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Reliability analysis of lifetime systems based on Weibull distribution

Mohammadreza Shahriari[a], Hooman Shahrasbi[b,*], Arash Zaretalab[c]

[a] *Faculty of Industrial Management, South Tehran Branch, Islamic Azad University, Tehran, Iran*

[b] *Department of Economics, Management, Industrial Engineering and Tourism, University of Aveiro, Aveiro, Portugal*

[c] *Department of Business Management, Shahr-e-Qods Branch, Islamic Azad University, Tehran, Iran*

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Abstract

Reliability analysis is crucial for understanding the performance and failure characteristics of lifetime systems. This paper presents a comprehensive study on the reliability analysis of lifetime systems using the Weibull distribution. The Weibull distribution, known for its flexibility in modeling failure times, provides a versatile framework for capturing diverse failure behaviors. A useful model for redundancy systems is proposed in this paper. The model consists of $(n+1)$ components, where n components serve as spare parts for the main component. The failure rate of the working component is time-dependent, denoted as $\lambda(t)$, while the failure rates of the non-working components are assumed to be zero. Whenever a component fails, one of the spare parts immediately takes over its role. The failed components in this model are considered non-repairable. To analyze this model, we establish the differential equations that describe the system states. By solving these equations, we calculate important parameters such as system reliability and mean time to failure (MTTF) in real-time scenarios. These parameters provide valuable insights into the performance and behavior of the system under study. By employing the Weibull distribution and the proposed model, this paper contributes to enhancing the understanding of reliability analysis in lifetime systems and enables the estimation of important reliability parameters for practical applications.

Keywords: reliability, Markov chain, redundancy, Weibull distribution
2020 MSC: 60K10, 60J10

1 Introduction

Reliability analysis based on the Weibull distribution is a widely employed approach to evaluate the reliability and lifetime characteristics of various systems. The Weibull distribution is a probabilistic model commonly used to represent the failure times of components or systems.

The Weibull distribution is characterized by two parameters: the shape parameter and the scale parameter. The shape parameter determines the failure mode of the system, while the scale parameter defines the characteristic life

*Corresponding author

Email addresses: shahriari.mr@gmail.com (Mohammadreza Shahriari), hooman@ua.pt (Hooman Shahrasbi), arash_zaretalab@yahoo.com (Arash Zaretalab)

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or scale of the distribution. This distribution offers flexibility and can effectively capture various failure behaviors, including early-life failures, constant failure rates, and wear-out failures.

Redundancy systems are of great practical importance in reliability analysis. In these systems, calculating the probability density function ($p.d.f$) of the subsystems can be complex, often requiring simulation-based approaches. In this paper, we investigate a useful model for redundancy systems and derive the $p.d.f$ of the model. This particular model consists of $(n+1)$ components, where n components serve as spare parts for the main component. The failure rate of the working component is time-dependent, following the function $\lambda(t)$, while the failure rates of the non-working components are assumed to be zero. When a component fails, one of the spare parts immediately takes over its function. It is important to note that the failed components in this model are considered non-repairable.

Reliability analysis often employs the k-out-of-n system as a common model to evaluate the reliability of systems composed of multiple components. In the k-out-of-n system, the system functions successfully if at least k out of the n components remain operational.

By employing the Weibull distribution and studying redundancy systems, this paper contributes to the field of reliability analysis and provides insights into system behavior and reliability under various scenarios.

The k-out-of-n system is a valuable approach used in various industries to model systems with redundant components, where the overall system's reliability depends on the functioning of a specified number of components. There are two main variations of this system: the k-out-of-n:G (series-parallel) system and the k-out-of-n:F (parallel-series) system.

