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Research Article

Free Vibration Analysis of Simply Supported and Clamped Functionally Graded Rectangular Plate Using Coupled Displacement Field Method

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ABSTRACT

In this paper, a coupled displacement field (CDF) method was proposed to examine the free vibration behavior of a functionally graded (FG) rectangular plate with simply supported (SSSS) and clamped (CCCC) boundary conditions. The composition of the functionally graded rectangular plate is ceramic on the top and metal on the bottom. According to the power-law exponent form, the rectangular plate material properties vary continuously in the thickness direction. The trial functions signifying the displacement constituents of the cross-sections of the plate are stated in simple algebraic polynomial forms. The lateral displacement field is derived in terms of the total rotations with the help of coupling equations. By utilizing the energy formulation, the undetermined coefficients are obtained. The frequency parameters with various aspect ratios, thickness ratios, and power-law for all edges are simply supported and clamped boundary conditions are derived. To validate the numerical results, a comparison of frequency parameters is done with other literature.

1. Introduction

Several engineering disciplines like automobile, aerospace, mechanical, and nuclear fields use complex structures made of structural members like plates and beams. Plates can be thick or thin, depending on the purpose. When these plates are subjected to internal or external force, they may vibrate with large amplitudes. The design of a structural member using a rectangular plate must consider the free vibration behavior under various environmental conditions. A functionally graded plate composition can be a metal, ceramic, or polymer. The properties of these materials continuously vary in the direction of thickness from one surface to another. The FG plate behavior will be analyzed under different boundaries to reduce vibrations. The fundamental frequency parameters of the plate are to be analyzed to prevent any damage caused by vibrations.

The first-order shear deformation theory (FSDT) is based on the displacement field, which uses shear correction factors to set the differences between the actual transverse shear stress distribution and those evaluated by using the FSDT kinematic relations. To find the frequencies of the FG rectangular plates, FSDT

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was used to analyze and derive the equations of motion [1]. Significant results on the behavior of the FG plate are found in the path of material gradient stiffness [2]. The vibration frequencies of the FG plate based on amplitude and volume fraction have significant effects [3]. The governing equations of the plates are derived analytically by using FSDT under consideration of transverse shear stresses and rotational inertial effects [4, 5]. By implementing Hamilton’s rule, fundamental governing equations are derived [6, 7]. The interpolation functions of higher order are utilized to separate spatial derivatives [8].

The Rayleigh-Ritz (RR) method and the CDF method were used for solving the Eigenvalue problem [9]. The RR method is used to develop admissible functions for the analysis of vibrations in thick plates with similar elastic edge constraints [10, 11]. The RR method is used to find frequencies based on Mindlin's theory [12]. The Mindilin theory is used for vibration analysis on plates that are rectangular and thick [13]. The characteristic functions are studied for isotropic rectangular thick plates [14]. The observation is done on governing equilibrium equations of forces and force-displacement relations that are reduced to three partial differential equations of motion with total deflection [15]. An elasticity solution of FG simply supported 3-D plate is obtained based on transverse loading [16]. By eliminating the integration constants from the projections of the general boundary conditions, the stiffness matrix has been derived [17]. An investigation is done on the nonlinear forced vibrations of thin FG circular plates under classical clamped-clamped boundary conditions [18]. The governing equations for the boundary conditions are derived by differential rules [19]. Based on the strain linear elasticity theory, 3-D vibration solutions are derived for FG rectangular plates under various boundary conditions [20]. Young's modulus varies throughout the direction of thickness, where Poisson's ratio is assumed to be constant [21]. Based on relative displacement and rotational degrees of freedom, the mass and stiffness matrix are derived [22].

To meet the outcome of the corresponding Kirchhoff frequencies, plates with various thickness ratios have been considered [23]. The vibration attributes of FG plates are verified based on power law, aspect, and thickness ratios [24]. Based on the numerical method, the mixed boundary conditions of a plate for differential equations are obtained [25-27]. Eigenfrequencies are obtained for a broad range of thicknesses and aspect proportions [28]. The ordinary differential equation is resolved from the Eigen differential equation [29]. The analysis is done on a functionally graded cantilever beam to perceive the behavior of deformation and variations in stress [30]. Without changing the shape parameters Meshfree method is used to analyze the vibration response of rectangular plates [31]. The effects of variations in the Poisson's ratio are studied [32].

