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Generalized Thermoelastic Interactions Using an Eigenvalue Technique in a Transversely Isotropic Unbounded Medium with Memory Having a Line Heat Source

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ABSTRACT

The present article looks over thermoelastic interactions in a homogeneous, linear, transversely isotropic unbounded continuum with the aid of memory-dependent derivatives in the presence of a line heat source. The exploration has been unifiedly carried out in the context of Green-Lindsay and Lord-Shulman models. A cylindrical polar coordinates system has been used to describe the problem and the eigenvalue technique has been adopted to solve the governing field equations in the transformed domain of Laplace. The solution for different thermophysical quantities is obtained in the real space-time domain using the Stehfest method for numerical Laplace inversion. The obtained numerical data for different thermophysical quantities are plotted in graphs to investigate the impacts of the time delay parameter, and the different kernel functions, and a comparison between the considered models has been accomplished. It is worth mentioning that the results of an analogous problem for isotropic material can be easily deduced from the corresponding results of this article. The adoption of generalized thermoelastic theory with memory-dependent derivative along with eigenvalue technique in analyzing the thermoelastic interactions is relatively fresh.

1. Introduction

The heat conduction equation of the classical theory of thermoelasticity does not accommodate any elastic term and it is parabolic in nature. It denies the practical observation of heat generation due to elastic changes and it also recommends boundless speed of the thermal wave propagation. The generalized theory of thermoelasticity prevails over these major imperfections of the classical theory of thermoelasticity. Towards the formulation of generalized theory of thermoelasticity, many pioneers have their valuable contributions of which we mention here some of them [1-9]. Ezzat [10] has solved a thermoelastic problem with two relaxation times in cylindrical regions. Youssef [11] has studied a thermoelastic problem in an infinite medium with a cylindrical cavity containing a moving heat source. Lotfy et al. [12, 13] have solved generalized thermoelastic problems for functionally graded and piezo-photo-thermoelastic materials respectively. Lotfy and Hassan [14] have investigated the propagation of thermoelastic waves within the purview of the Lord-Shulman two-temperature

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generalized thermoelasticity. Yasein et al. [15] have discussed the effect of variable temperature and thermal conductivity for semiconducting elastic medium in the context of dual-phase-lag and Lord-Shulman model. Mahdy et al. [16] have investigated the interaction impact between elastic waves, plasma waves, and thermal waves in hyperbolic generalized two-temperature theory. Abouelregal et al. [17–19] have solved various generalized thermoelastic problems considering Moore–Gibson–Thompson generalized thermoelastic theory. The generalized theory of thermoelasticity is widely implemented to tackle various problems with high heat flux for short time intervals which generally appears in nuclear reactors, energy channels, LASER units, etc.

Fractional calculus is an important tool for solving different physical problems in the field of generalized thermoelasticity because of the non-local property of the fractional operator. The fractional operator carries the previous state together with the current state of a dynamical system. Ezzat et al. [20] have introduced a unified mathematical model of heat conduction with three-phase lag thermoelasticity using fractional calculus. Abbas [21] has solved a thermoelastic problem for an infinite body with a spherical cavity in the context of fractional-order thermoelasticity. For an unbounded generalized thermoelastic media with a spherical cavity, Molla et al. [22] have studied the effects of fractional parameters on stress distributions. Lotfy et al. [23] have studied the effects of variable thermal conductivity in a magnetophotothermal elastic medium in the context of fractional calculus. Sufficient works in the field of generalized thermoelasticity have been performed by several researchers using fractional calculus, of which we mention [24-28] to name but a few.

Wang and Li [29] have launched the notion of memory-dependent derivatives in place of fractional derivatives [30]. The memory-dependent derivative (MDD) is defined in an integral form of a common derivative with a kernel function on a slipping interval. The delayed time intervals and the kernel function for MDD can be chosen freely whereas they are fixed in fractional derivatives. The order of the derivative in fractional calculus is necessarily a fraction but in MDD the order of the derivative is simply an integer which leads to easier numerical calculations. Taking all such points into consideration it is easy to guess that this kind of definition is better than a fractional one for reflecting the memory effect and this definition leads to recognizing a more intuitionistic physical explanation. According to them m-th order memory-dependent derivative of a differentiable function f(t) relative to the delay time $\omega > 0$ is defined as follows:

$$D_{\omega}^{(m)} f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} k(t,\xi) f^{(m)}(\xi)d\xi \qquad (1)$$

where $k(t,\xi)$ is the kernel function which can be chosen freely and the kernel $k(t,\xi)$ must be a differentiable function with respect to its arguments. When $k(t,\xi)$=1 and $\omega \to 0$ then $D_{\omega}^{(m)} f(t) \to f^{(m)}(t)$ i.e. m-th order common derivative of $f(t)$.

Yu et al. [31] have recently formulated a new generalized thermoelastic model using MDD [29]. El-karamany and Ezzat [32] have derived the variational principle, reciprocity theorem, and uniqueness of solutions in a thermodiffusive medium in the context of MDD. Ezzat et al. [33] have established a new generalized thermoelasticity model based on MDD. Sur et al. [34] have solved a physical problem in fiber-reinforced cylinders using MDD by finite element method. Sarkar et al. [35] have solved a two-dimensional problem in the context of memory-dependent two-temperature generalized thermoelastic model. Biswas et al. [36] examined the effect of the magnetic field in an orthotropic medium with a phase-lag model based on MDD. Abouelregal and Dargail [37] have introduced a new mathematical model for functionally graded thermoelastic nanobeams (FGNB) with MDD due to a periodic heat flux. Mondal and Sur [38] have studied the memory effects in a functionally graded magneto-thermoelastic rod. Mondal et al. [39] have studied the phase lag effect for a two-temperature generalized piezo thermoelastic problem in the context of memory-dependent derivatives. For more works using memory-dependent derivatives, one can go through the work of El-Attar et al. [40] Sarkar and Othman [41], Sur et al. [42], and Abouelregal et al. [43].