1. k-out-of-n: G (series-parallel) System: In the k-out-of-n:G system, the components are arranged in both series and parallel combinations. This means that the components are grouped together in parallel, and each group is then connected in series. The system is considered operational if there are at least k working components among the n total components.
   One way to envision this system is to imagine having several branches, each containing multiple components in parallel, and then these branches are connected in series. For the system to function properly, at least k branches need to have working components. If any of the branches fail to meet the minimum requirement (less than k working components), the entire system will be considered failed.
   The k-out-of-n: G system is often referred to as a "series-parallel" system due to the combination of these two arrangements. Analyzing the reliability of the k-out-of-n:G system involves evaluating various reliability metrics, such as the reliability function and failure rate, based on the individual component reliabilities and the specific system configuration.
2. k-out-of-n: F (parallel-series) System: In the k-out-of-n:F system, the components are grouped in parallel sets, and each set is then connected in series. The system is considered operational if there are at least k fully functional parallel groups among the n total groups.
   To visualize this system, think of multiple sets of components that are connected in parallel, and each set contains multiple components that are connected in series. For the system to operate correctly, at least k sets need to have all their components functioning. If any of the sets fail to meet the minimum requirement (less than k fully functional groups), the entire system will be considered failed.
   The k-out-of-n:F system is often referred to as a "parallel-series" system because it combines the parallel and series arrangements. Similar to the k-out-out-n:G system, analyzing the reliability of the k-out-of-n: F system involves evaluating the reliability function, failure rate, and other reliability metrics based on the individual component reliabilities and the specific system configuration.
   In both variations of the k-out-of-n system, redundancy plays a crucial role in ensuring system reliability. These configurations are commonly used in industries where system failures could have significant consequences, such as aerospace, telecommunications, and power distribution, among others. Reliability analysis of such systems helps engineers and decision-makers in designing robust and dependable systems that can withstand component failures and maintain overall functionality.

Table 1 summarizes some studies on k-out-of-n systems, as these systems are the focus of extensive research in the field of reliability. However, there is limited research specifically addressing the reliability of subsystems within these k-out-of-n systems. Therefore, further studies exploring the reliability of subsystems in k-out-of-n systems are needed to enhance our understanding and enable more comprehensive reliability analyses.

Note: The content of Table 1, mentioned in the text, is not provided in the given information.

Table 1: Classification of literature on $k-out-of-n$ systems

References	State	Consecutive	Repairable	$k-out-of-n$ Configuration	Achievement
[3]	Binary	✓	–	F	Exact expression for the failure probability
[10]	Binary	✓	–	G	Matrix formulation and solution
[19]	Binary	–	✓	F	Priority repair rule
[24]	Binary	–	–	F/G	Use the components subject to common shocks
[8]	Binary	✓	–	F/G	Computational algorithm
[18]	Binary	–	–	G	Compute the optimal D value in D-Policy
[12]	Binary	–	–	F/G	Consider the environmental effects
[5]	Binary	✓	–	F	Present a bound for n_k
[1]	Binary	–	–	G	Computational algorithm
[35]	Binary	✓	✓	F	Matrix formulation and solution
[21]	Binary	–	–	F/G	Present a limit reliability function of homogeneous series
[4]	Binary	–	–	F/G	Develop a data completion procedure
[17]	Binary	–	–	F/G	Achieve exact formula and bounds
[2]	Binary	–	–	G	Computational algorithm
[7]	Binary	–	–	F/G	Present exact and approximate approach
[9]	Binary	✓	–	F/G	Present a heuristic algorithm for replacement policies
[13]	Binary	✓	–	F	Present a formula for two-dimensional lower bound
[15]	Binary	✓	–	G	Prove some theorem
[6]	Binary	–	–	F	Investigate that where to allocate the spares
[20]	Binary	–	–	F/G	Derive some formulas
[11]	Binary	✓	–	F	Assume that the states are fuzzy
[22]	Multi	–	–	G	Use the markov chain
[14]	Multi	–	–	G	Present an analytic model
[16]	Multi	–	–	F/G	Approve analytical approach
[29, 36]	Multi	–	✓	F	Markov process imbedding method is used to analyze the number of working units in each sector for each model.
[30, 34]	Multi	–	–	F	A combination of finite Markov chain imbedding approach is presented.
[27, 33]	Binary	–	✓	G	Reliability measures have been computed for different types of configurations.
[23, 32]	Binary	–	✓	G	A condition-based model for determining the optimal inspection period for minimum long-run average cost rate.

