Int. J. Nonlinear Anal. Appl. In Press, 1–10
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2024.34032.5086

Evaluation of the inefficiency in the assembly and body line of Iran Khodro Group using two-stage non-cooperative data envelopment analysis

Sara Daneshmandi[a], Shokrollah Ziari[b,*], Ebrahim Niknaghsh[a], Amir Gholam Abri[c], Mehdi Amir Miandaragh[c]

[a] *Department of Industrial Management, Fi.C., Islamic Azad University, Firuzkoh, Iran*

[b] *Department of Mathematics, ST.C., Islamic Azad University, Tehran, Iran*

[c] *Department of Mathematics, Fi.C., Islamic Azad University, Firoozkooh, Iran*

(Communicated by Javad Vahidi)

Abstract

Data envelopment analysis (DEA) is one of the fundamental methods for analyzing the performance of a homogeneous set of decision-making units. Recently, DEA has been developed and applied to multi-stage processes. The essential feature of multi-stage processes is the presence of non-optimal intermediate outputs, which are usually not the final output of the system. This study used a new approach to analyze the reuse of non-optimal intermediate outputs in a two-stage production process with common resources. Ultimately, non-cooperative efficiency criteria were used to demonstrate unit efficiency. Finally, the unit under study is relatively inefficient from the perspective of the non-cooperative model related to the assembly and body production line of Iran Khodro.

Keywords: Data Envelopment Analysis (DEA), Two-stage DEA, Production line
2020 MSC: 90C08

1 Introduction

Intense competition among companies and production units is a defining feature of today's society. As a result, these organizations constantly assess their status and compare it to their competitors as a concern for managers. The essential step in improving the efficiency of an organization is identifying inefficiencies in the system. The organization's position can be improved by eliminating these inefficiencies. The first methods developed to examine the efficiency of organizations can be attributed to the year 1975 when Farrell [8] introduced a method for measuring the efficiency of production units with a simple and basic approach, only suitable for units with a single input and output. In other words, this method could not be used for units with multiple inputs and outputs. Therefore, unlike the previous model, Charnes et al. [5] introduced the data envelopment analysis model, which could calculate efficiency for units with more than one input and output.

Two widely used models in data envelopment analysis (DEA) are the CCR and BCC models, with the former operating under constant returns to scale and the latter under variable returns to scale. In both models, the nature of

*Corresponding author
Email addresses: sara.daneshmandi@iau.ac.ir (Sara Daneshmandi), sziari@iau.ac.ir (Shokrollah Ziari), ebrahim@azad.ac.ir (Ebrahim Niknaghsh), eniknaghsh@iau.ac.ir (Amir Gholam Abri), m_amirmiandaragh@iau.ac.ir (Mehdi Amir Miandaragh)

Received: May 2024 *Accepted:* July 2024

the output-oriented and input-oriented is preserved. The CCR model is one of the earliest developed models for DEA based on constant returns to scale. The acronym CCR is derived from the initials of the developers of this model. Six years after the development of the CCR model, in 1984, the second DEA model, named BCC, was developed by Banker, Charnes, and Cooper [3]. The CCR model assumed that returns are constant, a presumption emanating from the infinite range [14].

DEA can be classified as a non-parametric method based on linear programming to measure the efficiency of homogeneous decision-making units with several inputs and outputs. Different types of efficiency can be considered, including absolute, relative, and economic efficiency [10, 20, 24]. In traditional DEA models, decision-making units were considered as a series of black boxes, and each input to these black boxes led to the creation of a final output. In recent years, these classical methods have been developed, resulting in the development of two-step processes. In two-stage processes, the performance of subunits and intermediate products is also considered [12, 26, 28].

Various analyses have been presented, including overall efficiency decomposition into average separate outputs. Akbari et al. [1] employed the SBM model developed by the DEA network to assess the efficiency of bank branches. Their sample consisted of 31 branches from major commercial banks in Iran. Chen and Zhu [6] utilized an input-oriented model for the first stage and an output-oriented model for the second stage. Sangkyo and Jungnam developed the first two-stage DEA model considering auxiliary variables for inputs and outputs [22]. This model was implemented for a real-world case in the banking industry, and the results were compared with those obtained from previous two-stage DEA models.