Memory-dependent generalized thermoelasticity which provides an alternative approach to describe memory-dependence that has been commonly depicted by fractional generalized thermoelasticity is comparatively new. To the best of our knowledge, only a few works have been reported in the literature to date that adopt generalized thermoelasticity theory with memory-dependent derivative and eigenvalue approach in solving the generalized thermoelastic problems.

The intent of this article is to look over thermoelastic interactions in a homogeneous, linear, transversely isotropic unbounded continuum in the presence of a line heat source. The exploration has been unified and carried out in the context of Lord-Shulman [5] and Green-Lindsay [1] models with the aid of memory-dependent derivatives. A cylindrical polar coordinates system has been used to trace the problem and the eigenvalue technique [44] is employed to solve the governing field equations in the transformed domain of Laplace. Using the Stehfest method [45] for numerical Laplace inversion the solution for stress, displacement, and temperature is brought to the real space-time domain. The obtained numerical data for the above-mentioned thermophysical quantities are plotted in graphs to look over the impacts of the time delay parameter and the different kernel functions and a comparison between the considered models has been accomplished. Results of analogous problems with integer order thermoelasticity theory realizable from [46] can also be achievable as a special case of our results. Also, results of analogous problems for isotropic material can be easily deduced from the corresponding results of this article.

2. Basic Governing Equations

The governing equations for generalized thermoelasticity with memory-dependent effect and relaxation times for a linear, homogeneous, and anisotropic medium are [33, 47]

(a) strain-displacement relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \qquad i,j = 1,2,3, \tag{2}$$

(b) the constitutive equations:

$$\tau_{ij} = C_{ijkl}e_{kl} - \beta_{ij}(1+\alpha_1 D_\omega)\theta, \qquad i,j,k,l = 1,2,3, \tag{3}$$

(c) stress-equations of motion:

$$\tau_{ij,j} + \rho F_i = \rho \ddot{u}_i, \qquad i,j = 1,2,3, \tag{4}$$

(d) the heat conduction equation:

$$K_{ij}\theta_{ij} = (1+\alpha_0 D_\omega)\rho c_E\dot{\theta} + (1+\tau D_\omega)(\beta_{ij}\theta_0\dot{e}_{ij} - \rho Q), \qquad i,j = 1,2,3, \tag{5}$$

where F_i, ρ, u_i, e_{ij}, τ_{ij}, C_{ijkl} respectively stand for the body force per unit mass, the mass density of the medium, the mechanical displacement, the strain tensor, the stress tensor, elasticity tensor and β_{ij} is the thermal elastic coupling tensor; c_E is the specific heat at constant strain, Q is the heat source per unit mass, α_0, α_1, τ are thermal relaxation parameters, K_{ij} is the thermal conductivity tensor, θ is the change in temperature above the reference temperature θ_0, D_ω denotes first order memory dependent derivative, subscript comma represents material derivative with respect to space variable and the superscript dot represents derivative with respect to time.

Special Cases:

1. If we consider $\alpha_0 = \alpha_1 = \tau = 0$ then the governing equations (2) - (5) reduce to the governing equations of classical coupled thermoelasticity (CCTE) [48].
2. When $\alpha_1 = 0$, $\alpha_0 = \tau \neq 0$ then the governing equations (2) - (5) reduce to the governing equations of Lord- Shulman model which is also known as extended thermoelasticity (ETE) [5]. does $\alpha_0 \neq 0$, $\alpha_1 \neq 0$, $\tau = 0$ then the governing equations (2) - (5) reduce to the governing equations of Green-Lindsay model which is also known as temperature rate dependent thermoelasticity (TRDTE) [1].

3. Formulation of the Problem

We consider an unbounded, linear thermoelastic continuum that is homogeneous and transversely isotropic in nature and is influenced under the action of a line heat source acting along the z-axis of the cylindrical polar coordinates system (r,ϕ,z) used for the description of the problem. We assume that the problem is axisymmetric and hence the displacement, and temperature are functions of r and t only. Therefore the displacement vector possesses only radial component $u = u(r,t)$ and the stress tensor possesses only radial component τ_{rr} and transverse component $\tau_{\phi\phi}$.

As a consequence, the equations of motion, and heat conduction equation take the following forms respectively

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \beta\frac{\partial}{\partial r}(1+\alpha_1 D_\omega)\theta + \rho F_r = \rho\,\ddot{u}, \tag{6}$$

$$K\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) = \rho c_E(1+\alpha_0 D_\omega)\dot{\theta} + (1+\tau D_\omega)\left[\theta_0\beta\left(\frac{\partial\dot{u}}{\partial r} + \frac{\dot{u}}{r}\right) - \rho Q\right], \tag{7}$$

and constitutive equations can be obtained as follows

$$\tau_{rr} = C_{11}\frac{\partial u}{\partial r} + C_{12}\frac{u}{r} - \beta(1+\alpha_1 D_\omega)\theta, \tag{8}$$

$$\tau_{\phi\phi} = C_{12}\frac{\partial u}{\partial r} + C_{11}\frac{u}{r} - \beta(1+\alpha_1 D_\omega)\theta, \tag{9}$$

where C_{11}, C_{12} are the material constants, β is the coupling parameter.