This paper is structured into four main sections. Each section serves a specific purpose in presenting and analyzing the proposed model:

1. Literature Survey: The first part of the paper involves a comprehensive literature survey. In this section, the authors review and summarize existing research, studies, and relevant literature related to the subject matter of the model. This literature review helps establish the context and background of the research, highlighting the existing knowledge, gaps, and previous approaches used in the field.
2. Model Presentation: The second part of the paper is dedicated to presenting the model. Here, the authors detail the formulation and development of their proposed model. They explain the underlying principles, assumptions, and equations that constitute the model. This section provides readers with a clear understanding of the theoretical framework and methodology used in the research.
3. Numerical Example: In the third part of the paper, the authors provide a numerical example to demonstrate the practical application of the model. They select a specific case study or scenario and apply their proposed model to solve it. By doing so, they showcase how the model works in real-world situations and how it can be used to analyze and solve problems in the relevant domain.
4. Conclusion and Further Studies: The last part of the paper is devoted to drawing conclusions based on the findings from the literature survey, model presentation, and numerical example. The authors summarize the key results and insights obtained from their research. Additionally, they may discuss the limitations of their model and suggest potential areas for improvement or future research directions. This section provides closure to the paper and highlights the significance of the study's contributions to the field.

By dividing the paper into these four parts, the authors follow a structured and logical approach to effectively communicate their research process, findings, and implications to the readers. This format helps readers navigate the paper easily, understand the context and motivation for the proposed model, grasp the technical details of the model itself, and gain insights into its practical applicability through the numerical example. The conclusion and further studies section also encourage the advancement of knowledge in the field by identifying potential avenues for future research and exploration.

2 Proposed procedure

To perform reliability analysis using the Weibull distribution, several steps are typically followed to gain insights into the failure characteristics of lifetime systems [31]:

1. Data Collection: It is essential to gather failure time data from the system under consideration. This data should include the precise times at which failures occurred, enabling a comprehensive analysis of the system's reliability.
2. Data Preparation: Organizing the failure time data is crucial. This involves calculating the time-to-failure values, typically achieved by subtracting the start time from the recorded failure time. This step ensures that the data is properly structured for further analysis.
3. Data Visualization: Creating a scatter plot or histogram of the failure times helps visualize the distribution of the data. This graphical representation aids in identifying patterns and determining if the data aligns with the characteristics of the Weibull distribution. Patterns such as early-life failures, constant failure rates, or wear-out failures can be identified through data visualization.
4. Parameter Estimation: Once the data is prepared and visualized, the parameters of the Weibull distribution need to be estimated. Statistical methods like maximum likelihood estimation or least squares estimation is commonly employed to estimate these parameters accurately. This step allows for a more precise fitting of the Weibull distribution to the failure time data.
5. Reliability Metrics Calculation: With the estimated parameters, various reliability metrics can be calculated. These metrics include the reliability function, which represents the probability of a system surviving without failure up to a given time. The failure rate, indicating the rate of failures over time, provides insights into the likelihood of failure at different stages of the system's life. Additionally, the mean time to failure (MTTF) represents the average time until failure occurrence, serving as a key metric for understanding the system's reliability performance.

Reliability analysis based on the Weibull distribution empowers engineers and analysts to comprehend the failure characteristics of lifetime systems effectively. This understanding enables informed decisions to enhance system reliability and performance. The Weibull distribution is a valuable tool utilized in various industries, including manufacturing, electronics, automotive, and aerospace, where product reliability plays a pivotal role in customer satisfaction and operational success. By leveraging the Weibull distribution, reliability analysis aids in optimizing maintenance strategies, predicting failure rates, and making data-driven decisions to improve overall system reliability [25, 26, 28].

