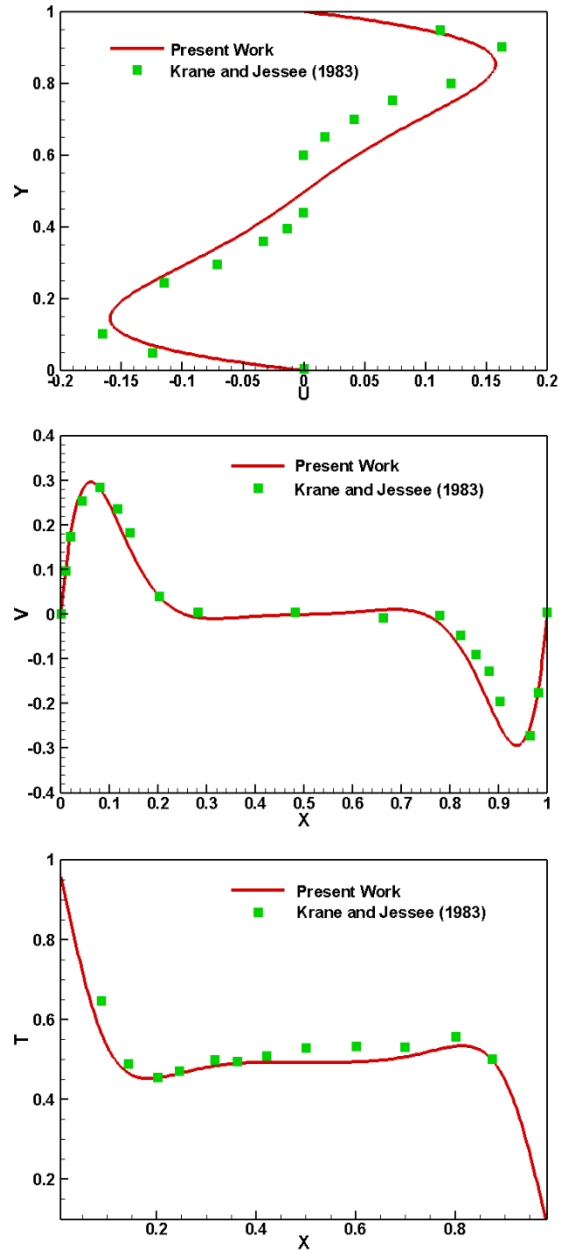


Fig. 6. Comparison between Krane and Jessee [41] experimental results and present work for the temperature, U-velocity and V-velocity distribution on centerline at  $Ra=1.89 \times 10^5$  and  $Pr=0.71$

#### 4. Results and Discussion

A numerical simulation has been performed to investigate the effects of different involved parameters in the present study, such as Rayleigh number  $Ra$ , Darcy number  $Da$ , porosity  $\epsilon$ , porous media thickness, power law index, Hartman number  $Ha$  and volume fraction of nanofluid on natural convection of non-Newtonian fluid flow in the cavity partially filled with a porous media.

The effects of  $Da$  number on the averaged Nusselt at  $\phi=0.05$ ,  $\delta/L=0.2$ ,  $Ra=1.5 \times 10^6$  and  $\epsilon=0.75$  are explained in Fig. 7. As shown in the Figure,  $Nu$  enhances by increasing  $Da$  number. The augmentation of  $Nu$  number is more precipitous at  $Da$  greater than  $10^{-4}$ . This is owing

to increasing the porous medium permeability and accordingly the severe fluid flow which causes the enhancing the convective heat from the hot wall.

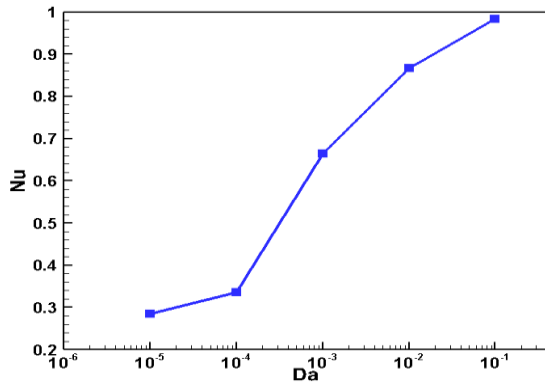


Fig. 7. The Nu number variation vs. Da number at  $\delta/L=0.2$ ,  $\varphi=0.05$ ,  $Ra=1.5*10^6$  and  $\varepsilon=0.75$

Table 1. The Nu number variation vs. Da number at  $\delta/L=0.2$ ,  $\varphi=0.05$ ,  $Ra=1.5*10^6$  and  $\varepsilon=0.75$

Da	10 <sup>-5</sup>	10 <sup>-4</sup>	10 <sup>-3</sup>	10 <sup>-2</sup>	10 <sup>-1</sup>
Nu	0.284	0.336	0.664	0.867	0.983

The isotherm lines at different Da numbers are compared at Fig. 8. The isotherm lines get more dense and perpendicular to the hot wall by increasing the Da number and results in the temperature gradients enhancement and this will lead to Nu number increase.

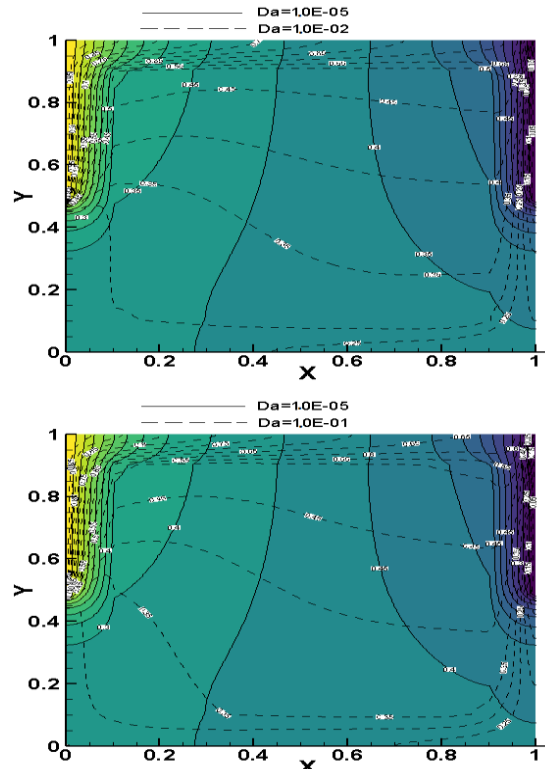
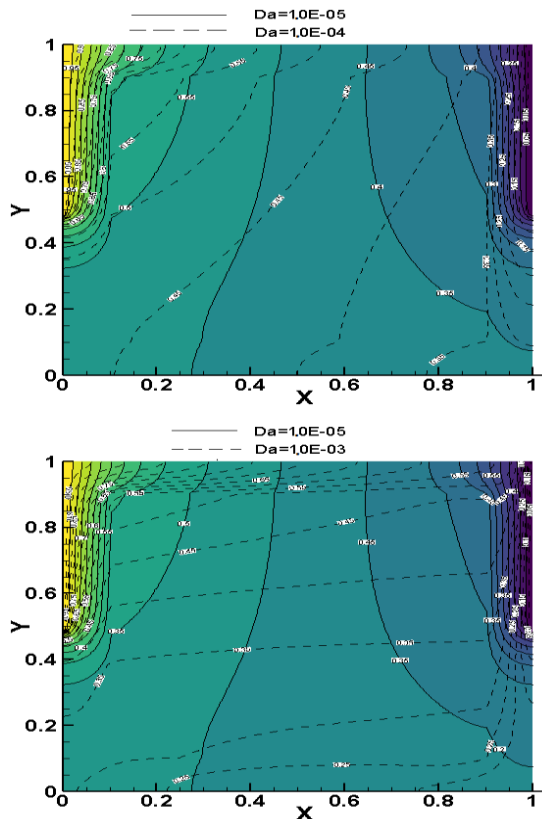


Fig. 8. Comparison of the isotherm lines at different Da numbers for  $\varphi=0.05$ ,  $\delta/L=0.2$ ,  $Ra=1.5*10^6$  and  $\varepsilon=0.75$

To investigate the impact of porous media thickness and the angles of inclination on Nu number and heat transfer performance Fig. 9 is presented at  $\varphi=0.05$ ,  $Ra=1.5*10^6$  and  $\varepsilon=0.75$ . According to Fig. 9, Nu increase almost 39%, 50% and 52% with enhancing porous media thickness from 0.1 to 0.6 at inclination angle of  $-30^\circ$ ,  $0^\circ$  and  $30^\circ$ , respectively.