In the CDF method, the fields for lateral displacement and total rotations are coupled through the static equilibrium equation [33]. The CDF method uses only one undetermined coefficient. In the CDF method, a single-term admissible function is used in the principle of conservation of total energy. The admissible trial function was assumed, where the lateral displacement function is attained by using coupling equations [34, 35]. The axial, bending, and shear displacements of a thick clamped-clamped functionally graded material under a uniform load are developed [36]. Due to the utilization of coupling equations, the transverse displacement distribution comprises the identical undetermined coefficient as existing in the rotation direction. Material properties vary continuously through thickness according to a power law distribution in terms of the volume fraction of the constituents [37, 38]. The RR method uses two undetermined coefficients, which are reduced to one determined coefficient in the CDF method, which significantly minimizes the complexity of vibrations. The effects of the power-law, aspect ratio, thickness-length ratio, and various boundary conditions on the vibration characteristics of the FG rectangular plate are examined [39-41]. Free vibration analysis of rectangular plates under various boundary conditions is done [42]. The results of a plate on the natural frequencies under clamped and simply supported conditions are observed [43].

The objective of the present work is to study the free vibration analysis of an FG plate subjected to simply supported and clamped boundary conditions using the CDF method. To satisfy the essential boundary conditions the trail functions that denote the displacement fields are expressed in simple algebraic polynomial forms. The results obtained under simply supported and clamped boundary conditions are compared with the frequencies obtained in 8, 23, 24, [26-29], 32 and [41-43] are found to be in good agreement.

2. Functionally Graded Plate

FG plate length (a), breadth (b), and thickness (h) are displayed in Fig. 1.

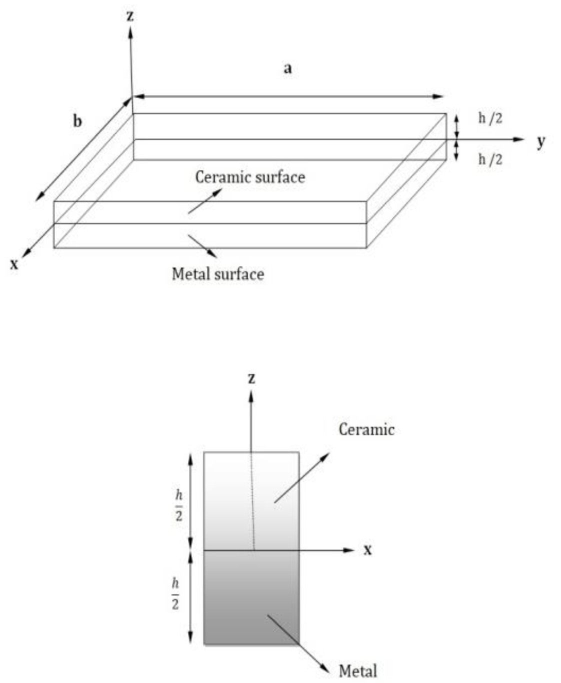

Fig. 1. Geometry of a functionally graded rectangular plate.

The FG plate used is a combination of ceramic on the upper and metal on the lower, where the mechanical attributes differ continuously in axis z. Since the thickness property varies, the upper surface $(z = h/2)$ and lower surface $(z = -h/2)$ are treated as ceramic and metal respectively. It is observed that the properties of the FG plate become pure ceramic at k = 0 and metallic at a very high equivalent of k.

The power-law function is written as

$$p(z) = (p_c - p_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + p_m \qquad (1)$$

where p_c and p_m are the attributes of ceramic and metal, h is thickness and k is the power-law exponent of the FG plate. Accordingly, E and M vary continuously along the z direction as shown below.

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m$$
$$M(z) = (M_c - M_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + M_m \qquad (2)$$

3. First-Order Shear Deformation Theory

The displacements u, v and w are given by

$$u(x,y,z,t) = u_0(x,y,t) + z\alpha_x(x,y) \qquad (3)$$

$$v(x,y,z) = v_0(x,y,t) + z\alpha_y(x,y) \qquad (4)$$

$$w(x,y,z,t) = w_0(x,y,t) \qquad (5)$$

where $(u_0, v_0, w_0, \alpha_x, \alpha_y)$ are unknown functions that are to be resolved. (u_0, v_0, w_0) indicates the displacements of the mid-plane $(z = 0)$ and t denotes the time. α_x and α_y denotes rotations of the transverse normal about the y and x axis.