In this section, the notations used in this paper are presented as follows. Then the presented methodology is analyzed. The notations used in this paper are as follows:

n : Number of elements,

$\beta.e^{\alpha.t}$: Failure rate of each element at time t,

$P_i(t)$: Probability that the system is in state with i spare element at time t,

$R(t)$: Probability that system works at time t,

$MTTF$: Mean time to failure of the system,

Assume a system with $(n+1)$ elements that the elements start working after the failure of former element. It means than the system has n spare elements. The failure rate of working elements is time dependent as $\lambda.t$. The state of the system is shown in figure 1. In this figure, the node number $\{i; \quad i = 0, 1, ..., n\}$ denotes the number of elements that do not start working and the nude (f) is denoting that system is failed. In this figure $h(t) = \beta e^{\alpha.t}$

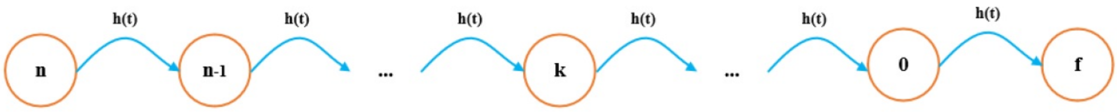

Figure 1: The structure of the model

3 Modeling

The failure rates of non-working elements are zero and when an element start working has the same failure rate of other worked elements. In this system we have:

$$R(t) = \sum_{i=0}^{n} P_i(t) \tag{3.1}$$

For the node number (0) to (n) in figure 1 we have:

$$\begin{cases} P_n(t+\Delta t) = P_n(t) - \beta e^{\alpha t}\Delta t P_n(t) \\ P_k(t+\Delta t) = P_k(t) - \beta e^{\alpha t}\Delta t P_k(t) + \beta e^{\alpha t}\Delta t P_{k+1}(t); \quad k < n. \end{cases} \tag{3.2}$$

By solving the equation number (3.2) we have:

$$\begin{cases} \lim_{\Delta t\to 0} \frac{P_n(t+\Delta t)-P_n(t)}{\Delta t} = P_n'(t) = -\beta e^{\alpha t}P_n(t) \Rightarrow P_n'(t) + \beta . e^{\alpha t}P_n(t) = 0 \\ \lim_{\Delta t\to 0} \frac{P_k(t+\Delta t)-P_k(t)}{\Delta t} = P_k'(t) = -\beta e^{\alpha t}P_k(t) + \beta e^{\alpha t}P_{(k+1)}(t) \Rightarrow P_k'(t) + \beta e^{\alpha t}P_k(t) = \beta e^{\alpha t}P_{(k+1)}(t); \quad k < n \end{cases} \tag{3.3}$$

To solve equation (3.3), we consider the following procedure. For the node number (n) we have $P_n'(t)+\beta e^{\alpha t}P_n(t) = 0$ and $e^{\int \beta e^{\alpha t}dt} = e^{\frac{\beta}{\alpha}e^{\alpha t}}$. Then $e^{\frac{\beta}{\alpha}e^{\alpha t}}P_n(t) = C$ and so $P_n(t) = Ce^{-\frac{\beta}{\alpha}e^{\alpha t}}$. This implies that $P_n(t) = Ce^{-\frac{\beta}{\alpha}e^{\alpha t}}$ and $P_n(0) = 1$. Hence, we have $1 = e^{-\frac{\beta}{\alpha}}C$. Thus, $P_n(t) = e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)}$ and for the node number $(n-1)$ we can calculate the $P_{(n-1)}(t)$. From $P_{(n-1)}'(t) + \beta e^{\alpha t}P_{(n-1)}(t) = \beta e^{\alpha t}P_n(t) = \beta e^{\alpha t}e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)}$ and $e^{\int \beta e^{\alpha t}dt} = e^{\frac{\beta}{\alpha}e^{\alpha t}}$, we have

$$e^{\frac{\beta}{\alpha}e^{\alpha t}}P_{(n-1)}(t) = \int e^{\frac{\beta}{\alpha}e^{\alpha t}}dt = \int \beta e^{\alpha t}e^{\frac{\beta}{\alpha}}dt = \left(\frac{\beta}{\alpha}\right)e^{\alpha t}e^{\frac{\beta}{\alpha}} + C.$$