Kao and Huang [16] investigated the efficiency in a two-stage production system and used the relationship between the two stages in measuring system efficiency, demonstrating that overall efficiency is the product of the efficiencies of the two subunits. Identifying the appropriate approach to determine the improved points (image on the efficient frontier) for inefficient DMUs in the framework of two-stage DEA under constant returns to scale (CRS) was examined by Chen et al. [7]. Lim and Cho [19] presented a linear parametric model using Charles-Cooper transformations. An innovative linear DEA model was developed for measuring the efficiency of a two-stage system with common inputs under constant returns to scale by Talou et al. [27]. Amirteimoori and Yang [2] developed a DEA approach for measuring the efficiency of decision processes which can be divided into two stages. In this trend, we modified the proposed model in [2] to measure the efficiency of the two-stage production system.

The rest of the paper is organized as follows. Section 2 reviews the proposed two-stage model in [2]. Section 3 presents a two-stage network DEA model with a shared input including additive efficiency measures. Section 4, applies the new approach to the 17 prefabricated cabin plants. In Section 4, an example of 17 prefabricated cabin plants given in [2] is performed to illustrate our proposed model. Section 4 applies the proposed model to the 7 products of Irankhodroo and SIPA company cabin plants. Conclusions are presented in Section 6.

2 Review the Two-stage model proposed by Amirteymoori and Yang [2]

Consider a two-stage production process shown in Figure 1.

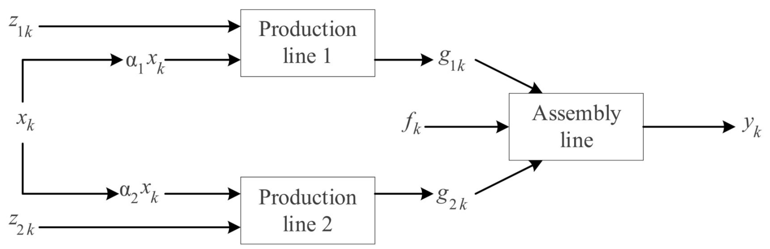

Figure 1: The production system.

Suppose that we have n DMUs and each DMU$_j$, $j = 1, ..., n$ consists of two parallel production lines and an assembly line. The first and second production lines consume inputs $Z_j^{(1)} = \left(z_{1j}^{(1)}, z_{2j}^{(1)}, ..., z_{Dj}^{(1)}\right)^T$ and $Z_j^{(2)} = \left(z_{1j}^{(2)}, z_{2j}^{(2)}, ..., z_{Hj}^{(2)}\right)^T$, respectively. Also, it is assumed that DMU$_j$ has m shared inputs $x_j = (x_{1j}, x_{2j}, ..., x_{mj})^T$ among the two production lines. The observed shared input to production lines 1 and 2 can be denoted as $x_j = x_j^{(1)} + x_j^{(2)}$, where $x_j^{(1)} = \left(x_{1j}^{(1)}, x_{2j}^{(1)}, ..., x_{ij}^{(1)}\right)^T$ and $x_j^{(2)} = \left(x_{1j}^{(2)}, x_{2j}^{(2)}, ..., x_{ij}^{(2)}\right)^T$. In optimality, some portion $0 \leq \alpha_i^{(1)} < 1$ of the shared inputs x_{ij} is allocated to the first line, and the remainder $\alpha_i^{(2)} = 1 - \alpha_i^{(1)}$ is allocated to the second line. So, the first component consumes inputs $\alpha_i^{(1)} x_{ij},\ i = 1, 2, ..., m$ and $z_{dj}^{(1)},\ d = 1, 2, ..., D$ to produce $g_{pj}^{(1)},\ p = 1, 2, ..., P$, and the second line consumes inputs $\alpha_i^{(2)} x_{ij},\ i = 1, 2, ..., m$ and $z_{hj}^{(2)},\ h = 1, 2, ..., H$ to produce $g_{qj}^{(2)},\ q = 1, 2, ..., Q$. The assembly line is used by a mixture of inputs $g_p^{(1)}$ and $g_q^{(2)}$ and an external input $f_j = (f_{1j}, f_{2j}, ..., f_{Bj})$ and the terminal product is $y_j = (y_{1j}, y_{2j}, ..., y_{sj})^T$.