Axial strain and shear strain are

$$\varphi_{xx} = z\frac{\partial \alpha_x}{\partial x}, \quad \varphi_{yy} = z\frac{\partial \alpha_y}{\partial y}, \quad \varphi_{zz} = 0 \qquad (6)$$

$$\psi_{xy} = z\left(\frac{\partial \alpha_x}{\partial y} + \frac{\partial \alpha_y}{\partial x}\right), \quad \psi_{xz} = \alpha_x + \frac{\partial w}{\partial x}, \quad \psi_{yz} = \alpha_y + \frac{\partial w}{\partial y} \qquad (7)$$

here, φ_{xx}, φ_{yy} and φ_{zz} indicate normal strains whereas ψ_{xy}, ψ_{xz} and ψ_{yz} indicate shear strains. Strain and kinetic energies represented by U and T are

$$U = \frac{1}{2}\int_0^b\int_0^a \frac{1}{1-\nu^2}\left\{\begin{array}{l}\left(\frac{\partial \alpha_x}{\partial x}\right)^2 + \left(\frac{\partial \alpha_y}{\partial y}\right)^2 + 2\nu\left(\frac{\partial \alpha_x}{\partial x}\frac{\partial \alpha_y}{\partial y}\right) \\ +2(1-\nu)\left(\frac{\partial \alpha_x}{\partial y}\frac{\partial \alpha_y}{\partial x}\right) dxdy \int_{-h/2}^{h/2} E(z) z^2 dz\end{array}\right\}$$
$$+\frac{1}{2}\int_0^b\int_0^a \frac{k}{2(1+\nu)}\left\{\left(\alpha_x + \frac{\partial w}{\partial x}\right)^2 + \left(\alpha_y + \frac{\partial w}{\partial y}\right)^2\right\} dxdy \int_{-h/2}^{h/2} E(z) dz \qquad (8)$$

$$T = \frac{\omega^2}{2}\int_0^b\int_0^a \left[w^2 + \frac{h^2}{12}\left(\alpha_x^2 + \alpha_y^2\right)\right] dxdy \int_{-h/2}^{h/2} M(z) dz \qquad (9)$$

Using the above equations, The undetermined coefficients are derived by $\partial(U - T) = 0$

4. Coupled Displacement Field Method

By considering α_x and α_y, estimate the transverse displacement denoted by w along the x and y directions.

$$\frac{\partial w}{\partial x} = -\alpha_x + S\left[\frac{12}{5(1-\nu)}\left(\frac{\partial^2 \alpha_x}{\partial x^2} + \nu\frac{\partial^2 \alpha_y}{\partial y \partial x}\right) + \frac{6}{5}\left(\frac{\partial^2 \alpha_x}{\partial y^2} + \frac{\partial^2 \alpha_y}{\partial y \partial x}\right)\right] \qquad (10)$$

$$\frac{\partial w}{\partial y} = -\alpha_y + S\left[\frac{12}{5(1-\nu)}\left(\frac{\partial^2 \alpha_y}{\partial y^2} + \nu\frac{\partial^2 \alpha_x}{\partial y \partial x}\right) + \frac{6}{5}\left(\frac{\partial^2 \alpha_y}{\partial x^2} + \frac{\partial^2 \alpha_x}{\partial y \partial x}\right)\right] \qquad (11)$$

where

$$S=\frac{\int_{-h/2}^{h/2}E(z)z^2dz}{\int_{-h/2}^{h/2}E(z)\,dz}$$

$$\alpha_x=\sum_{i=1}^{n}c_i\zeta_i \quad (12)$$

$$\alpha_y=\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j \quad (13)$$

Transverse lateral displacement w is obtained by applying Eqs. (12) and (13) in Eqs. (10) and (11). After integration and evaluation of the constant, we get

$$w=\sum_{i=1}^{n}\sum_{k=1}^{n}c_i\varsigma_k \quad (14)$$

here, c_i is the undetermined coefficient and ζ_i, ξ_j and ς_k are the admissible functions.

$$\begin{aligned}\zeta_i&=\Upsilon\eta_i\\ \xi_j&=\Upsilon\eta_j\\ \varsigma_k&=\Upsilon\eta_k\end{aligned} \quad (15)$$

here, $i,j,k=1,2,3.....n$, where n indicates the number of polynomials. The boundaries are controlled by the exponent's r, s, t, and u of the function $\Upsilon=x^r y^s(a-x)^t(b-y)^u$ which can be 0, 1, or 2. Here, 0 indicates free (F), 1 indicates simply supported (S) and 2 indicates clamped (C). Using Pascal's triangle, $\eta_{i,j,k}$ parameters are given in Table 1.