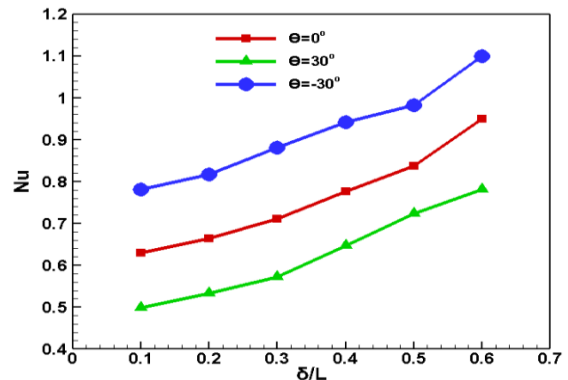


Fig. 9. The variation of the Nu number vs. porous media thickness at different angles of inclination at for  $\varphi=0.05$ ,  $Ra=1.5*10^6$ ,  $k_s/k_f=67$  and  $\varepsilon=0.75$

As can be seen in Fig. 10, increasing porous media thickness  $\delta/L$  substantially weakens natural convection heat transfer and convection field is confined in non-porous region. However, by comparing the isotherm lines of  $\delta/L=0.1$  and  $\delta/L=0.5$ , the isotherm lines become more uniform which indicates the dominance of conduction heat transfer mechanism at higher

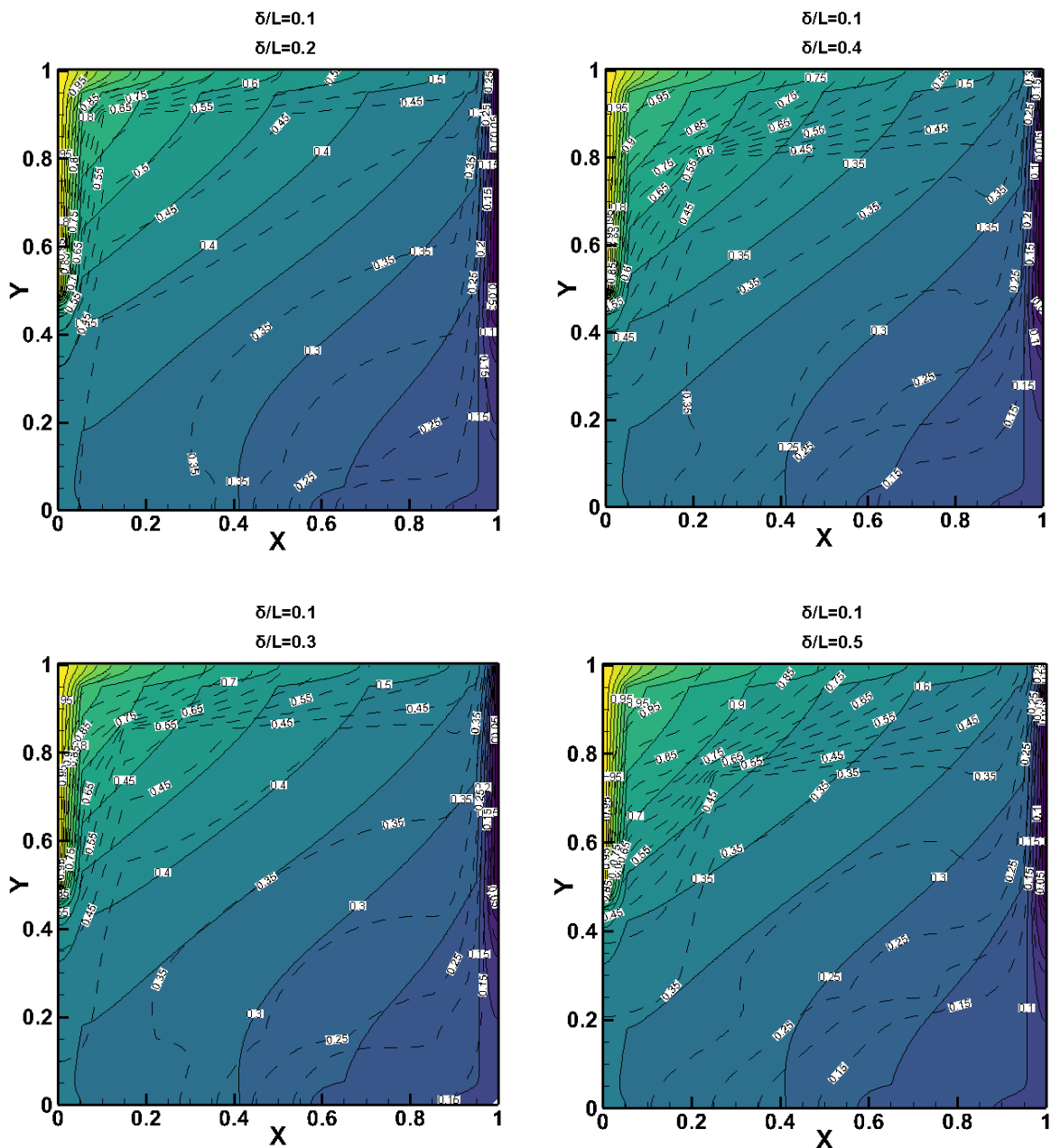


thicknesses. Dominance of conduction heat transfer will lead to Nu increment due to high value of conductivity ratio. The Nu reduces with enhancement of inclination angle from  $-30^\circ$  to

$+30^\circ$ . With increasing the inclination angle, the gravitational component will reduce and consequence the buoyancy force reduction and Nu number decline.

**Table 2.** The variation of the Nu number vs. porous media thickness at different angles of inclination at for  $\phi=0.05$ ,  $Ra=1.5 \times 10^6$ ,  $k_s/k_f=67$  and  $\epsilon=0.75$

$\delta/L$	0.1	0.2	0.3	0.4	0.5	0.6
<b>Nu (<math>\theta=0^\circ</math>)</b>	0.782	0.816	0.881	0.942	0.982	1.151
<b>Nu (<math>\theta=30^\circ</math>)</b>	0.633	0.664	0.711	0.777	0.838	0.951
<b>Nu (<math>\theta=-30^\circ</math>)</b>	0.499	0.534	0.572	0.647	0.723	0.781



**Fig. 10.** Comparison of the isotherm lines at different porous media thicknesses for  $\phi=0.05$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  and  $\epsilon=0.75$