The object in [2] was to determine the relative efficiencies of the two production lines and assembly line along with an overall efficiency of the whole system. The proposed model in [2] is based on an additive model. In the assessment of production lines 1 and 2, the output measures $g^{(1)}$ and $g^{(2)}$ should be increased. On the other hand, these measures are considered as inputs to the assembly line and they should be decreased. If it is treated the system's operation as a black box, ignoring the intermediate measures may yield an efficient DMU with inefficient production lines and/or assembly lines. In the proposed model, the intermediate measures $g^{(1)}$ and $g^{(2)}$ are considered to be free variables, and they will be increased or decreased to make the whole system as efficient.

To provide a realistic picture of DMU's performance, some restrictions are imposed on the variables $\alpha^{(1)}$ and $\alpha^{(2)}$. Ratio constraints of the form $l_i^\alpha \leq \frac{\alpha^{(1)}}{\alpha^{(2)}} \leq u_i^\alpha$ are imposed. These constraints reflect the relative importance of the shared resources that are divided between two production lines. Consider the assessment of $\text{DMU}_j,\ j = 1, ..., n$ in additive form. Taking into account $\alpha_i^{(1)} + \alpha_i^{(2)} = 1,\ i = 1, 2, ..., m$, the proposed model in [2] can be written as follows:

$$max \quad E_k = \sum_{i=1}^{m} s_i^{x(1)} + \sum_{i=1}^{m} s_i^{x(2)} + \sum_{p=1}^{P} s_p^{g(1)} + \sum_{d=1}^{D} s_d^{z(1)} + \sum_{h=1}^{H} s_h^{z(2)} + \sum_{q=1}^{Q} s_q^{g(2)} + \sum_{b=1}^{B} s_b^{f} + \sum_{r=1}^{s} s_r^{y} \tag{2.1}$$

$$s.t.$$

$$\text{Line 1:} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{(1)} + s_i^{x(1)} = \alpha_i^{(1)} \left(x_{ik}^{(1)} + x_{ik}^{(2)} \right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{dj}^{(1)} + s_d^{z(1)} = z_{dk}^{(1)}, \quad \forall\ d,$$

$$\sum_{j=1}^{n} \lambda_j g_{pj}^{(1)} + s_p^{g(1)} = g_{pk}^{(1)}, \quad \forall\ p,$$

$$\text{Line 2:} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{(2)} + s_i^{x(2)} = \alpha_i^{(2)} \left(x_{ik}^{(1)} + x_{ik}^{(2)} \right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{hj}^{(2)} + s_h^{z(2)} = z_{hk}^{(2)}, \quad \forall\ h,$$

$$\sum_{j=1}^{n} \lambda_j g_{qj}^{(2)} + s_q^{g(2)} = g_{qk}^{(2)}, \quad \forall\ q,$$

Assembly Line:

$$\sum_{j=1}^{n} \lambda_j g_{pj}^{(1)} + s_p^{g(1)} = g_{pk}^{(1)}, \quad \forall\ p,$$

$$\sum_{j=1}^{n} \lambda_j g_{qj}^{(2)} + s_q^{g(2)} = g_{qk}^{(2)}, \quad \forall\ q,$$

$$\sum_{j=1}^{n} \lambda_j f_{bj} + s_b^{f} = f_{bk}, \quad \forall\ b,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^{y} = y_{rk}, \quad \forall\ r,$$

General constraint:

$$l_i^{\alpha} \leq \frac{\alpha_i^{(1)}}{\alpha_i^{(2)}} \leq u_i^{\alpha}, \quad \forall\ i,$$

$$\alpha_i^{(1)} + \alpha_i^{(2)} = 1, \quad \forall\ i,$$

$$\alpha_i^{(1)},\ \alpha_i^{(2)},\ \lambda_j \geq 0, \quad \forall\ i,\ j,$$

$$s_i^{x(1)},\ s_i^{x(2)},\ s_d^{z(1)},\ s_h^{z(2)},\ s_p^{g(1)},\ s_q^{g(2)},\ s_b^{f},\ s_r^{y} \geq 0, \quad \forall\ i,\ d,\ h,\ p,\ q,\ b,\ r.$$