Table 1. Ten parameters of $\eta_{i,j,k}$ [24]

i	1	2	3	4	5	6	7	8	9	10
$\eta_{i,j,k}$	1	x	y	x^2	xy	y^2	x^3	x^2y	xy^3	y^3

Using Eqs. (12), (13), and (14) in Eqs. (8) and (9) we get

$$U_1=\frac{1}{2}\int_0^b\int_0^a\frac{1}{1-\nu^2}\left\{\begin{aligned}&\left(\frac{\partial}{\partial x}\sum_{i=1}^{n}c_i\zeta_i\right)^2+\left(\frac{\partial}{\partial y}\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j\right)^2\\&+2\nu\left(\frac{\partial}{\partial x}\sum_{i=1}^{n}c_i\zeta_i\;\frac{\partial}{\partial y}\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j\right)\\&+2(1-\nu)\left(\frac{\partial}{\partial y}\sum_{i=1}^{n}c_i\zeta_i\;\frac{\partial}{\partial x}\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j\right)\end{aligned}\right\}dxdy\int_{-h/2}^{h/2}E(z)z^2dz \quad (16)$$

$$U_2=\frac{1}{2}\int_0^b\int_0^a\frac{k}{2(1+\nu)}\left\{\begin{aligned}&\left(\sum_{i=1}^{n}c_i\zeta_i+\frac{\partial}{\partial x}\sum_{i=1}^{n}\sum_{k=1}^{n}c_i\varsigma_k\right)^2+\\&\left(\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j+\frac{\partial}{\partial y}\sum_{i=1}^{n}\sum_{k=1}^{n}c_i\varsigma_k\right)^2\end{aligned}\right\}dxdy\int_{-h/2}^{h/2}E(z)dz \quad (17)$$

$$T=\frac{\omega^2}{2}\int_0^b\int_0^a\left[\begin{aligned}&\left(\sum_{i=1}^{n}\sum_{k=1}^{n}c_i\varsigma_k\right)^2+\\&\frac{h^2}{12}\left(\left(\sum_{i=1}^{n}c_i\zeta_i\right)^2+\left(\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\xi_j\right)^2\right)\end{aligned}\right]dxdy\int_{-h/2}^{h/2}M(z)dz \quad (18)$$

Reducing the lagranzian concerning c_i

$$\frac{\partial(U_x-T)}{\partial c_i}=0;\quad i=1,2,3....n \quad (19)$$

where $U_x=U_1+U_2$

The governing equation is given by

$$[\mathrm{A}]-\lambda^2[\mathrm{B}]\{\Delta\}=0 \quad (20)$$

$[\mathrm{A}]$ and $[\mathrm{B}]$ indicate stiffness and inertia matrices and $\{\Delta\}$ represent unknown coefficients in the column vector. where

$$\{\Delta\}=[c_1,c_2,c_3,c_4,........n]^T$$

The frequency parameters obtained by Eq. (21) are discussed in the next chapter.

5. Results and Analysis

The behavior of vibrations in an FG rectangular plate using CDF with respect to thickness ratio (h/a) is obtained. The FG plate Non-dimensional frequency parameters may be expressed as

$$\lambda=\omega a^2\sqrt{\frac{\rho_c h}{D_c}} \quad (21)$$

The properties of the materials used in the FG plate differ, i.e., for aluminum E_m = 70 GPa, ρ_m = 2700 kg/m^3 and ν_m = 0.3 and for alumina E_c = 380 GPa, ρ_c = 3800 kg/m^3 and ν_c = 0.3 respectively.

Table 2. Frequency parameters for all edges of the SSSS FG plate with k = 0 and h/a = 0.001 using CDF.