We introduce the following dimension-free variables

$$\begin{gathered} r^* = \frac{\Omega}{V}r,\ t^* = \Omega t,\ u^* = \frac{\Omega V\rho}{\beta\theta_0}u,\ \theta^* = \frac{\theta}{\theta_0}, \\ \alpha_0{}^* = \Omega\alpha_0, \alpha_1{}^* = \Omega\alpha_1,\ \tau^* = \Omega\tau, \gamma = \frac{C_{12}}{C_{11}}, \\ Q^* = \frac{Q}{Q_0}, F_r{}^* = \frac{F_r}{F_0}, \omega^* = \Omega\omega, \\ \tau_{rr}{}^* = \frac{\tau_{rr}}{\beta\theta_0},\ \tau_{\phi\phi}{}^* = \frac{\tau_{\phi\phi}}{\beta\theta_0}, \end{gathered} \tag{10}$$

where

$\Omega = \frac{C_{11}c_E}{K}, V^2 = \frac{C_{11}}{\rho}, Q_0 = \frac{K\theta_0\Omega^2}{\rho V^2}, F_0 = \frac{\Omega\beta\theta_0}{\rho V}.$

Using these dimension free variables we get the above equations (suppressing the asterisks for simplicity) as

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} - \frac{\partial}{\partial r}(1+\alpha_1 D_\omega)\theta + F_r = \ddot{u}, \quad (11)$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = (1+\alpha_0 D_\omega)\dot{\theta} + (1+\tau D_\omega)\left[\varepsilon\left(\frac{\partial \dot{u}}{\partial r} + \frac{\dot{u}}{r}\right) - Q\right], \quad (12)$$

where $\varepsilon = \frac{\beta^2\theta_0}{\rho^2 c_E V^2}$,

$$\tau_{rr} = \frac{\partial u}{\partial r} + \gamma\frac{u}{r} - (1+\alpha_1 D_\omega)\theta, \quad (13)$$

$$\tau_{\phi\phi} = \gamma\frac{\partial u}{\partial r} + \frac{u}{r} - (1+\alpha_1 D_\omega)\theta. \quad (14)$$

Here we choose the kernel function $k(t-\xi)$ as follows:

$$k(t-\xi) = 1 - \frac{2b}{\omega}(t-\xi) + \frac{a^2}{\omega^2}(t-\xi)^2 = \begin{cases} 1 & if\ a=b=0, \\ 1-\left(\frac{t-\xi}{\omega}\right) & if\ a=0, b=\frac{1}{2}, \\ 1-(t-\xi) & if\ a=0, b=\frac{\omega}{2}, \\ \left(1-\frac{t-\xi}{\omega}\right)^2 & if\ a=b=1, \end{cases} \quad (15)$$

a and b being constants.

4. Solution of the Problem

To solve the problem we take the Laplace transform defined by

$$\bar{f}(r,p) = \int_0^\infty f(r,t)e^{-pt}dt, \mathrm{Re}(p) > 0, \quad (16)$$

On equations (11), (12), (13) and (14) to get

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - \frac{\bar{u}}{r^2} - \frac{d}{dr}\left(1+\frac{\alpha_1}{\omega}G_\omega\right)\bar{\theta} + \bar{F}_r = p^2\bar{u}, \quad (17)$$

$$\frac{d^2\bar{\theta}}{dr^2} + \frac{1}{r}\frac{d\bar{\theta}}{dr} = p\left(1+\frac{\alpha_0}{\omega}G_\omega\right)\bar{\theta} + \left(1+\frac{\tau}{\omega}G_\omega\right)\left[\varepsilon p\left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r}\right) - \bar{Q}\right], \quad (18)$$

$$\bar{\tau}_{rr} = \frac{d\bar{u}}{dr} + \gamma\frac{\bar{u}}{r} - \left(1+\frac{\alpha_1}{\omega}G_\omega\right)\bar{\theta}, \quad (19)$$

$$\bar{\tau}_{\phi\phi} = \gamma\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r} - \left(1+\frac{\alpha_1}{\omega}G_\omega\right)\bar{\theta}, \quad (20)$$

where

$$G_\omega(p) = 1 - \frac{2b}{\omega p} + \frac{2a^2}{\omega^2 p^2} - e^{-p\omega}\left[(1-2b+a^2) + \frac{2(a^2-b)}{\omega p} + \frac{2a^2}{\omega^2 p^2}\right]. \quad (21)$$

Using equation (17), equation (18) can be written as

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\frac{d\bar{\theta}}{dr} = \varepsilon p^3\left(1+\frac{\tau}{\omega}G_\omega\right)\bar{u} + p\left[\varepsilon\left(1+\frac{\tau}{\omega}G_\omega\right)\left(1+\frac{\alpha_1}{\omega}G_\omega\right) + \left(1+\frac{\alpha_0}{\omega}G_\omega\right)\right]\frac{d\bar{\theta}}{dr} - \left(1+\frac{\tau}{\omega}G_\omega\right)\left(\varepsilon p\bar{F}_r + \frac{d\bar{Q}}{dr}\right). \quad (22)$$

We now write equations (17) and (22) in vector-matrix differential equation as follows:

$$L\tilde{v} = A\tilde{v} + \tilde{f}, \quad (23)$$

where

$$L \equiv \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} \quad (24)$$

is the Bessel operator, $\tilde{v} = \begin{bmatrix}\bar{u} \\ \frac{d\bar{\theta}}{dr}\end{bmatrix}$, A= $\begin{bmatrix}a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}$,

$\tilde{f} = \begin{bmatrix} -\bar{F}_r \\ -\left(1+\frac{\tau}{\omega}G_\omega\right)\left(\varepsilon p\bar{F}_r + \frac{d\bar{Q}}{dr}\right)\end{bmatrix}$,

and

$$a_{11} = p^2, a_{12} = \left(1+\frac{\alpha_1}{\omega}G_\omega\right), \quad a_{21} = \varepsilon p^3\left(1+\frac{\tau}{\omega}G_\omega\right), \quad a_{22} = p\left[\varepsilon\left(1+\frac{\tau}{\omega}G_\omega\right)\left(1+\frac{\alpha_1}{\omega}G_\omega\right) + \left(1+\frac{\alpha_0}{\omega}G_\omega\right)\right]. \quad (25)$$