In applying the model described in [2], attention is paid to the additive model. In the assessment of production lines 1 and 2, the output measures $k^{(1)}$ and $k^{(2)}$ should be increased. On the other hand, these measures are considered as inputs to the assembly line and they should be decreased. If they treat the system's operation as a black box, ignoring the intermediate measures may yield an efficient DMU with inefficient production lines and/or assembly lines. In the model proposed in [2], the intermediate measures $k^{(1)}$ and $k^{(2)}$ are considered to be free variables, and they will be increased or decreased to make the whole system as efficient. To provide a realistic picture of DMU's performance, some restrictions are imposed on the variables $\alpha^{(1)}$ and $\alpha^{(2)}$. Ratio constraints of the form $l_i^{\alpha} \leq \frac{\alpha_i^{(1)}}{\alpha_i^{(2)}} \leq u_i^{\alpha}$ on the portion variables $\alpha^{(1)}$ and $\alpha^{(2)}$ are imposed. These constraints reflect the relative importance of the shared resources that are split between two production lines.

3 Proposed Two-stage model

According to the imposing constraint $l_i^{\alpha} \leq \frac{\alpha_i^{(1)}}{\alpha_i^{(2)}} \leq u_i^{\alpha}$, for all$i$ into the Model (2.1), the number of efficient DMUs can be changed by changing the l_i^{α} and u_i^{α} and so, to obtain the unique number of efficient DMUs in all lines of the production, we modify the proposed model in [2] by discarding the mentioned constraint. For this purpose, consider a two-stage production process shown in Figure 1. Suppose we have n DMUs and each DMU$_j,\ j = 1, ..., n$ consists of two parallel production lines and an assembly line. The first and second production lines consume inputs $Z_j^{(1)} = \left(z_{1j}^{(1)}, z_{2j}^{(1)}, ..., z_{Dj}^{(1)}\right)^T$ and $Z_j^{(2)} = \left(z_{1j}^{(2)}, z_{2j}^{(2)}, ..., z_{Hj}^{(2)}\right)^T$, respectively. Also, it is assumed that DMU$_j$ has m shared inputs $x_j = (x_{1j}, x_{2j}, ..., x_{mj})^T$ among the two production lines. We assume that x_{ij} are split into $\alpha^{(1)} x_{ij}$ and $(1 - \alpha^{(1)}) x_{ij},\ i = 1, 2, ..., m,\ 0 \leq \alpha^{(1)} \leq 1$ for each DMU$_j,\ j = 1, ..., n$, corresponding to the portions of shared inputs are allocated to the first and second production lines, respectively. In optimality, some portion $0 \leq \alpha_i^{(1)} < 1$ of the shared inputs x_{ij} is allocated to the first line, and the remainder $\alpha_i^{(2)} = 1 - \alpha_i^{(1)}$ is allocated to the second line. So, the first component consumes inputs $\alpha_i^{(1)} x_{ij},\ i = 1, 2, ..., m$ and $z_{dj}^{(1)},\ d = 1, 2, ..., D$ to produce $g_{pj}^{(1)},\ p = 1, 2, ..., P$, and the second line consumes inputs $\alpha_i^{(2)} x_{ij},\ i = 1, 2, ..., m$ and $z_{hj}^{(2)},\ h = 1, 2, ..., H$ to produce $g_{qj}^{(2)},\ q = 1, 2, ..., Q$. The assembly line is used by a mixture of inputs $g_p^{(1)}$ and $g_q^{(2)}$ and an external input $f_j = (f_{1j}, f_{2j}, ..., f_{Bj})$ and the terminal product is $y_j = (y_{1j}, y_{2j}, ..., y_{sj})^T$. The object in [2] was to determine the relative efficiencies of the two production lines and assembly lines along with the overall efficiency of the whole system. The proposed model in this work is based on an additive model. In the assessment of production lines 1 and 2, the output measures $g^{(1)}$ and $g^{(2)}$ should be increased. On the other hand, these measures are considered as inputs to the assembly line and they should be decreased. In the proposed model, the intermediate measures $g^{(1)}$ and $g^{(2)}$ are considered to be free variables, and they will be increased or decreased to make the whole system as efficient. To provide a realistic picture of DMU's performance, some restrictions are imposed on the variables $\alpha^{(1)}$ and $\alpha^{(2)}$. Constraint of the form $\alpha^{(1)} + \alpha^{(2)} = 1$ is imposed. This constraint reflects the relative importance of the shared resources that are divided between two production lines. Consider the assessment of DMU$_j,\ j = 1, ..., n$ in additive form. Taking into account $\alpha_i^{(1)} + \alpha_i^{(2)} = 1,\ i = 1, 2, ..., m$, the proposed model can be written as follows:

$$\max \quad E_k = \sum_{i=1}^{m} s_i^{x(1)} + \sum_{i=1}^{m} s_i^{x(2)} + \sum_{p=1}^{P} s_p^{g(1)} + \sum_{d=1}^{D} s_d^{z(1)} + \sum_{h=1}^{H} s_h^{z(2)} + \sum_{q=1}^{Q} s_q^{g(2)} + \sum_{b=1}^{B} s_b^{f} + \sum_{r=1}^{s} s_r^{y} \tag{3.1}$$

$s.t.$

$$\text{Line 1:} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{(1)} + s_i^{x(1)} = \alpha_i^{(1)} \left(x_{ik}^{(1)} + x_{ik}^{(2)}\right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{dj}^{(1)} + s_d^{z(1)} = z_{dk}^{(1)}, \quad \forall\ d,$$

$$\sum_{j=1}^{n} \lambda_j g_{pj}^{(1)} + s_p^{g(1)} = g_{pk}^{(1)}, \quad \forall\ p,$$

Line 2: $$\sum_{j=1}^{n} \lambda_j x_{ij}^{(2)} + s_i^{x(2)} = \alpha_i^{(2)} \left(x_{ik}^{(1)} + x_{ik}^{(2)} \right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{hj}^{(2)} + s_h^{z(2)} = z_{hk}^{(2)}, \quad \forall\ h,$$

$$\sum_{j=1}^{n} \lambda_j g_{qj}^{(2)} + s_q^{g(2)} = g_{qk}^{(2)}, \quad \forall\ q,$$

Assembly Line:

$$\sum_{j=1}^{n} \lambda_j g_{pj}^{(1)} + s_p^{g(1)} = g_{pk}^{(1)}, \quad \forall\ p,$$

$$\sum_{j=1}^{n} \lambda_j g_{qj}^{(2)} + s_q^{g(2)} = g_{qk}^{(2)}, \quad \forall\ q,$$

$$\sum_{j=1}^{n} \lambda_j f_{bj} + s_b^{f} = f_{bk}, \quad \forall\ b,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^{y} = y_{rk}, \quad \forall\ r,$$

General constraint:

$$\alpha_i^{(1)} + \alpha_i^{(2)} = 1, \quad \forall\ i,$$

$$\alpha_i^{(1)},\ \alpha_i^{(2)},\ \lambda_j \geq 0, \quad \forall\ i,\ j,$$

$$s_i^{x(1)},\ s_i^{x(2)},\ s_d^{z(1)},\ s_h^{z(2)},\ s_p^{g(1)},\ s_q^{g(2)},\ s_b^{f},\ s_r^{y} \geq 0, \quad \forall\ i,\ d,\ h,\ p,\ q,\ b,\ r.$$

According to the two first constraints in the part of the Assembly line of the above model are redundant, by removing these constraints, the above model can be rewritten as follows:

$$\max \quad E_k = \sum_{i=1}^{m} s_i^{x(1)} + \sum_{i=1}^{m} s_i^{x(2)} + \sum_{p=1}^{P} s_p^{g(1)} + \sum_{d=1}^{D} s_d^{z(1)} + \sum_{h=1}^{H} s_h^{z(2)} + \sum_{q=1}^{Q} s_q^{g(2)} + \sum_{b=1}^{B} s_b^{f} + \sum_{r=1}^{s} s_r^{y} \tag{3.2}$$