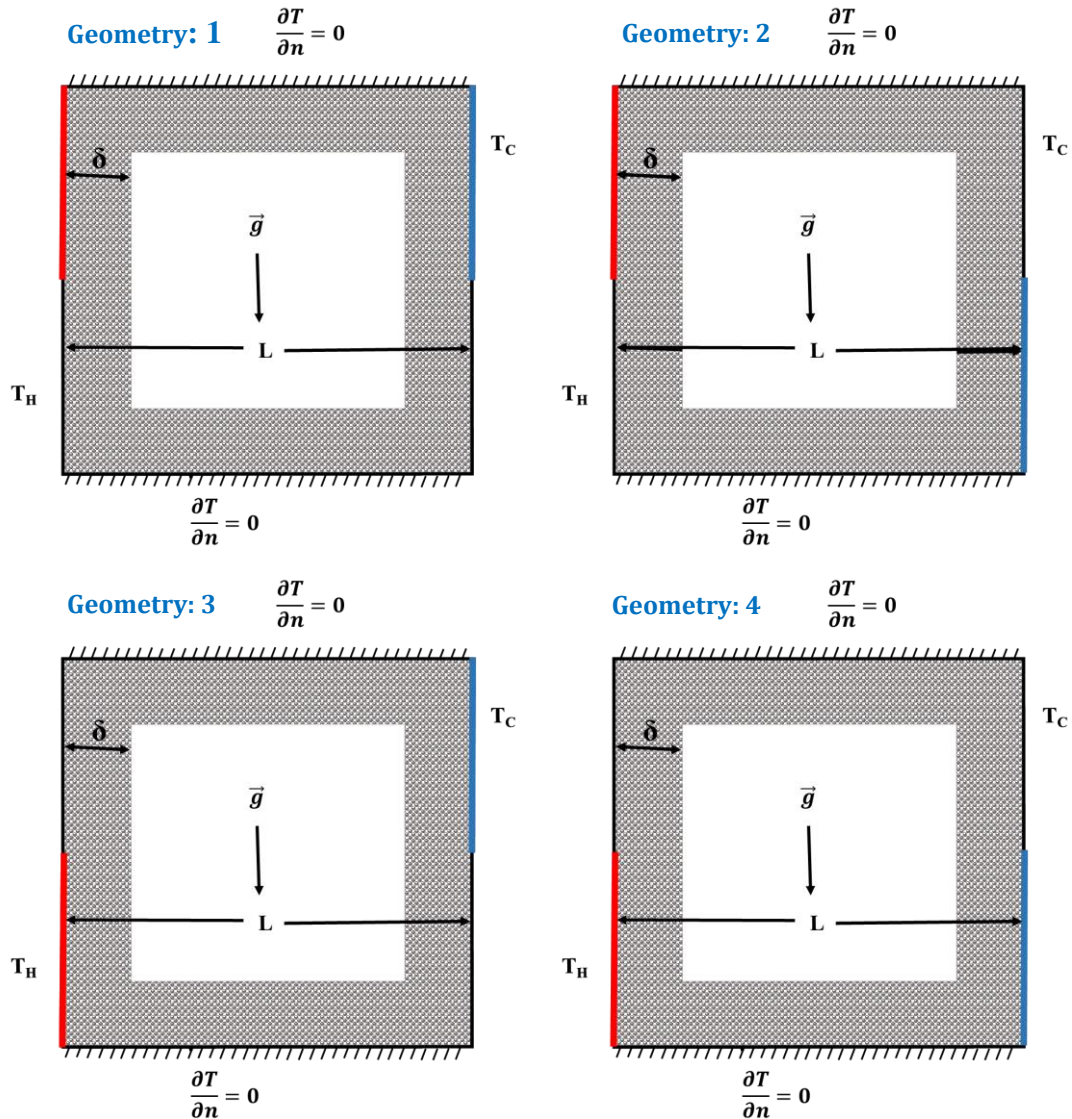


Fig. 11. The different arrangements of hot and cold sections in the cavity

The Nu number variation versus volume fraction of nanofluid at different geometries (Fig. 11) is depicted in Fig. 12. Nu linearly enhances by increasing volume fraction of nanoparticle at different location of hot and cold surfaces. Thermal conductivity of nanofluid augments with increment of nanoparticle volume fraction and leads in Nu number rise.

The Nu number enhances by setting the hot surface at the lower part of the cavity. The vorticity pattern in cavity for different positions of hot and cold surfaces is demonstrated in Fig. 13. When the hot surface is in the upper half of cavity at the geometry 1 and 2, two vorticities are formed which one of them is smaller in left side and the other in right side of the cavity. However, when the hot surface is located in the lower part of the cavity at geometry 3 and 4, one vorticity with more strength is created in the enclosure which will result in the Nu enhancement.

To investigate the variation trend of Nu number versus porosity at different Rayleigh numbers, the graph presented in Fig. 14 is provided. As obviously shown, for all the cases, enhancing the buoyancy force coming from augmentation of Ra boosts average Nusselt number. In fact, decreasing porosity will lead in declining of convection heat transfer and Nu reduction. The isotherm line for various porosity are depicted in Fig. 15. Convection is strengthened by increasing porosity and isotherm lines are getting more affected by advection fluid flow and denser next to the hot wall.

The Nu number variation versus Ha number at different power law indexes is demonstrated in Fig. 16. Nu decreases almost 10.5%, 7.6% and 5.2% by increasing Ha number from 0.0 to 40.0 at viscosity power law index of 0.6, 1.0 and 1.4, respectively.

Fig. 17 provides isotherm lines for different power law indexes at  $\phi=0.05$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$ . It is noticed that the power-law index enhancement make the fluid isotherms density on the hotter wall to enhance. Actually, it reveals that the rise of power-law index augments the convection heat transfer in general. The impact is most evident in the isotherm near the upper left side of the enclosure where the enhancement in the power-law index induces the movement of the isotherm between the hot and cold walls to increase. The comparison between different power law indices in the isotherms elucidates that the Dilatant nanofluid ( $n>1$ ) isotherms leans more toward the hot wall than the pseudoplastic nanofluid ( $n<1$ ) and thus proves that heat transfer augments with enhancing power law index.

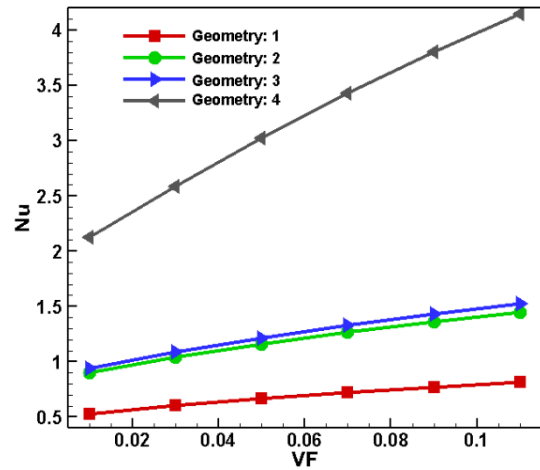


Fig. 12. The Nu variation vs. volume fraction of nanofluid with different positions of hot and cold surfaces at  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  and  $\epsilon=0.75$