If χ_1 and χ_2 be the eigenvalues of the matrix, then the eigenvectors V_1 and V_2 corresponding to the respective eigenvalues χ_1 and χ_2 are given by

$$V_1 = \begin{bmatrix}-a_{12} \\ a_{11}-\chi_1\end{bmatrix}, V_2 = \begin{bmatrix}-a_{12} \\ a_{11}-\chi_2\end{bmatrix}. \quad (26)$$

Let

$$V = [V_1 \quad V_2] = \begin{bmatrix}-a_{12} & -a_{12} \\ a_{11}-\chi_1 & a_{11}-\chi_2\end{bmatrix}$$

then

$$AV = V\Lambda, \quad (27)$$

where $\Lambda = \begin{bmatrix}\chi_1 & 0 \\ 0 & \chi_2\end{bmatrix}$,

and hence

$$A = V\Lambda V^{-1}. \quad (28)$$

Now equations (23) and (28) together yield

$$L\tilde{y} = \Lambda\tilde{y} + V^{-1}\tilde{f}, \tag{29}$$

which can also be written in scalar form as follows

$$\frac{d^2 y_q}{dr^2} + \frac{1}{r}\frac{dy_q}{dr} - \left(\chi_q + \frac{1}{r^2}\right) y_q = P_q, (q = 1,2), \tag{30}$$

where we have assumed

$$V^{-1}\tilde{f} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \tag{31}$$

$$\tilde{y} = V^{-1}\tilde{v} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{32}$$

Let $u_{q1}(r)$, $u_{q2}(r)$ (q= 1, 2) be two linearly independent solutions of the homogeneous equations corresponding to the equations in (30), then

$$\begin{aligned} u_{11} &= K_1(\lambda_1 r), \quad u_{12} = I_1(\lambda_1 r), \\ u_{21} &= K_1(\lambda_2 r), \quad u_{22} = I_1(\lambda_2 r), \end{aligned} \tag{33}$$

and the complementary functions y_{1c} and y_{2c} of the equations in (30) are given by

$$\begin{aligned} y_{1c} &= a_1 K_1(\lambda_1 r) + b_1 I_1(\lambda_1 r), \\ y_{2c} &= a_2 K_1(\lambda_2 r) + b_2 I_1(\lambda_2 r), \end{aligned} \tag{34}$$

where $\chi_q = {\lambda_q}^2 (q = 1, 2)$ and I_1, K_1 respectively denote the modified Bessel functions of first and second kind of order one.

In absence of body force (i.e when $F_r = 0$), the equation (31) gives

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{a_{12}\left({\lambda_2}^2 - {\lambda_1}^2\right)} \begin{bmatrix} a_{11} - {\lambda_2}^2 & a_{12} \\ -\left(a_{11} - {\lambda_1}^2\right) & -a_{12} \end{bmatrix} \times \begin{bmatrix} 0 \\ -\left(1 + \frac{\tau}{\omega} G_\omega\right)\frac{d\bar{Q}}{dr} \end{bmatrix},$$

which implies

$$P_1 = \frac{\left(1 + \frac{\tau}{\omega} G_\omega\right)}{\left({\lambda_1}^2 - {\lambda_2}^2\right)}\frac{d\bar{Q}}{dr}, P_2 = \frac{\left(1 + \frac{\tau}{\omega} G_\omega\right)}{\left({\lambda_2}^2 - {\lambda_1}^2\right)}\frac{d\bar{Q}}{dr}. \tag{35}$$

Particular integrals y_{1p} and y_{2p} corresponding to the equations in (30) are given by

$$y_{qp} = -u_{q1}\int \frac{u_{q2} P_q}{W_q} dr + u_{q2}\int \frac{u_{q1} P_q}{W_q} dr, \quad (q = 1,2) \tag{36}$$

where

$$W_q = \begin{vmatrix} u_{q1} & u_{q2} \\ u'_{q1} & u'_{q2} \end{vmatrix} = \frac{1}{r}, \quad (q = 1,2).$$

We now choose the line heat source in the form,

$$Q = \frac{q_0}{2\pi r}\delta(r - c)H(t), \tag{37}$$

which acts along $r = c$; $\delta(t)$, $H(t)$ representing the well-known Dirac delta function and Heaviside unit step function respectively and q_0 is a constant.

Taking Laplace transform of the equation (37) we get

$$\bar{Q} = \frac{1}{p}\frac{q_0}{2\pi r}\delta(r - c), \tag{38}$$

and hence particular integrals are obtained from (36) using (33) and (35) as follows

$$\begin{aligned} y_{1p} &= -u_{11}\int \frac{u_{12} P_1}{W_1} dr \\ &= -\frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_1}{2\pi p\left({\lambda_1}^2 - {\lambda_2}^2\right)} I_0(\lambda_1 c) K_1(\lambda_1 r), \\ y_{2p} &= -u_{21}\int \frac{u_{22} P_2}{W_2} dr \\ &= -\frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_2}{2\pi p\left({\lambda_2}^2 - {\lambda_1}^2\right)} I_0(\lambda_2 c) K_1(\lambda_2 r), \end{aligned} \tag{39}$$

where we have disregarded the second term in (36) for the sake of bounded solution. Again, since the heat source is acting along the z axis, taking $c \to 0$ the particular integrals of equations in (30) are obtained from (39) as follows

$$\begin{aligned} y_{1p} &= \frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_1}{2\pi p\left({\lambda_1}^2 - {\lambda_2}^2\right)} K_1(\lambda_1 r), \\ y_{2p} &= \frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_2}{2\pi p\left({\lambda_2}^2 - {\lambda_1}^2\right)} K_1(\lambda_2 r). \end{aligned} \tag{40}$$