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	10.264	10.264③	11.466	11.449③	13.504	13.495③	16.423	16.433③	28.319	28.514③
0.5	12.337	12.337③	19.74	19.739 ③	32.423	32.421③	41.945	41.947③	49.654	49.659③
		12.337⑦		19.739⑦		32.076⑦		41.945⑦		49.348⑦
1	19.739	19.739①	49.34	49.348①	49.345	49.348①	79.386	78.956①	100.17	98.696①
		19.739③		49.348③		49.348③		79.401③		100.17③
		19.739⑤		49.349⑤		49.349⑤		78.9633⑤		98.719⑤
		19.739⑥		49.347⑥		49.3475⑥		78.955⑥		98.694⑥
		19.739⑧		49.348⑧		49.348⑧		78.956⑧		98.696⑧
		19.739⑨		49.348⑨		49.348⑨		78.957⑨		99.304⑨
		19.74⑩		49.35⑩		-		79.03⑩		99.25⑩
		19.74⑪		49.35⑪		49.35⑪		78.96⑪		-
1.5	32.076	32.078②	61.684	61.688②	98.698	98.697②	111.48	111.03②	129.06	128.31②
		32.076⑧		61.685⑧		98.696⑧		111.03⑧		128.30⑧
		32.08⑩		61.71⑩		98.76⑩		111.57⑩		-
2	49.353	49.348③	78.942	78.958③	129.69	129.68③	167.77	167.79③	198.63	198.63③
		49.348④		78.957④		128.30④		167.78④		197.39④
		49.348⑦		78.956⑦		128.30⑦		167.78⑦		197.39⑦
2.5	71.558	71.556⑧	101.161	101.16⑧	151.83	150.51⑧	220.74	219.59⑧	256.62	256.60⑧
		71.555⑨		101.16⑨		150.99⑨		222.91⑨		256.61⑨
		71.55⑩		101.19⑩		150.95⑩		219.71⑩		-

①,②,③,④,⑤,⑥,⑦,⑧,⑨,⑩,⑪ parameters are captured from RR, Ref. Papers [8, 23, 24, 26, 27, 28, 29, 32, 41, 42, 43].

Table 3. Frequency parameters for all edges of the CCCC FG plate with k = 0 and h/a = 0.001 using CDF**.**

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	22.633	22.633③	23.443	23.440③	24.625	24.877③	26.752	27.039③	62.046	30.816③
0.5	24.585	24.579③	31.831	31.829③	44.954	44.819③	64.021	63.598③	64.868	63.986③
1	35.996	35.997②	73.385	73.432②	73.42	73.432②	108.28	108.38②	132	131.65②
		35.989③		73.399③		73.399③		108.27③		131.89③
		35.992⑧		73.413⑧		73.413⑧		108.27⑧		131.64⑧
		35.985⑨		73.395⑨		73.395⑨		108.22⑨		131.78⑨
		35.99⑩		73.41⑩		-		108.26⑩		131.66⑩
		37.22⑪		76.24⑪		76.24⑪		113.4⑪		-
1.5	60.813	60.782②	93.72	93.901②	148.8	148.85②	149.86	149.76②	179.63	179.86②
		60.762③		98.841③		148.78③		149.68③		179.57③
		60.772⑧		93.860⑧		148.82⑧		149.74⑧		179.66⑧
		60.762⑨		93.835⑨		148.78⑨		149.85⑨		179.57⑨
		60.77⑩		93.87⑩		148.83⑩		149.88⑩		-
2	98.28	98.318③	127.4	127.32③	179.32	179.28③	255.19	254.39③	255.78	255.95③
2.5	147.73	147.8⑧	174.04	173.85⑧	221.62	221.54⑧	291.36	291.89⑧	394.29	384.71⑧
		147.78⑩		173.84⑩		221.52⑩		291.87⑩		-

②,③,⑧,⑨,⑩,⑪ parameters are captured from RR, Ref. Papers [23, 24, 32, 41, 42, 43].

Table 4. Frequency parameters for all edges of the SSSS FG plate with k = 1 and h/a = 0.001 using CDF.

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	8.54	8.5405③	9.5408	9.5260③	11.236	11.228③	13.675	13.673③	23.563	23.725③
0.5	10.265	10.265③	16.425	16.424③	26.976	26.976③	35.039	34.902③	41.315	41.319③
1	16.423	16.424③	41.065	41.061③	41.066	41.061③	66.061	66.065③	83.353	83.349③
1.5	26.688	-	51.3238	-	82.124	-	93.572	-	149.085	-
2	41.061	41.060③	65.692	65.697③	107.93	107.9③	139.61	139.61③	165.249	165.27③
		41.059④		65.697④		107.93④		139.61④	-	165.27④
2.5	59.54	-	84.177	-	126.37	-	126.37	-	213.53	-

,③,④, parameters are captured from RR, Ref. Papers [24, 26].

Table 5. Frequency parameters for all edges of the CCCC FG plate with k = 1 and h/a = 0.001 using CDF.