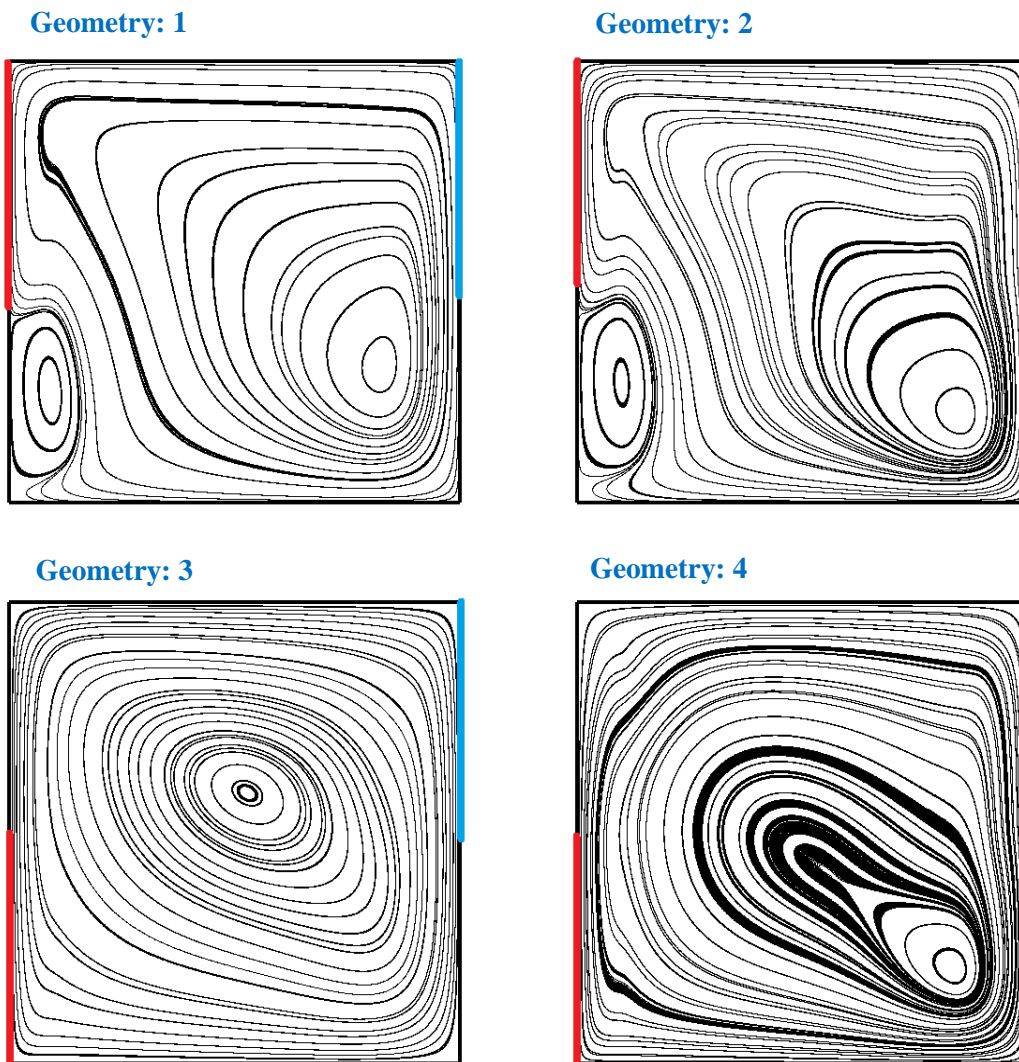
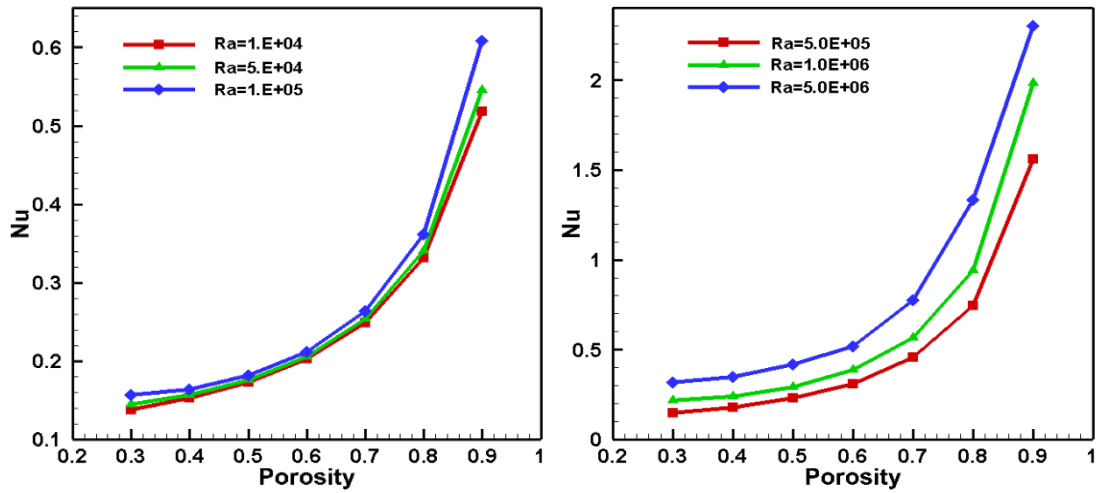


Fig. 13. The stream line at  $\phi=0.01$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  and  $\epsilon=0.75$  for different location of hot and cold surfaces

**Table 3.** The Nu variation vs. volume fraction of nanofluid with different positions of hot and cold surfaces at  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  and  $\varepsilon=0.75$

VF	0.01	0.03	0.05	0.07	0.09	0.11
<b>Nu (Geomety 1)</b>	0.529	0.601	0.664	0.720	0.769	0.813
<b>Nu (Geomety 2)</b>	0.903	1.039	1.161	1.269	1.364	1.447
<b>Nu (Geomety 3)</b>	0.941	1.087	1.215	1.329	1.431	1.523
<b>Nu (Geomety 4)</b>	2.126	2.584	3.022	3.429	3.804	4.144



**Fig. 14.** The Nu number variation vs. porosity at  $\phi=0.01$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  for different Ra numbers

**Table 4.** The Nu number variation vs. porosity at  $\phi=0.01$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^6$  for different Ra numbers

$\varepsilon$	0.3	0.4	0.5	0.6	0.7	0.8	0.9
<b>Nu (Ra=1.E+4)</b>	0.138	0.153	0.173	0.202	0.248	0.332	0.518
<b>Nu (Ra=5.E+4)</b>	0.145	0.157	0.176	0.205	0.257	0.341	0.545
<b>Nu (Ra=1.E+5)</b>	0.157	0.164	0.181	0.212	0.264	0.361	0.608
<b>Nu (Ra=5.E+5)</b>	0.171	0.180	0.230	0.310	0.456	0.746	1.562
<b>Nu (Ra=1.E+6)</b>	0.216	0.237	0.2916	0.388	0.567	0.941	1.985
<b>Nu (Ra=5.E+6)</b>	0.322	0.351	0.421	0.520	0.776	1.334	2.302