Furthermore, since the medium is infinte and $K_1(r)$, $I_1(r)$ become unbounded as $r \to 0$ and as $r \to \infty$ respectively, we set $a_i = b_i = 0$ $(i = 1, 2)$ in equation (34). Thus the solutions for the equations in (30) are given by

$$\begin{aligned} y_1 &= \frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_1}{2\pi p\left({\lambda_1}^2 - {\lambda_2}^2\right)} K_1(\lambda_1 r), \\ y_2 &= \frac{q_0\left(1 + \frac{\tau}{\omega} G_\omega\right)\lambda_2}{2\pi p\left({\lambda_2}^2 - {\lambda_1}^2\right)} K_1(\lambda_2 r). \end{aligned} \tag{41}$$

Now the solution of (23) are obtained using (32) in the following form

$$\tilde{v} = \begin{bmatrix} \bar{u} \\ \frac{d\bar{\theta}}{dr} \end{bmatrix} = \begin{bmatrix} -\left(1 + \frac{\alpha_1}{\omega} G_\omega\right) y_1 - \left(1 + \frac{\alpha_1}{\omega} G_\omega\right) y_2 \\ \left(p^2 - {\lambda_1}^2\right) y_1 + \left(p^2 - {\lambda_2}^2\right) y_2 \end{bmatrix}. \tag{42}$$

From this we get

$$\bar{u} = -M\{\lambda_1 K_1(\lambda_1 r) - \lambda_2 K_1(\lambda_2 r)\}, \tag{43}$$

$$\frac{d\bar{\theta}}{dr} = \frac{M}{\left(1+\frac{\alpha_1}{\omega}G_\omega\right)}\{\lambda_1 K_1(\lambda_1 r)\left(p^2-{\lambda_1}^2\right) -\lambda_2 K_1(\lambda_2 r)\left(p^2-{\lambda_2}^2\right)\}, \tag{44}$$

where

$$M = \frac{q_0\left(1+\frac{\alpha_1}{\omega}G_\omega\right)\left(1+\frac{\tau}{\omega}G_\omega\right)}{2\pi p\left({\lambda_1}^2-{\lambda_2}^2\right)}.$$

Integrating equation (44) we obtain

$$\bar{\theta} = -\frac{M}{\left(1+\frac{\alpha_1}{\omega}G_\omega\right)}\{K_0(\lambda_1 r)\left(p^2-{\lambda_1}^2\right) -K_0(\lambda_2 r)\left(p^2-{\lambda_2}^2\right)\}. \tag{45}$$

Using equations (43) and (45) in (19), (20) we get respectively

$$\bar{\tau}_{rr} = -M\left[\frac{(\gamma-1)}{r}\{\lambda_1 K_1(\lambda_1 r)-\lambda_2 K_1(\lambda_2 r)\} - p^2\{K_0(\lambda_1 r)-K_0(\lambda_2 r)\}\right] \tag{46}$$

and

$$\bar{\tau}_{\phi\phi} = -M\left[\frac{(1-\gamma)}{r}\{\lambda_1 K_1(\lambda_1 r)-\lambda_2 K_1(\lambda_2 r)\} + \{{\lambda_1}^2 K_0(\lambda_1 r) - {\lambda_2}^2 K_0(\lambda_2 r)\} - p^2\{K_0(\lambda_1 r)-K_0(\lambda_2 r)\}\right]. \tag{47}$$

All results of [46] corresponding to the fractional parameter α = 1 can be derived from our results by taking $\omega \to 0$ and $k(t,\xi) = 1$.

Again the results for isotropic material can be derived from our results by simply taking $\gamma = \frac{\lambda}{\lambda+2\mu}, \varepsilon = \frac{\beta^2\theta_0}{\rho c_E(\lambda+2\mu)}$ and $\beta = (3\lambda+2\mu)\alpha_t$ in our calculations, where λ, μ are Lame's constant, α_t is the coefficient of linear thermal expansion for isotropic material.

5. Numerical Results and Discussion

For numerical solutions, we have considered a single crystal of zinc and the parameters are chosen as follows [21]:

$C_{11} = 1.628 \times 10^{11}$N/m², $\alpha_0 = 0.05$, $q_0 = 1$,

$C_{12} = 0.362 \times 10$N/m², $\alpha_1 = 0.1$, $\theta_0 = 296$K,

$K = 1.24 \times 10^2$W/mK, $\beta = 5.75 \times 10^6$N/m²K,

$\varepsilon = 2.21 \times 10^{-2}$, $\rho = 7.14 \times 10^3$Kg/m³.

Using MATLAB software numerical solutions for displacement, temperature and radial stress are obtained in the time domain by the Stehfest method [45]. The numerically obtained solutions are plotted in graphs to study the influence of the delay time parameter and the kernel functions on the above-mentioned thermophysical quantities.

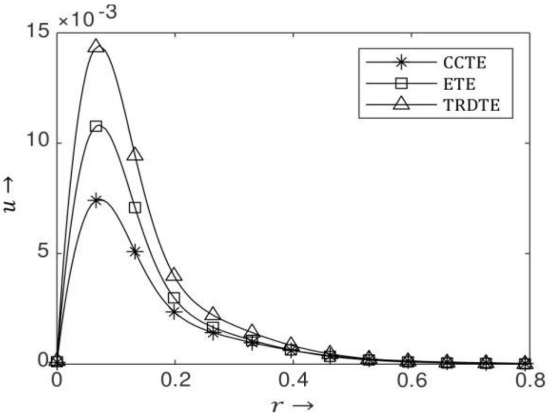

Fig. 1. Variation of displacement u with r for different theories at ω = 0.5, t = 0.1, a = 1, b = 1.