$s.t.$

Line 1: $$\sum_{j=1}^{n} \lambda_j x_{ij}^{(1)} + s_i^{x(1)} = \alpha_i^{(1)} \left(x_{ik}^{(1)} + x_{ik}^{(2)} \right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{dj}^{(1)} + s_d^{z(1)} = z_{dk}^{(1)}, \quad \forall\ d,$$

$$\sum_{j=1}^{n} \lambda_j g_{pj}^{(1)} + s_p^{g(1)} = g_{pk}^{(1)}, \quad \forall\ p,$$

Line 2: $$\sum_{j=1}^{n} \lambda_j x_{ij}^{(2)} + s_i^{x(2)} = \alpha_i^{(2)} \left(x_{ik}^{(1)} + x_{ik}^{(2)} \right), \quad \forall\ i,$$

$$\sum_{j=1}^{n} \lambda_j z_{hj}^{(2)} + s_h^{z(2)} = z_{hk}^{(2)}, \quad \forall\ h,$$

$$\sum_{j=1}^{n} \lambda_j g_{qj}^{(2)} + s_q^{g(2)} = g_{qk}^{(2)}, \quad \forall\ q,$$

Assembly Line:

$$\sum_{j=1}^{n} \lambda_j f_{bj} + s_b^{f} = f_{bk}, \quad \forall\ b,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^{y} = y_{rk}, \quad \forall\ r,$$

General constraint:

$$\alpha_i^{(1)} + \alpha_i^{(2)} = 1, \quad \alpha_i^{(1)},\ \alpha_i^{(2)},\ \lambda_j \geq 0, \quad \forall\ i,\ j,$$

$$s_i^{x(1)},\ s_i^{x(2)},\ s_d^{z(1)},\ s_h^{z(2)},\ s_p^{g(1)},\ s_q^{g(2)},\ s_b^{f},\ s_r^{y} \geq 0, \quad \forall\ i,\ d,\ h,\ p,\ q,\ b,\ r.$$

Theorem 3.1. The LP model (3.2) is feasible.

Proof . It is obvious that the following solution is a feasible solution to this problem.

$$\lambda_k = 1, \quad \lambda_j = 0,\ j = 1, 2, ..., n,\ j \neq k,$$

$$s_i^{x(1)} = s_i^{x(2)} = s_d^{z(1)} = s_h^{z(2)} = s_p^{g(1)} = s_q^{g(2)} = s_b^{f} = s_r^{y} = 0, \quad \forall\ i,\ d,\ h,\ p,\ q,\ b,\ r,$$

$$\alpha_i^{(1)} = \frac{x_{ik}^{(1)}}{x_{ik}^{(1)} + x_{ik}^{(2)}}, \quad i = 1, 2, ..., m, \qquad \alpha_i^{(2)} = \frac{x_{ik}^{(2)}}{x_{ik}^{(1)} + x_{ik}^{(2)}}, \quad i = 1, 2, ..., m.$$

□

Definition 3.2. DMU$_k$ is said to be additive efficient if and only if $E_k = 0$.

Definition 3.3. DMU$_k$ is said to be additive efficient in stage 1 if and only if

$$E_k^{(1)} = \sum_{i=1}^{m} s_i^{x(1)} + \sum_{p=1}^{P} s_p^{g(1)} + \sum_{d=1}^{D} s^{z(1)} = 0.$$

Definition 3.4. DMU$_k$ is said to be additive efficient in stage 2 if and only if

$$E_k^{(2)} = \sum_{i=1}^{m} s_i^{x(2)} + \sum_{p=1}^{P} s_p^{g(2)} + \sum_{d=1}^{D} s^{z(2)} = 0.$$

Definition 3.5. DMU$_k$ is said to be additive efficient in assembly line if and only if

$$E_k^{(A)} = \sum_{p=1}^{P} s_p^{g(1)} + \sum_{p=1}^{P} s_p^{g(2)} + \sum_{b=1}^{B} s_b^{f} + \sum_{r=1}^{s} s_r^{y} = 0.$$