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	18.829	18.832③	19.506	19.503③	20.489	20.699③	22.258	22.498③	51.626	25.642③
0.5	20.452	20.451③	26.493	26.484③	37.391	37.292③	53.263	52.916③	58.976	53.239③
1	29.959	29.945③	61.049	61.072③	61.049	61.072③	90.084	90.082③	100.82	100.75
1.5	50.512	-	78.11	-	123.79	-	124.77	-	149.45	-
2	81.857	81.805③	106.01	105.93③	149.17	149.17③	212.12	211.67③	213.24	212.96③
2.5	123.17	-	144.16	-	184.16	-	244.28	-	327.99	-

③ parameters are captured from RR, Ref. Paper [24].

Table 6. Frequency parameters for all edges of the SSSS FG plate with k = 2 and h/a = 0.001 using CDF.

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	8.1635	8.1639③	9.164	9.106③	10.733	10.733③	17.111	13.069③	22.736	22.679③
0.5	9.1823	9.8125③	15.701	15.7③	25.788	25.787③	33.496	33.363③	39.493	39.497③
1	15.701	15.699③	39.247	39.25③	39.25	39.251③	63.149	63.153③	79.675	79.674③
1.5	25.511	-	49.071	-	78.502	-	89.463	-	102.64	-
2	39.25	39.249③	62.808	62.8③	103.16	103.15③	133.45	133.45③	157.98	157.99③
2.5	56.918	-	80.429	-	120.77	-	179.67	-	204.09	-

③ parameters are captured from RR, Ref. Paper [24].

Table 7. Frequency parameters for all edges of the CCCC FG plate with k = 2 and h/a = 0.001 using CDF.

a/b	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value	CDF	Ref. value
0.2	18.001	18.002③	18.642	18.643③	19.71	19.786③	21.231	21.506③	49.315	24.512③
0.5	19.556	19.549③	25.326	25.316③	35.764	35.648③	50.933	50.583③	56.506	50.893③
1	28.635	28.624 ③	58.373	58.379③	58.402	58.379③	86.108	86.111③	104.96	104.91③
1.5	48.343	-	74.67	-	118.43	-	119.21	-	142.87	-
2	78.161	78.199③	101.25	101.26③	142.6	142.59③	202.81	202.33③	203.44	203.57③
2.5	117.44	-	138.4	-	176.31	-	233.23	-	313.84	-

③ parameters are captured from RR, Ref. Paper [24].

Table 8. Frequency parameters for all edges of the SSSS FG plate with k = 0 for using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3	1	1.5	2	2.5	3	5
0.001	10.263	11.448	12.336	14.266	19.738	32.075	49.353	71.558	98.705	256.61
0.01	10.206	11.433	12.32	14.247	19.731	32.067	49.336	71.539	98.676	256.56
	-	11.446⑥	12.33⑥	14.252⑥	19.732⑥	32.057⑥	49.304⑥	71.463⑥	-	-
0.02	9.8526	11.374	11.689	14.221	19.709	32.041	49.305	71.493	98.615	256.41
0.03	9.2671	11.210	11.688	14.163	19.638	31.99	49.242	71.412	98.508	256.15
0.04	8.594	10.903	11.688	14.047	19.593	31.791	48.705	71.277	98.317	255.72
0.05	7.7076	10.518	11.671	13.841	19.476	31.792	48..690	70.847	97.992	255.12

⑥ parameters are captured from RR, Ref. Paper [28].

Table 9. Frequency parameters for all edges of the CCCC FG plate with k = 0 for using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3.	1	1.5	2	2.5	3	5
0.001	22.633	23.648	24.585	27.006	35.996	60.813	98.28	147.73	208.84	568.16
0.01	22.181	23.521	24.496	26.969	35.956	60.751	98.307	147.75	208.78	565.96
0.02	20.890	23.155	24.241	26.804	35.843	60.670	98.258	147.72	208.76	566.12
0.03	19.091	22.555	23.813	26.504	35.663	60.545	98.182	147.61	208.73	566.55
0.04	17.160	21.743	23.237	26.107	35.388	60.279	97.884	146.18	208.59	566.92
0.05	15.337	20.809	22.535	25.609	35.025	60.065	97.706	145.45	207.98	566.90