Figure 1 exhibits the variation of displacement u along r for different theories in the range 0 < r < 0.8 for ω = 0.5, t = 0.1, a = 1, b = 1 and it shows that the magnitude of the displacement component u corresponding to TRDTE model is larger compare to ETE model which has again greater magnitude than CCTE model. In all the curves magnitude of u gradually increases in 0 < r < 0.1 (approx) and then decreases to vanish.

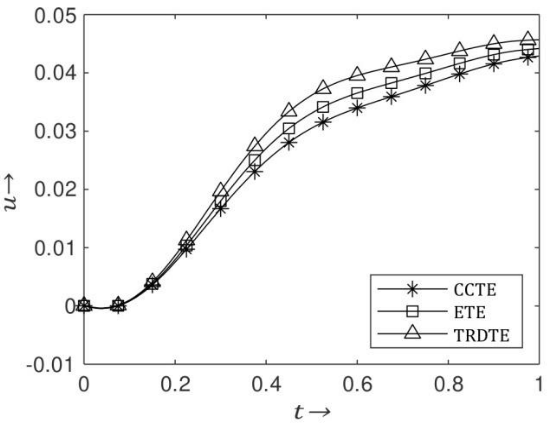

Fig. 2. Variation of displacement u with t for different theories at ω = 0.5, r = 0.3, a = 0, b = 0.

Figure 2 indicates the variation of displacement u with the time t for different theories when ω = 0.5, a = 0, b = 0, r = 0.3 in the range 0 < t < 1. We see that the initial value of u for each of the three models is the same. With the increase of time t the values of u gradually increase to achieve larger values in the TRDTE model compared to the ETE model which again has larger values compared to the CCTE model and finally becomes stable in all cases.

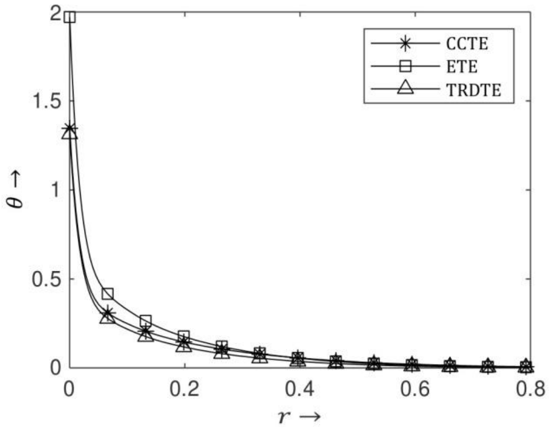

Fig. 3. Variation of temperature θ with r for different theories at ω = 0.5, t = 0.1, a = 1, b = 1.

Figure 3 shows the variation of temperature θ along r for different theories in the range $0 < r < 0.8$ when $\omega = 0.5$, $a = 1$, $b = 1$, $t = 0.1$. From the graph, it is clear that the magnitude of θ corresponding to the ETE model is the greatest and that corresponding to the TRDTE model is the least.

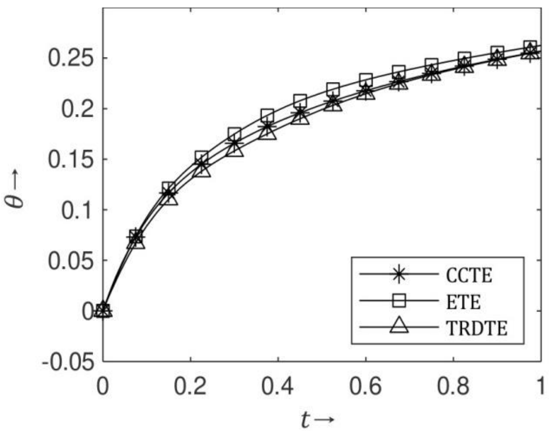

Fig. 4. Variation of temperature θ with t for different theories at ω = 0.5, r = 0.3, a = 0, b = 0.

Figure 4 represents the variation of temperature θ with the time t for different theories when $\omega = 0.5$, $r = 0.3$, $a = 0$, $b = 0$ in the range $0 < t < 1$. From the figure, we see that the magnitude of the corresponding to the ETE model is the greatest and that corresponding to the TRDTE model is the least.

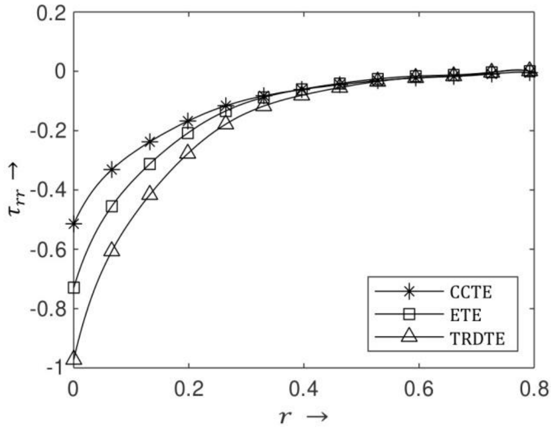

Fig. 5. Variation of radial stress τ_{rr} with r for different theories at ω = 0.5, t = 0.1, a = 1, b = 1.

Figure 5 indicates the variation of radial stress τ_{rr} with r for different theories at $\omega = 0.5$, $t = 0.1$, $a = 1$, $b = 1$ in the range $0 < r < 0.8$. We observe that the magnitude of radial stress τ_{rr} for each model is decreasing in nature and eventually vanishes. The maximum of the radial stress τ_{rr} for each model is attained near the center, which is the largest for the TRDTE model and the smallest for the CCTE model.