This implies that $P_{(n-1)}(t) = \left(\frac{\beta}{\alpha}\right)e^{\alpha t}e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} + Ce^{-\frac{\beta}{\alpha}e^{\alpha t}}$ and $P_{(n-1)}(0) = 0$. These follow that $0 = \left(\frac{\beta}{\alpha}\right) + Ce^{-\frac{\beta}{\alpha}}$. Thus,

$$P_{(n-1)}(t) = \left(\frac{\beta}{\alpha}\right)e^{\alpha t}e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} - \left(\frac{\beta}{\alpha}\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} = \left(\frac{\beta}{\alpha}\right)\left(e^{\alpha t}-1\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} \tag{3.4}$$

Moreover, we have $P_{(n-2)}'(t) + \beta e^{\alpha . t}P_{(n-2)}(t) = \beta e^{\alpha t}P_{(n-1)}(t) = \frac{\beta^2}{\alpha}\left(e^{2\alpha t} - e^{\alpha t}\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)}$ and $e^{\int \beta e^{\alpha t}dt} = e^{\frac{\beta}{\alpha}e^{\alpha t}}$. Then,

$$\begin{aligned} e^{\frac{\beta}{\alpha}e^{\alpha t}}P_{(n-2)}(t) &= \int e^{\frac{\beta}{\alpha}e^{\alpha t}}\frac{\beta^2}{\alpha}\left(e^{2\alpha t} - e^{\alpha t}\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)}dt = \int \frac{\beta^2}{\alpha}\left(e^{2\alpha t} - e^{\alpha . t}\right)e^{\frac{\beta}{\alpha}}dt \\ &= \left(\frac{\beta}{\alpha}\right)^2\left(\frac{e^{2\alpha t}}{2} - e^{\alpha t}\right)e^{\frac{\beta}{\alpha}} + C. \end{aligned}$$

Hence, we have $P_{(n-2)}(t) = \left(\frac{\beta}{\alpha}\right)^2\left(\frac{e^{2\alpha t}}{2} - e^{\alpha t}\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} + Ce^{-\frac{\beta}{\alpha}e^{\alpha . t}}$ and $P_{(n-2)}(0) = 0$. These imply that $C = \frac{1}{2}\left(\frac{\beta}{\alpha}\right)^2 e^{\frac{\beta}{\alpha}}$ and we have

$$P_{(n-2)}(t) = \left(\frac{\beta}{\alpha}\right)^2\left(\frac{e^{2\alpha t}}{2} - e^{\alpha t}\right)e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} + \frac{1}{2}\left(\frac{\beta}{\alpha}\right)^2 e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)} = \left(\frac{\beta}{\alpha}\right)^2\frac{\left(e^{\alpha t}-1\right)^2}{2!}e^{-\frac{\beta}{\alpha}\left(e^{\alpha t}-1\right)}. \tag{3.5}$$

Assume for the nude number $P_{(k+1)}(t)$ we obtain:

$$P_{(k+1)}(t) = \left(\frac{\beta}{\alpha}\right)^{(n-k-1)} . \frac{(e^{\alpha.t}-1)^{(n-k-1)}}{(n-k-1)!} . e^{-\frac{\beta}{\alpha}.(e^{\alpha.t}-1)}. \tag{3.6}$$

Then for the nude number (k) we calculate the $P_k(t)$. From

$$P'_k(t) + \beta e^{\alpha t} P_k(t) = \beta e^{\alpha t} \left(\frac{\beta}{\alpha}\right)^{(n-k-1)} \left(\frac{\beta}{\alpha}\right)^{(n-k-1)} \frac{(e^{\alpha t}-1)^{(n-k-1)}}{(n-k-1)!} e^{-\frac{\beta}{\alpha}(e^{\alpha t}-1)}$$

and $e^{\int \beta e^{\alpha.t}.dt} = e^{\frac{\beta}{\alpha}e^{\alpha.t}}$, we have