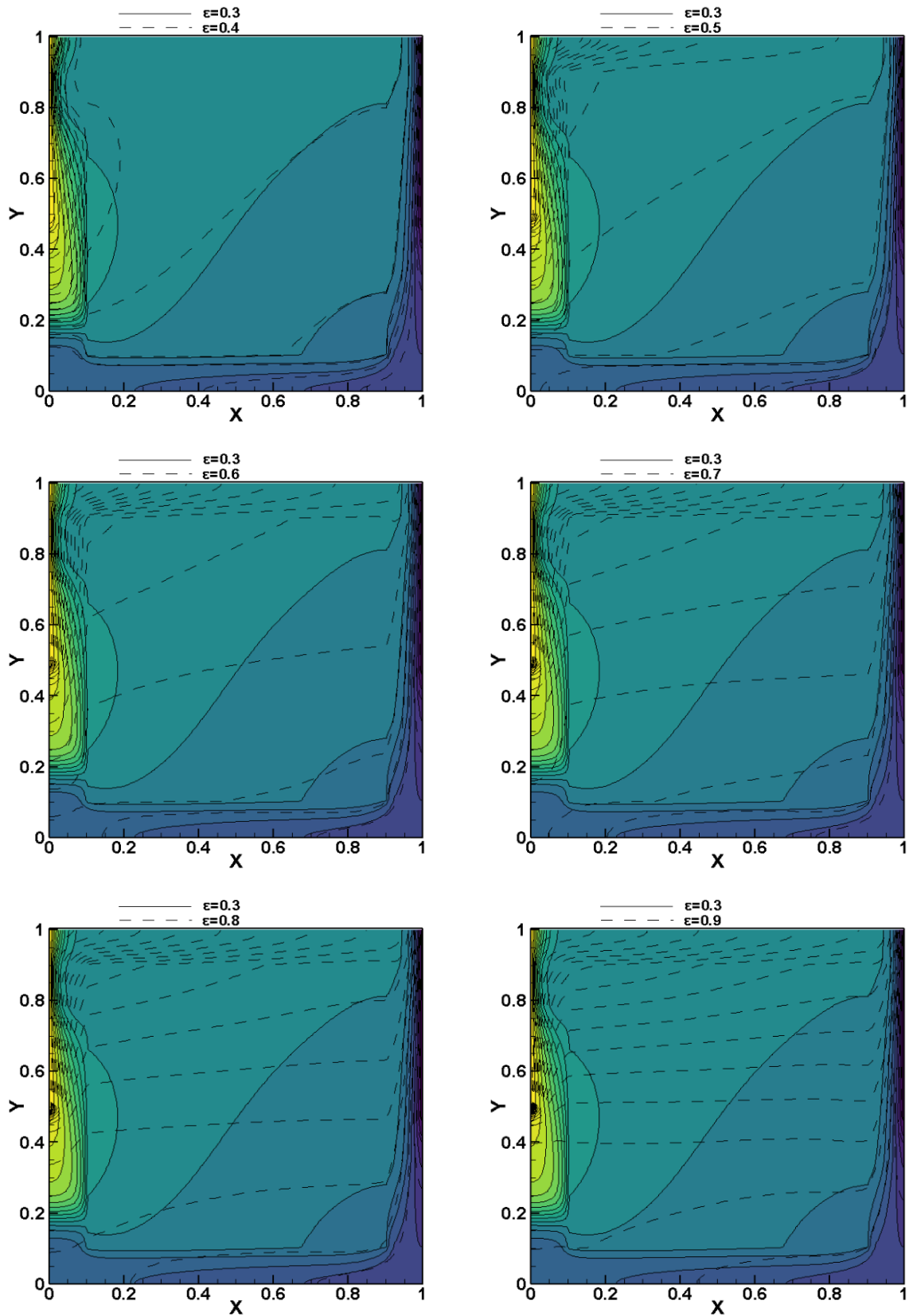


Fig. 15. Comparison of the isotherm lines at different porosities for  $\phi=0.05$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

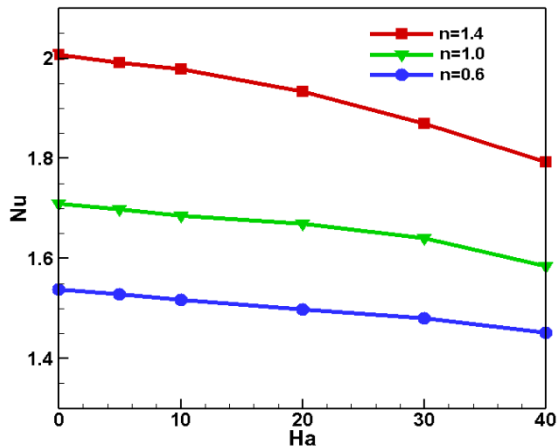


Fig. 16. Nu number variation vs. Ha number in different viscosity power law indexes at  $\phi=0.05$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

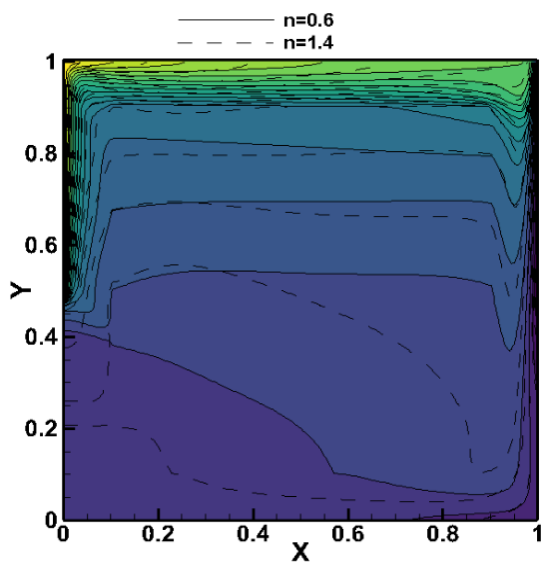
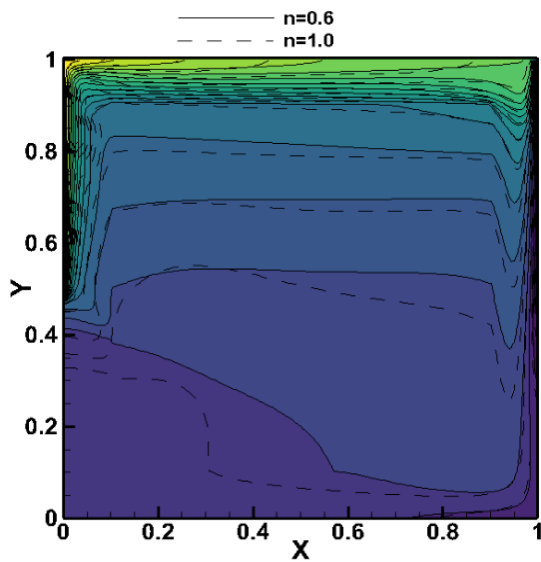


Fig. 17. Comparison of the isotherm lines at different viscosity power law indexes for  $\phi=0.05$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

Fig. 18 illustrates Nu number increases by enhancement of nanoparticle volume fraction. It reveals Nu increases almost 36%, 47% and 53% at inclination angle of  $30^\circ$ ,  $0^\circ$  and  $-30^\circ$  by enhancing volume fraction from 0.02 to 0.12, respectively. Nanoparticle effect is more dominant at angle of  $-30^\circ$ . Because, in this situation, buoyancy forces boosts natural convection and this is supplemented by nanoparticle conductivity augmentation in nanofluid.

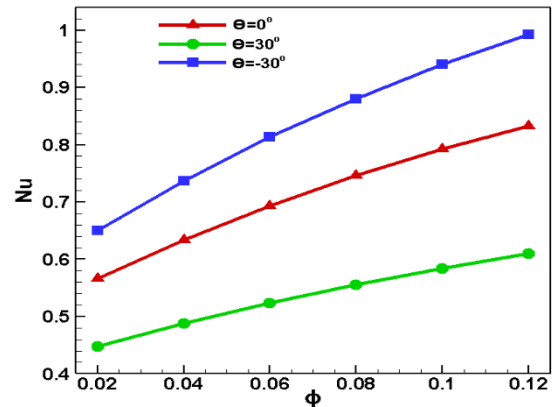


Fig. 18. Nu number vs. nanoparticle volume fraction at different angle of inclinations for  $\epsilon=0.75$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

The Bejan number is defined the ratio of heat transfer irreversibility to total irreversibility due to heat transfer and fluid friction. The variation of magnetic entropy generation and Be number is demonstrated in Fig. 19. Magnetic entropy generation and Be number enhances by increasing Da number. Convection heat transfer and fluid flow will be more dominant by augmentation of permeability and Da number and this will lead to more entropy generation by fluid flow and less entropy generation and irreversibility by magnetic field and convective heat transfer process.