4 Applied example [2]

In this section, we review the two-stage production process discussed in [2] with the analysis of the manufacturing company's activities. A limited company in Golestan, Iran, has 17 plants that produce prefabricated cabins. Each manufactory consists of two production lines arranged in series: the structure production line and the doors and windows (D&W) production line. The structure production line uses steels ($z_1^{(1)}$) and some portion of woods ($x^{(1)}$) to produce structures ($k^{(1)}$). Parallel to this line, the D& W production line uses glasses ($z_2^{(2)}$), some portion of woods ($x^{(2)}$) and aluminum ($z_1^{(2)}$) to produce doors and windows ($k^{(2)}$). The produced structures, doors and windows will be assembled in the assembly line to produce the final products which are prefabricated cabins (y). The assembly line uses two external inputs: corrugated plate (Asbestos cement) (f_1) and concrete (f_2). The production process is

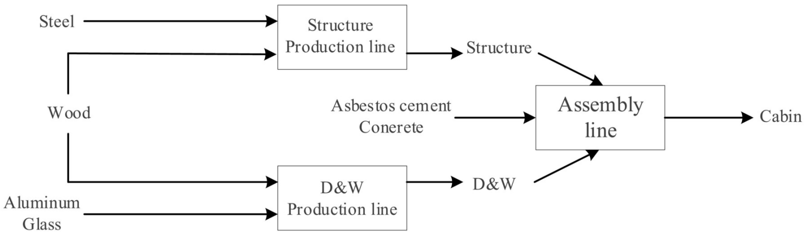

Figure 2: The production process of prefabricated cabin.

demonstrated in Figure 2. The data for six months is displayed in Table 2. The results from the model in [2] i.e. Model (2.1) are reported in Table 3 where the columns are the inefficiency slacks obtained from Model (2.1). As the table shows, eight plants are efficient in the overall sense. Also, the obtained results from the proposed model i.e. Model (3.2) are illustrated in Table 3 where the columns are the inefficiency slacks obtained from Model 2. As the table specifies, twelve plants are efficient in the overall sense.

Table 1: Input and output data taken [2].

DMU_j	$x^{(1)}$	$x^{(2)}$	$z_1^{(1)}$	$z_1^{(2)}$	$z_2^{(2)}$	$g^{(1)}$	$g^{(2)}$	y_1	f_1	f_2
1	5	5.5	12700	9	8	168	328	160	310	1600
2	4.6	4.4	11500	9	4	120	226	108	290	1100
3	3.5	5	11300	11	7	84	220	84	160	800
4	2	3.2	8000	12	6	65	156	69	165	750
5	6.3	3.7	13000	12	11	144	268	132	260	1400
6	4.7	5.8	13500	22	23	158	280	149	290	1500
7	4.3	5.2	12000	29	31	144	268	130	230	1350
8	6.8	4.2	13450	13	9	168	326	158	300	1600
9	5	3.5	11010	28	11	120	240	112	260	1100
10	4.1	3.4	10500	19	12	89	178	84	160	900
11	4.8	5.2	12350	10	9	144	284	132	235	1300
12	4.4	5.6	13000	29	17	144	262	129	225	1350
13	3.8	4.2	11505	9	11	108	200	99	215	1000
14	5	3.5	9550	22	21	96	178	82	165	850
15	5.2	6.3	13800	24	11	168	330	157	315	1600
16	5.4	5.1	13500	22	21	141	312	144	300	1500
17	6.8	5.7	13505	24	11	153	318	150	295	155