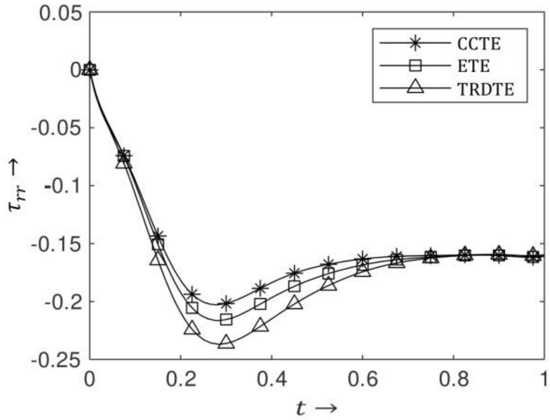

Fig. 6. Variation of radial stress τ_{rr} with t for different theories at ω = 0.5, r = 0.3, a = 0, b = 0.

Figure 6 exhibits the variation of radial stress τ_{rr} with t for different theories when $\omega = 0.5$, $r = 0.3$, $a = 0$, $b = 0$ in the interval $0 < t < 1$. We see that the magnitude of τ_{rr} corresponding to the CCTE model is the least and that corresponding to the TRDTE model is the greatest. Graphs corresponding to different thermoelastic models are distinguishable in the interval $0.1 \leq t \leq 0.8$ (approx) and after which they become stable.

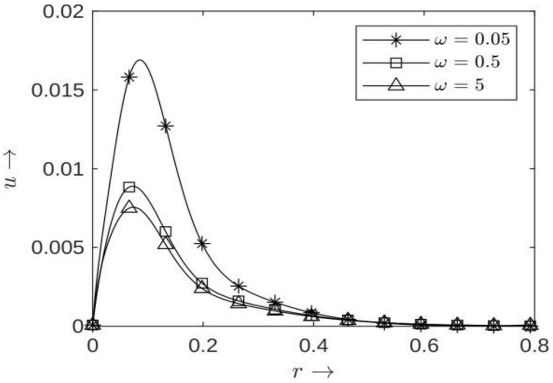

Fig. 7. Variation of displacement u with r for different values of ω in TRDTE model at t = 0.1, a = 0, b = 0.

Figure 7 represents the variation of displacement u along r for different values of ω in the TRDTE model at ω = 0.1, a = 0, b = 0 in the range 0 < r < 0.8. For any choice of ω distribution of u have increasing nature near the origin and after that they decay to vanish.

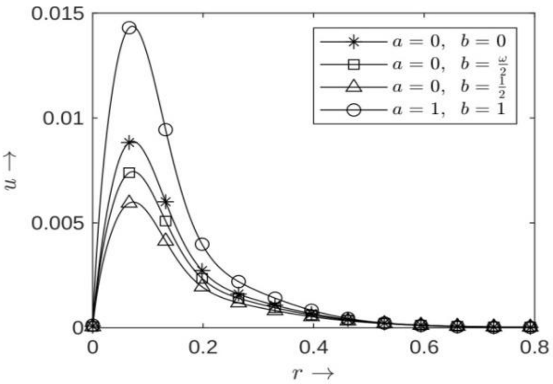

Fig. 8. Variation of displacement u with r for different kernel functions in TRDTE model at ω = 0.5, t= 0.1.

Figure 8 demonstrates that the variation of displacement u with r for different kernel functions in the TRDTE model at ω = 0.5, t = 0.1 in 0 < r < 0.8. For every choice of the kernel function, the distribution of u has a uniform increasing nature in 0 < r ≤ 0.1 (approx) and has a uniform decreasing nature in 0.1 ≤ r ≤ 0.66 (approx). It is observed that the curve corresponding to the nonlinear kernel (a = 1, b = 1) has greater magnitude compared to all linear kernels. For graphs corresponding to linear kernels, we see that magnitudes are greater for smaller values of the parameter b.

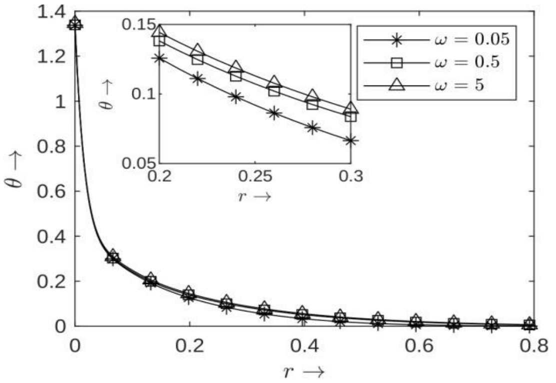

Fig. 9. Variation of temperature θ with r for different delay time parameter ω in TRDTE model at t = 0.1, a = 0, b = 0.

Figure 9 indicates the variation of temperature θ along r for different delay time parameter ω in the TRDTE model when t = 0.1, a = 0, b = 0 for the interval 0 < r < 0.8. We see that the value of θ for each chosen delay time parameter is the same near origin and with larger value of ω the curve travels the larger distance vanishes.

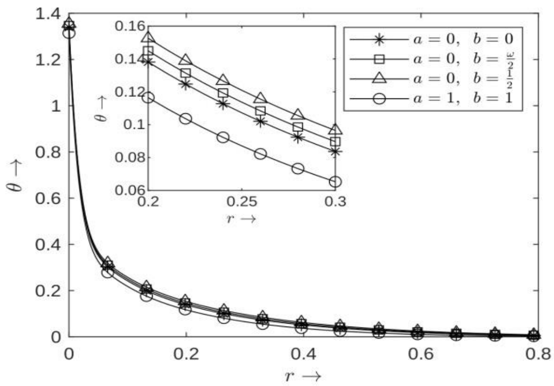

Fig. 10. Variation of temperature θ with r for different kernel functions in TRDTE model at ω = 0.5, t = 0.3.