$$\begin{aligned} e^{\frac{\beta}{\alpha}e^{\alpha t}} P_k(t) &= \int e^{\frac{\beta}{\alpha}e^{\alpha t}} \beta e^{\alpha t} \left(\frac{\beta}{\alpha}\right)^{(n-k-1)} \frac{e^{-\frac{\beta}{\alpha}(e^{\alpha t}-1)}}{(n-k-1)!} \sum_{i=0}^{n-k-1} \{(-1)^i C_i^{(n-k-1)} e^{i\alpha t}\} dt \\ &= \int \beta e^{\frac{\beta}{\alpha}} \left(\frac{\beta}{\alpha}\right)^{(n-k-1)} \sum_{i=0}^{n-k-1} \left\{(-1)^i C_i^{(n-k-1)} \frac{e^{(i+1)\alpha t}}{(n-k-1)!}\right\} dt \\ &= \left(\frac{\beta}{\alpha}\right)^{(n-k)} e^{\frac{\beta}{\alpha}} \sum_{i=0}^{n-k-1} \left\{\left((-1)^i C_i^{(n-k-1)} \frac{e^{(i+1)\alpha t}}{(i+1)(n-k-1)!}\right)\right\} + C. \end{aligned}$$

This implies that $0 = \left(\frac{\beta}{\alpha}\right)^{(n-k)} \sum_{i=0}^{n-k-1} \left\{(-1)^i . C_i^{(n-k-1)} \frac{1}{(i+1)(n-k-1)!}\right\} + Ce^{-\frac{\beta}{\alpha}}$ and so

$$C = \left(\frac{\beta}{\alpha}\right)^{(n-k)} \sum_{i=0}^{n-k-1} \left\{(-1)^i C_i^{(n-k-1)} \frac{1}{(i+1)(n-k-1)!}\right\} e^{-\frac{\beta}{\alpha}}.$$

Hence,

$$\begin{aligned} P_k(t) &= \left(\frac{\beta}{\alpha}\right)^{(n-k)} \frac{e^{-\frac{\beta}{\alpha}(e^{\alpha t}-1)}}{(n-k-1)!} \left\{ \sum_{i=0}^{n-k-1} \left\{(-1)^i C_i^{(n-k-1)} \frac{e^{(i+1)\alpha t}}{(i+1)}\right\} + \sum_{i=0}^{n-k-1} \left\{(-1)^i C_i^{(n-k-1)} \frac{1}{(i+1)}\right\} \right\} \\ &= \left(\frac{\beta}{\alpha}\right)^{(n-k)} \frac{e^{-\frac{\beta}{\alpha}(e^{\alpha t}-1)}}{(n-k-1)!} \sum_{i=0}^{n-k} \{(-1)^i C_i^{(n-k)} e^{i\alpha t}\} \\ &= \left(\frac{\beta}{\alpha}\right)^{(n-k)} \frac{\{e^{\alpha t}-1\}^{(n-k)}}{(n-k)!} e - \frac{\beta}{\alpha}\left(e^{\alpha t}-1\right). \end{aligned} \tag{3.7}$$

After solving differential equation number (3.3) the following equations obtains:

$$P_k(t) = \left(\frac{\beta}{\alpha}\right)^{(n-k)} e - \frac{\beta}{\alpha}\left(e^{\alpha t}-1\right) \frac{\{e^{\alpha t}-1\}^{(n-k)}}{(n-k)!}; \quad k = 0, 1, 2, ..., n. \tag{3.8}$$

The solution of this equation is provided in appendix. Using equation number (3.1) we earn:

$$R(t) = \sum_{i=0}^{n} P_i(t) = \sum_{i=0}^{n} \left(\frac{\beta}{\alpha}\right)^{(n-i)} e - \frac{\beta}{\alpha}\left(e^{\alpha t}-1\right) \frac{\{e^{\alpha t}-1\}^{(n-i)}}{(n-i)!}. \tag{3.9}$$