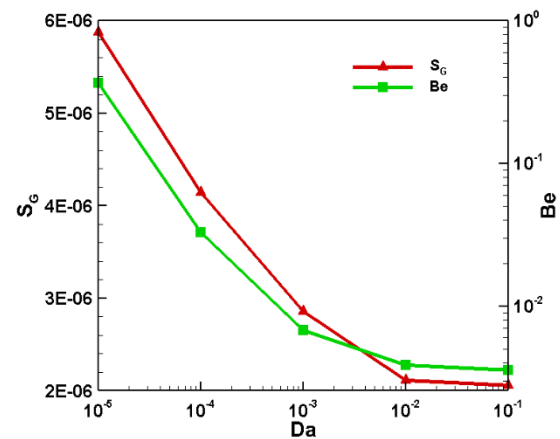


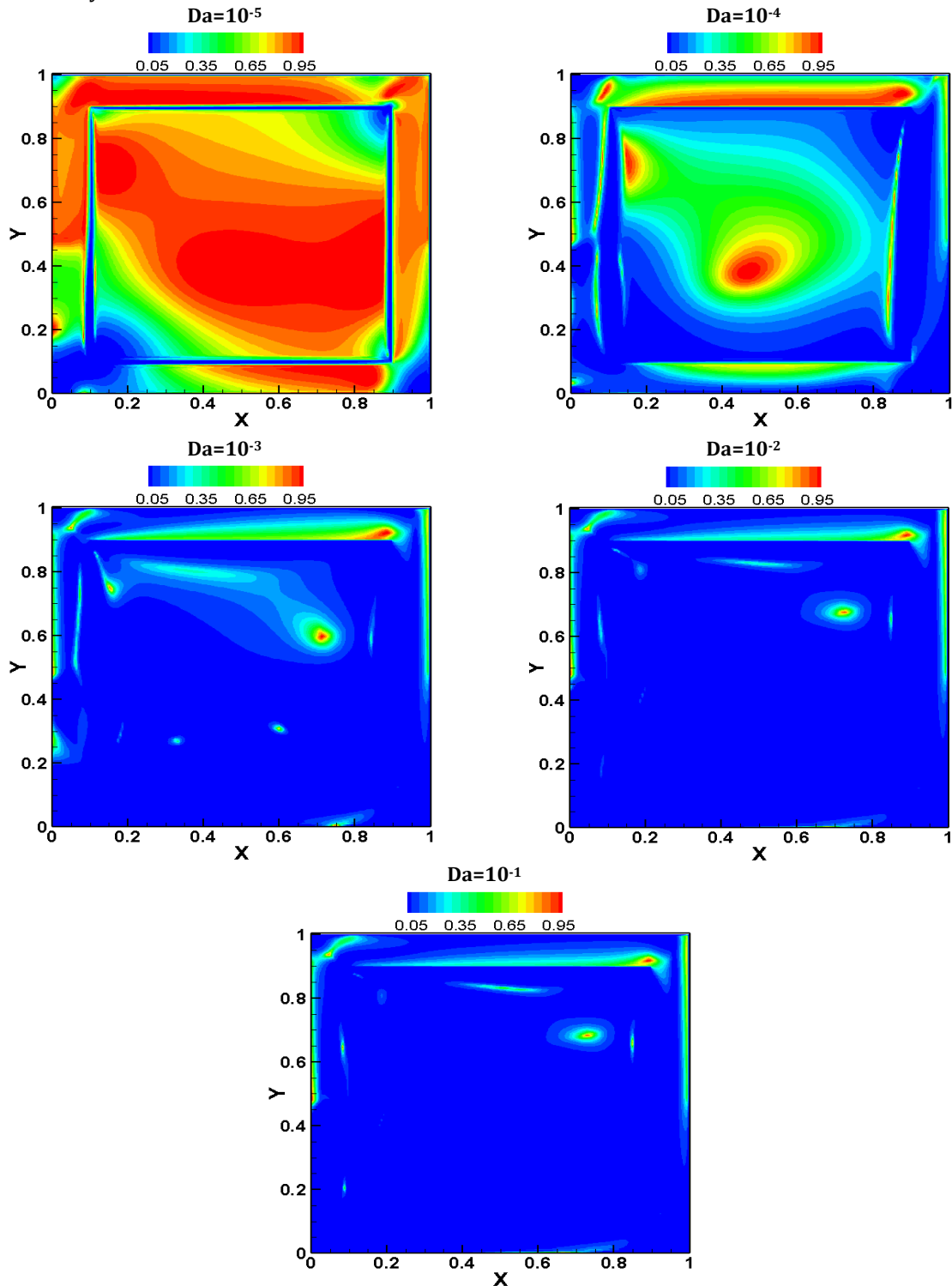
Fig. 19. The magnetic entropy production and Bejan number variation vs. Darcy number at  $\phi=0.05$ ,  $\delta/L=0.2$ ,  $Ra=1.5 \times 10^5$  and  $\epsilon=0.75$

**Table 5.** Nu number vs. nanoparticle volume fraction at different angle of inclinations for  $\epsilon=0.75$ ,  $Da=0.001$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

$\varphi$	0.1	0.2	0.3	0.4	0.5	0.6
<b>Nu (<math>\theta=0^\circ</math>)</b>	0.566	0.633	0.693	0.745	0.792	0.833
<b>Nu (<math>\theta=30^\circ</math>)</b>	0.447	0.487	0.523	0.555	0.583722	0.609476
<b>Nu (<math>\theta=-30^\circ</math>)</b>	0.649	0.737	0.813	0.880	0.939825	0.992884

The contour of Be number versus Da number is also depicted in Fig. 20. In low Da number irreversibility spreads all over the region and it indicates a dominance of the heat transfer irreversibility over the fluid friction

irreversibility. However, in higher Da number, fluid flow entropy generation is the main cause of irreversibility and Be number is substantially suppressed in fluid domain.



**Fig. 20.** Be number contours at different Da numbers for  $\epsilon=0.75$ ,  $\varphi=0.05$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

At Fig. 21, fluid flow and magnetic field entropy generation decrease and heat transfer irreversibility and Be number rises by porous thickness ratio. It is obvious that Ra number reduces by increasing porous thickness ratio and natural convection decrease consequently and so that fluid flow friction entropy generation and magnetic field effect on irreversibility will diminish. However, temperature gradient enhances in solid wall especially in porous section and as a result entropy generation will increase as depicted in Fig. 22.

As it is seen in Fig. 23, Be number and thermal entropy generation increases with porosity. However, irreversibility due to fluid friction and magnetic field decreases. By increasing porosity fluid can flow more freely and therefore fluid friction reduces and this will result in SF reduction. Conversely, convection heat transfer is strengthened by increasing porosity and this will lead to enhancement of temperature gradient in the enclosure particularly in the middle zone and the upper porous section as revealed in Fig. 24.