Table 10. Frequency parameters for all edges of the SSSS FG plate with k = 1 using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3	1	1.5	2	2.5	3	5
0.001	8.54	9.5259	10.264	11.862	16.423	26.688	41.061	59.545	82.113	213.47
0.01	8.4918	9.513	10.256	11.854	16.417	26.681	41.050	59.524	82.103	213.47
0.02	8.1979	9.4640	9.7259	11.832	16.399	26.660	41.025	59.485	82.052	213.34
0.03	7.7107	9.3278	10.142	11.784	16.340	26.623	40.971	59.418	81.964	213.13
0.04	7.1506	9.0723	9.9741	11.688	16.302	26.452	36.364	59.306	81.805	212.77
0.05	6.1131	8.7518	9.7114	11.516	16.205	26.452	16.205	58.948	81.534	212.27

Table 11. Frequency parameters for all edges of the CCCC FG plate with k = 1 using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3.	1	1.5	2	2.5	3	5
0.001	18.829	19.664	20.452	22.478	29.959	60.511	81.856	123.16	173.92	471.93
0.01	18.455	19.571	20.382	22.431	29.917	50.548	81.795	122.95	173.72	470.89
0.02	16.816	19.266	20.169	22.302	29.823	50.480	81.756	122.91	173.70	471.84
0.03	15.885	18.767	19.814	22.053	29.674	50.377	81.692	122.82	173.67	471.80
0.04	14.278	18.091	19.334	21.723	29.445	50.155	81.444	123.29	173.55	471.70
0.05	12.761	17.314	18.750	21.308	29.142	49.977	81.297	123.52	173.05	471.69

Table 12. Frequency parameters for all edges of the SSSS FG plate with k = 2 using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3	1	1.5	2	2.5	3	5
0.001	8.1635	9.1058	9.1823	11.339	15.700	25.511	39.25	56.918	78.494	204.09
0.01	8.1098	9.0924	9.8025	11.331	15.693	25.507	39.24	56.898	78.481	204.05
0.02	7.7901	9.037	9.7650	11.309	15.674	23.507	32.24	50.898	78.481	203.05
0.03	7.2882	8.8843	9.6739	11.254	11.254	15.621	56.788	56.788	78.333	203.71
0.04	7.1507	9.0723	9.9741	11.688	16.302	26.452	36.364	59.306	81.805	212.77
0.05	5.8457	8.2744	9.2028	10.947	15.448	25.256	38.915	56.470	77.918	202.57

Table 13. Frequency parameters for all edges of the CCCC FG plate with k = 2 using CDF.

h/a	Aspect ratio(a/b)									
	0.2	0.4	1/2	2/3.	1	1.5	2	2.5	3	5
0.001	18.001	18.809	19.555	21.487	28.635	48.343	78.260	117.44	165.80	458.66
0.01	17.600	18.684	19.475	21.434	28.595	48.312	78.194	117.37	166.08	450.63
0.02	16.468	18.369	19.251	21.287	28.521	48.228	78.139	117.36	166.07	450.36
0.03	14.928	17.451	18.875	21.036	28.341	48.109	77.781	117.34	166.06	450.02
0.04	13.311	17.138	18.367	20.695	28.117	47.910	77.614	117.32	166.04	450.01
0.05	11.817	16.332	17.748	20.248	27.767	20.431	77.521	117.30	165.03	450.00

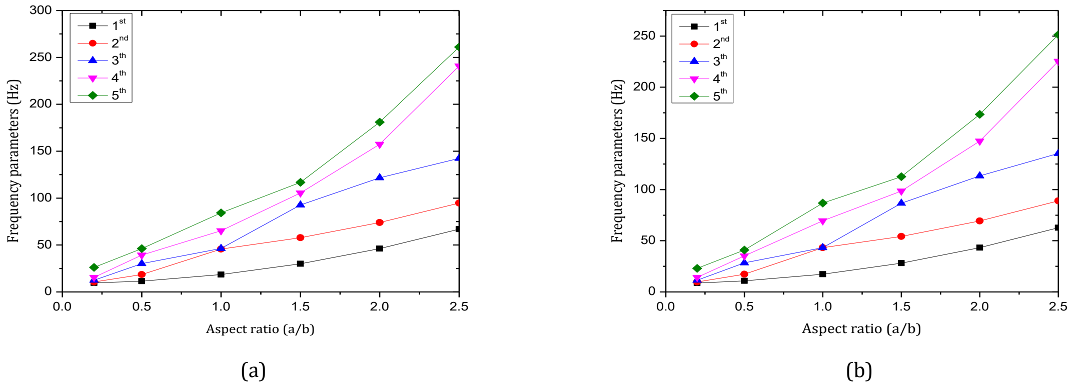

Fig. 2. Effect of aspect ratio on frequency parameters (The first five frequencies) of functionally graded simply-supported plate with k = 0.2 and h/a with (a) 0.01 (b) 0.02