The mean time to failure of the system is also calculated as follow:

$$MTTF = \int_{t=0}^{+\infty} R(t)dt \tag{3.10}$$

4 Numerical example

Suppose a system with 4 elements that 3 elements are spare parts for the main elements. The failure rates or working elements are $0.01e^{0.01t}$. We can calculate the reliability function of the system as:

$$P_k(t) = \left(\frac{0.01}{0.01}\right)^{(3-k)} \frac{\{e^{0.01t}-1\}^{(3-k)}}{(3-k)!} e^{-\frac{0.01}{0.01}\left(e^{0.01t}-1\right)} = e^{\left(1-e^{0.01t}\right)} \frac{\{e^{0.01t}-1\}^{(3-k)}}{(3-k)!}; \quad k = 0, 1, 2. \tag{4.1}$$

And the probability that system working more than 100 ours is:

$$R(100) = \sum_{i=0}^{3} e^{\left(1-e^{0.01\times 100}\right)} \frac{\{e^{0.01\times 100}-1\}^{(3-k)}}{(3-k)!} = e^{\left(1-e^{1}\right)} \left\{ \frac{(e^1-1)^3}{3!} + \frac{(e^1-1)^2}{2!} + \frac{(e^1-1)^1}{1!} + \frac{(e^1-1)^0}{0!} \right\} = 0.9041 \tag{4.2}$$

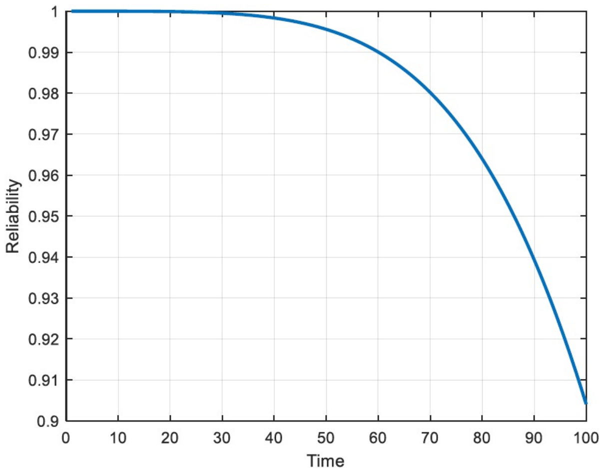

Figure 2: Reliability trends from 0 to 100-time units

5 Conclusion

In summary, the presented methodology for reliability analysis utilizing the Weibull distribution offers invaluable insights for industries prioritizing the dependability of lifetime systems, including manufacturing, electronics, automotive, and aerospace. By harnessing the power of the Weibull distribution, engineers and analysts can gain profound insights into system failures, enabling them to make informed decisions aimed at enhancing reliability and performance.

This study focused on a specific model comprising $(n+1)$ components, where n components acted as spare parts for the main component. The failure rate of the active component was assumed to be time-dependent, following a specified function, while the failure rates of the inactive components were considered zero. Whenever a component failed, one of the spare parts immediately took over its function. Notably, the failed components were deemed non-repairable.

By formulating the differential equations governing the system states and subsequently solving them, we were able to calculate essential parameters such as system reliability and mean time to failure (MTTF) in real-time scenarios. These results are particularly valuable for redundancy systems, providing insights that can significantly contribute to achieving improved system performance and reliability.

The proposed methodology and the attained results hold practical implications for industries striving to enhance the dependability and performance of their systems. Decision-makers can leverage the Weibull distribution and the analytical approach presented in this research to optimize maintenance strategies, forecast failure rates, and implement measures to bolster overall system reliability.

For future studies, it would be beneficial to explore additional system configurations, consider repairable components, or investigate the impact of different distribution assumptions. Moreover, applying the proposed methodology to real-world case studies could further validate its effectiveness and extend its practical utility across various industries.

To broaden the scope of future research, nonworking components' failure rates can be considered non-zero, allowing for the modeling of redundancy systems and yielding more precise results. Additionally, including repairable components in the model can offer a more comprehensive understanding of system behavior and provide a deeper exploration of system performance and maintenance optimization.

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