Figure 10 exhibits the variation of temperature θ along r for different kernel functions in the TRDTE model at ω = 0.5, t = 0.3 for the interval 0 < r < 0.8. It is clear that for every chosen kernel function, the temperature θ attains the greatest value near the origin and uniformly decreases nature. For the nonlinear kernel (a = 1, b = 1) θ travels the smaller distance to vanish. Among the linear kernels the magnitudes are greater for greater value of the parameter b and travels the larger distance to vanish for larger value of the parameter b.

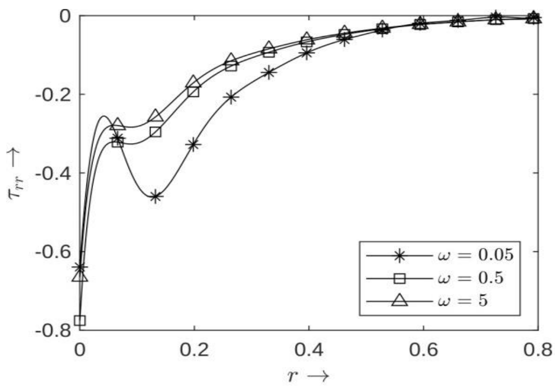

Fig. 11. Variation of the radial stress τ_{rr} with r for different delay time parameter ω in TRDTE model at t = 0.1, a = 0, b = 0.

Figure 11 shows that the variation of the radial stress τ_{rr} versus radial distance r for different delay time parameter ω in TRDTE model at t = 0.1, a = 0, b = 0 in the range 0 < r < 0.8. We see that the radial stress τ_{rr} is compressive in nature for all considered ω. Each values of ω curve has oscillatory behavior near the origin and then uniformly decrease to vanish. With larger values of ω the length of the intervals in which oscillatory behavior appears diminishes.

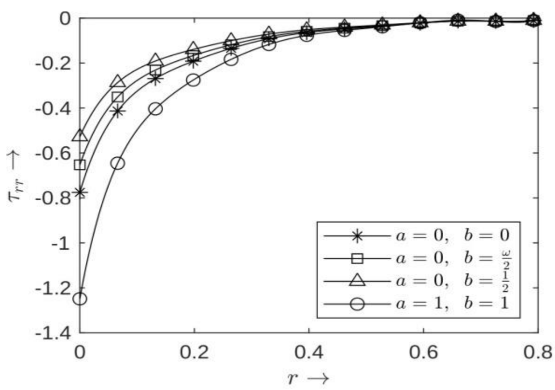

Fig. 12. Variation of the radial stress τ_{rr} with r for different kernel functions in TRDTE model at ω = 0.5, t = 0.1.

Figure 12 indicates the variation of the radial stress τ_{rr} versus radial displacement r for different kernel functions in TRDTE model at ω = 0.5, t = 0.1 in the range 0 < r < 0.8. From the graph we see that for any choice of the kernel function magnitude of the radial stress τ_{rr} is decreasing in nature and eventually vanishes. Its magnitude has larger value for non linear kernel (a = 1, b = 1). For linear kernels magnitudes are greater for smaller values of the parameter b.

6. Conclusions

The problem of investigating the displacement (u), temperature (θ) and the radial stress (τ_{rr}) in linear, homogeneous and transversely isotropic infinitely extended thermoelastic continuum under the influence of line heat source has been studied for different models in the context of memory dependent derivative with aid of eigenvalue approach. Cylindrical polar coordinates system (r, ϕ, z) has been employed for the description of the problem and solution is obtained in transformed domain of Laplace. Stehfest's [45] method is used for numerical Laplace inversion of the above-said solution. The analysis of the obtained results permits us to derive the following conclusions:

1. Significant effects of different considered models, delay time parameters, and different choices of kernel functions are observed in the distribution of displacement component (u), temperature (θ) and the radial stress component (τ_{rr}).
2. In the region of consideration of r the magnitude of displacement component (u) corresponding to the TRDTE model is greatest and that corresponding to the CCTE model is the smallest. A similar observation appears in the considered interval of time.
3. The magnitude of temperature (θ) corresponding to the ETE model is the largest and that corresponding to the TRDTE model is the smallest and this observation is true in both the region of consideration as well as in the interval of consideration.
4. Radial stress component (τ_{rr}) is compressive in nature. The magnitude of τ_{rr} corresponding to the TRDTE model is the largest and that corresponding to the CCTE model is the smallest. This observation is true in the region of consideration as well as in the interval of consideration.
5. Displacement component (u) and the radial stress component (τ_{rr}) travels longest distance to vanish for the nonlinear kernel ($a = 1, b = 1$) but this phenomenon happens for temperature distribution (θ) for the case of linear kernel function when a = 0, $b = \frac{1}{2}$.
6. Displacement component (u) and the radial stress component (τ_{rr}) travels longest distance to vanish for the smallest value of ω, but for temperature distribution (θ) the curve corresponding to largest value of ω vanishes at longest distance.
7. As the paper deals with thermoelastic interactions in the context of memory-dependent derivative, it describes the behavior of an elastic material more realistically than the theory of thermoelasticity with fractional as well as integer one.
8. All results of [46] corresponding to the fractional parameter α = 1 can be derived from our results by taking $\omega \rightarrow 0$ and $k(t, \xi) = 1$.

9. The results for isotropic material can be derived from our results by simply taking $\gamma = \frac{\lambda}{\lambda+2\mu}, \varepsilon = \frac{\beta^2\theta_0}{\rho c_E(\lambda+2\mu)}$ and $\beta = (3\lambda + 2\mu)\alpha_t$ in our calculations, where λ, μ are Lame's constant, α_t is the coefficient of linear thermal expansion for isotropic material.

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript.

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