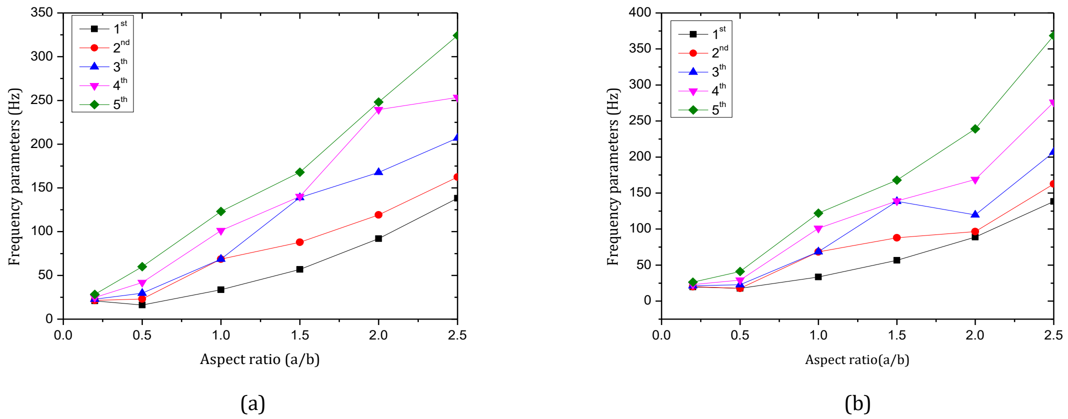

Fig. 3. Effect of aspect ratio on frequency parameters (The first five frequencies) of the functionally graded clamped plate with k = 0.2 and h/a with (a) 0.01 (b) 0.02

The vibration behavior of functionally graded plates was evaluated with different thickness ratios (h/a), aspect ratios (a/b), and power law index (k) subjected to different boundary conditions. The fundamental frequencies for all edges are simply supported and clamped with a thickness ratio of h/a = 0.001, different aspect ratios and power law index are presented in Tables 2-7. The results obtained in the present method are compared with the RR method [24] and it was observed that they are accurate with a maximum variation of 0.05%, which shows the efficacy of the proposed method.

The fundamental frequencies for all edges simply supported and clamped with different thickness ratios, aspect ratios, and power law indexes are presented in Tables 8-13. It is observed that the fundamental frequency parameters decrease with an increase in the plate thickness ratio and frequencies increase with an increase in the aspect ratio. Fundamental frequencies are decreasing with an increase in power-law for a fixed aspect ratio, irrespective of boundary conditions.

The effect of aspect ratios (a/b) on frequency parameters (The first five frequencies) of a simply supported functionally graded plate and a clamped functionally graded plate is plotted in Figs. 2 and 3, respectively with k = 0.2 and different h/a. It is observed that the frequency parameters increase with the increase in aspect ratio.

6. Conclusions

The vibration characteristics are investigated for an FG rectangular plate subjected to all edges SSSS and CCCC boundary conditions using the CDF method. The energy formulations in the CDF method contain half the number of undetermined coefficients when compared with the RR method. To inspect the vibration characteristics of the FG rectangular plate, various aspect ratios, thickness ratios, and power-law indexes are utilized. It is observed that the frequency parameters are decreasing with increasing k and increasing with increasing aspect ratios. The numerical results acquired from the present work are validated with other literature and are found to be similar.

Other shear deformation plate theories can be easily handled in the above analysis to compare the results obtained from FSDT. Further, the CDF method can be extended to study the free vibration behavior of isotropic shells, cylindrical panels, laminate composite plates, and non-linear dynamic responses of the structures.

Nomenclature

a	Dimension of the plate in x direction
b	Dimension of the plate in y direction
h	Thickness of the plate
k	Material variation profile
E_m	Metal Young's modulus
E_c	Ceramic Young's modulus
G	Shear modulus at functionally graded material
M_c	Density of Ceramic
M_m	Density of Metal
α_x	y-axis rotation
α_y	x-axis rotation
w	Transverse displacement
a/b	Aspect ratio
h/a	Thickness ratio
k	Shear correction factor (= 5/6)
U	Strain energy
T	Kinetic energy
ν	Poisson's ratio

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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