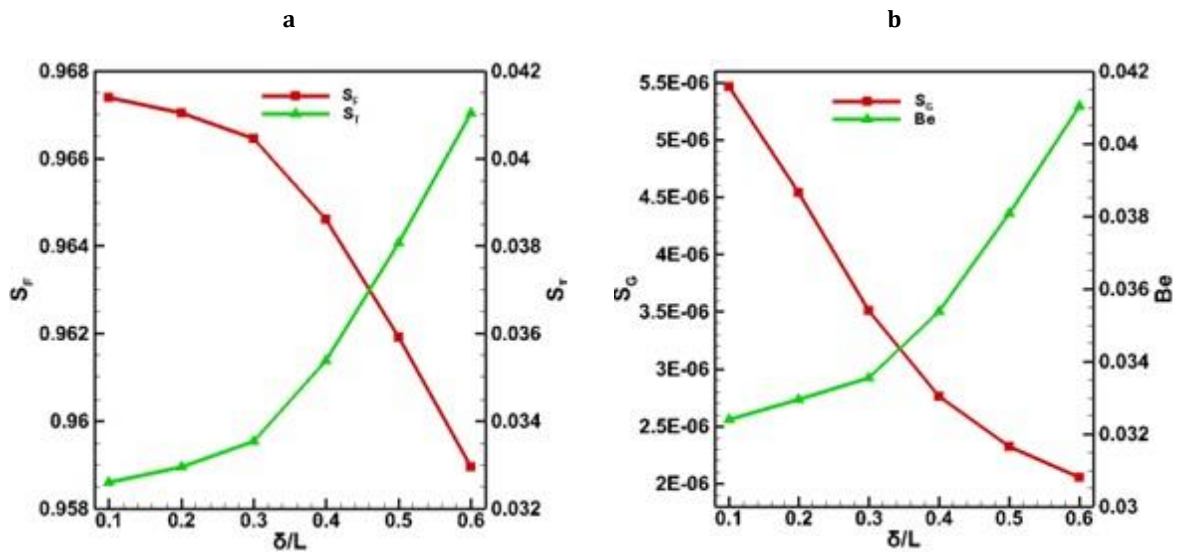
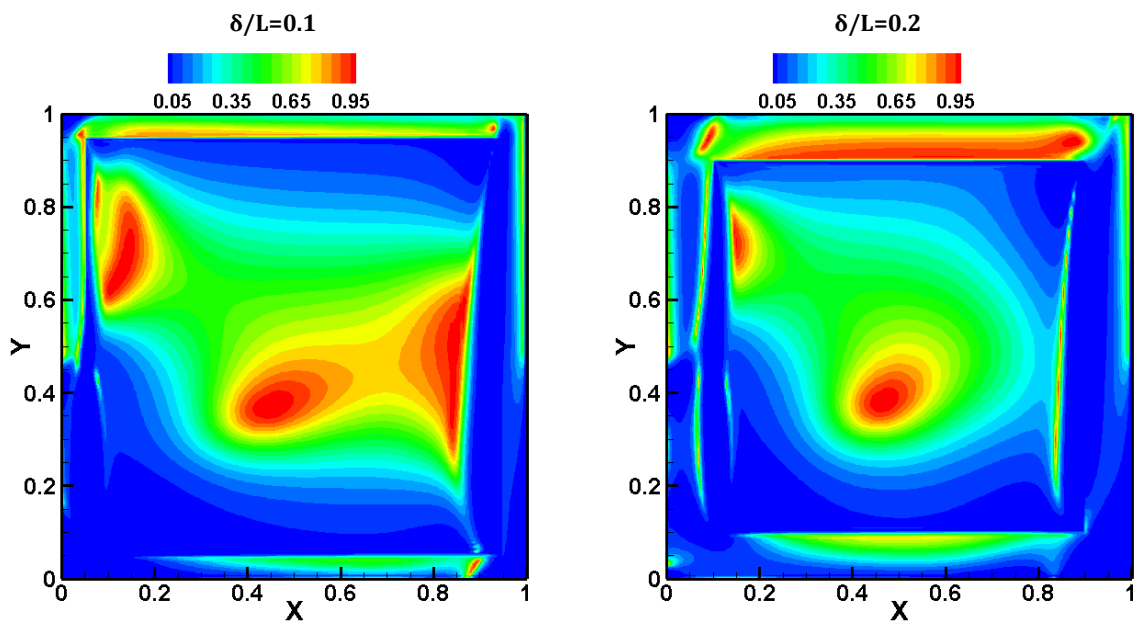


Fig. 21. The a) fluid flow and thermal entropy production and b) magnetic entropy production and Bejan number variation vs. thickness ratio at  $Da=0.001$ ,  $\phi=0.05$ ,  $Ra=1.5 \times 10^5$  and  $\epsilon=0.75$





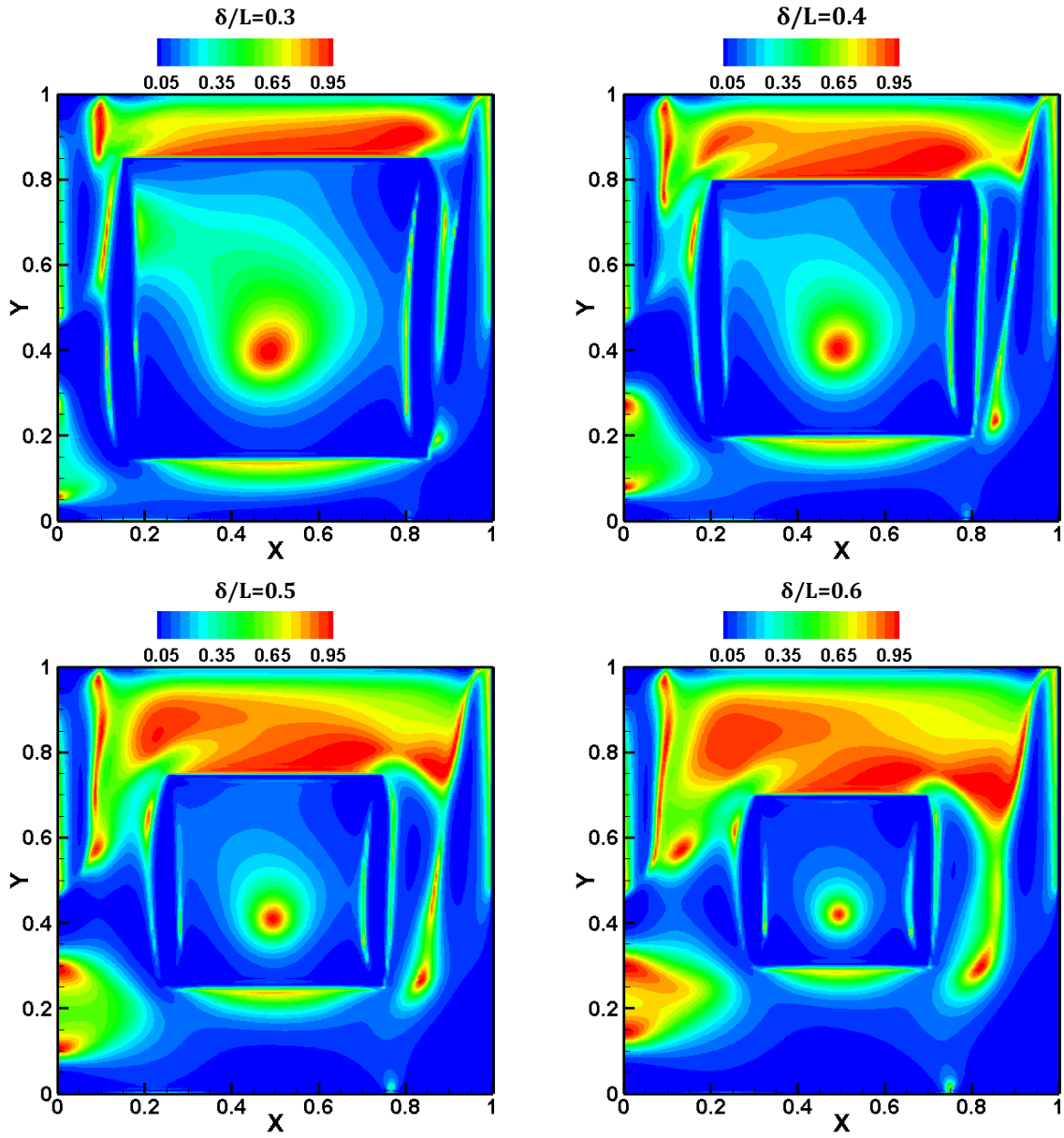


Fig. 22. Be number contours at different thickness ratios for  $\epsilon=0.75$ ,  $\phi=0.05$ ,  $Ra=1.5 \times 10^5$  and  $Da=0.001$

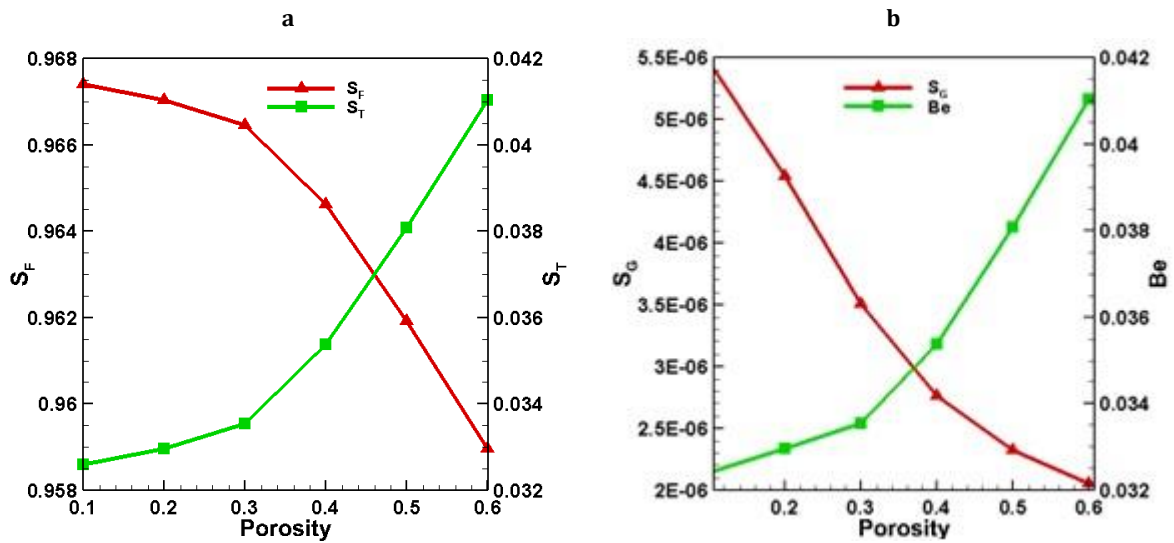


Fig. 23. The a) fluid flow and thermal entropy production and b) magnetic entropy production and Bejan number variation vs. porosity at  $Da=0.001$ ,  $\phi=0.05$ ,  $Ra=1.5 \times 10^5$  and  $\delta/L=0.2$

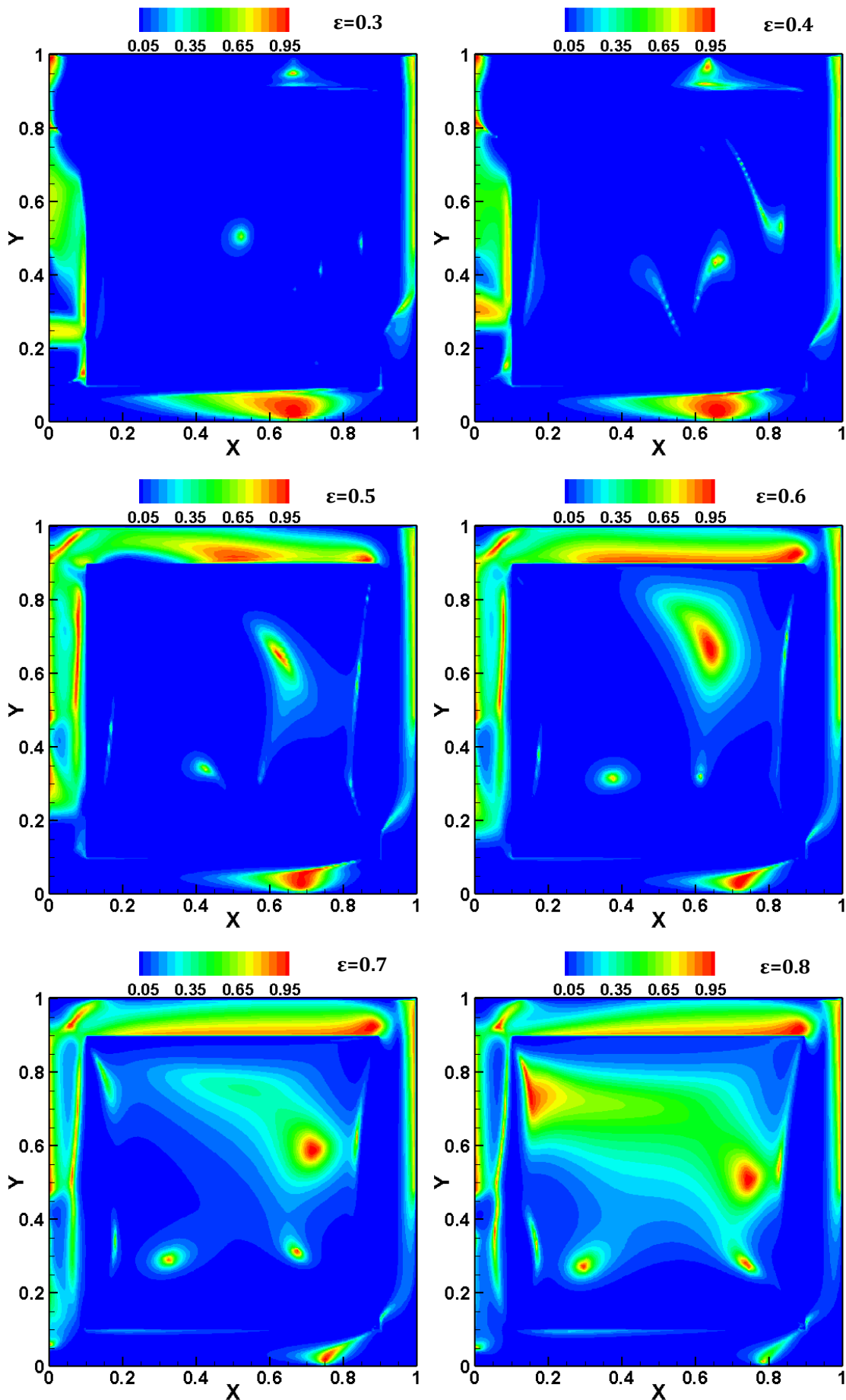


Fig. 24. Be number contours at different porosities for  $\delta/L=0.2$ ,  $\varphi=0.05$ ,  $Ra=1.5 \cdot 10^5$  and  $Da=0.001$

## 5. Conclusion

In this study, MHD of hybrid Cu-Al<sub>2</sub>O<sub>3</sub> non-Newtonian nanofluid with partially heated wall in an inclined partially porous cavity was investigated. The irreversibility and average Nusselt number were obtained by using CVFVM at the presence of magnetic field. The effect of main parameters such as porosity, porous thickness ratio, position of heated wall, Da number, Ha number and the angle of inclination was evaluated on the entropy generation and average Nu number. The important results can be summarized as follows:

1. Nu number increase almost 39%, 50% and 52% with enhancing porous media thickness from 0.1 to 0.6 at inclination angle of -30°, 0° and 30°, respectively.
2. The Nu number increases by setting the hot surface at the lower part of the cavity.
3. Convection heat transfer and fluid flow will be more dominant by augmentation of permeability and Da number and this will lead to more entropy production by fluid flow and less entropy production and irreversibility by magnetic field and convective heat transfer process.
4. Fluid flow and magnetic field entropy generation decrease and heat transfer irreversibility and Be number rises by porous thickness ratio.

## Nomenclature

$B$	Magnetic field (T)
$C_f$	Specific heat at constant pressure (kJ/kg.K)
Da	Darcy number
Ha	Hartmann number ( $Ha = BL\sqrt{\sigma/\nu}$ )
$k$	Heat conductivity (W/m.K)
$L$	Length of cavity (m)
$n$	Power-law index
$N$	The consistency coefficient
Nu	Nusselt number ( $Nu = hL/k$ )
$p$	Pressure (Pa)
Pr	Prandtl number ( $Pr = \nu/\alpha$ )
Re	Reynolds number ( $Re = UL/\nu$ )
$t$	Time (s)
$T$	Temperature (K)
$u$	x-velocity (m/s)
$v$	y-velocity (m/s)
$x, y$	Cartesian coordinate (m)

## Greek letters

$\rho$	Density (kg/m <sup>3</sup> )
$\mu$	Viscosity (Pa.s)
$\sigma$	The electrical conductivity (S/m)
$\varphi$	Volume fraction
$\tau$	Shear stress (N/m <sup>2</sup> )
$\varepsilon$	Porosity
$\theta$	Angle (degree)

## Subscripts

C	Cold
H	Hot
nf	Nanofluid

## Funding Statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

## Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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