5 Empirical study in Iran Khodro Automotive and SIPA Automotive Groups

Iran Khodro, branded as IKCO, is an Iranian automaker headquartered in Tehran. The public company manufactures vehicles, including Samand, Peugeot and Renault cars, trucks, minibuses and buses. Currently, this automotive group produces Tara, Dena, Peugeot 207 and Rana cars in mass production. Also, SIPA is an Iranian automaker headquartered in Tehran. Its products in recent years have been mostly Qick, Sina and Sahand cars. In the supply chain of large automotive manufacturing industrial groups such as Iran Khodro and SIPA by joining the producers of body parts of automobiles such as Peugeot Pars, Peugeot 206, Peugeot 207, Dena, Tiba, Aria, etc. and by providing employment opportunities for nearly 7000 workers. This section focuses on a case study of the two-stage static production process related to the assembly line and body unit of Iran Khodro Industrial and SIPA groups. In this section, we perform the two-stage production process described with the analysis of the assembly line and body unit of Iran Khodro Industrial and SIPA companies activities. Iran Khodro and SIPA Industrial groups have 4 and 3 plants respectively, these produce cars. We consider the assembly line and body unit of Iran Khodro Industrial and SIPA groups. Each manufactory consists of two production lines arranged in series: skeleton production line and doors (including front and rear doors and trunk lid door) and hood (D&H) production line. The skelrton production line uses number of workers ($z_1^{(1)}$) and some portion of materials ($x^{(1)}$) to produce number of skeletons ($g^{(1)}$). Parallel to this line, the D&H production line uses number of workers ($z_1^{(2)}$) and some portion of materials ($x^{(2)}$) to produce doors and hoods ($g^{(2)}$). The produced structures, doors and hoods will be assembled in the assembly line to produce the final products which are the body of cars (y). The assembly line uses two external inputs: number of workers (Assembly line workers) (f_1) and coloring materials (f_2). The production process is illustrated in Figure (3). Table 4 provides the data for the above industrial production process for the 7 cars (D_1:TARA, D_2:DENA, D_3:Peugeot

Table 2: Results from model 1 in [2] (Model (2.1)).

DMU_j	E_k	$E_k^{(1)}$	$E_k^{(2)}$	$E_k^{(A)}$	α_1	α_2
1	0	0	0	0	0.4762	0.5238
2	0	0	0	0	0.5111	0.4889
3	0	0	0	0	0.4118	0.5882
4	2632.8781	2516.3469	25.2187	98.4125	0.5439	0.4561
5	2619.8625	2529.2375	6.375	87.05	0.5462	0.4538
6	0	0	0	0	0.4476	0.5524
7	0	0	0	0	0.4526	0.5474
8	387.9808	361.9602	1.3839	23.4801	0.4884	0.5116
9	0	0	0	0	0.5882	0.4118
10	3698.46	3622.8657	21.88	59.1829	0.6027	0.3973
11	0	0	0	0	0.48	0.52
12	0	0	0	0	0.44	0.56
13	3692.1469	3652.4281	6.5312	34.2875	0.5746	0.4254
14	3134.5812	3054.2687	44.1875	55.925	0.6684	0.3316
15	1409.7531	1342.4719	26.4687	52.1125	0.5307	0.4693
16	2186.35	2060.85	44.5	87.6	0.5286	0.4714
17	1680.8438	1596.9063	29.5625	60.375	0.5875	0.4125

Table 3: Results from model proposed model (Model (3.2)).

DMU_j	E_k	$E_k^{(1)}$	$E_k^{(2)}$	$E_k^{(A)}$	α_1	α_2
1	0	0	0	0	0.4762	0.5238
2	0	0	0	0	0.5111	0.4889
3	0	0	0	0	0.4118	0.5882
4	0	0	0	0	0.3846	0.6154
5	2452.4082	2356.8134	7.2588	88.3360	0.4129	0.5871
6	0	0	0	0	0.4476	0.5524
7	0	0	0	0	0.4526	0.5474
8	0	0	0	0	0.6182	0.3818
9	0	0	0	0	0.5882	0.4118
10	3687.8958	3598	25.9492	63.9467	0.3590	0.6410
11	0	0	0	0	0.48	0.52
12	0	0	0	0	0.44	0.56
13	3500.0737	3456.8075	7.5196	35.7466	0.3874	0.6126
14	3154.3812	3051.1500	47.3062	55.9250	0.3015	0.6985
15	1421.0531	1341.2750	27.6656	52.1125	0.4266	0.5734
16	0	0	0	0	0.5143	0.4857
17	0	0	0	0	0.544	0.456

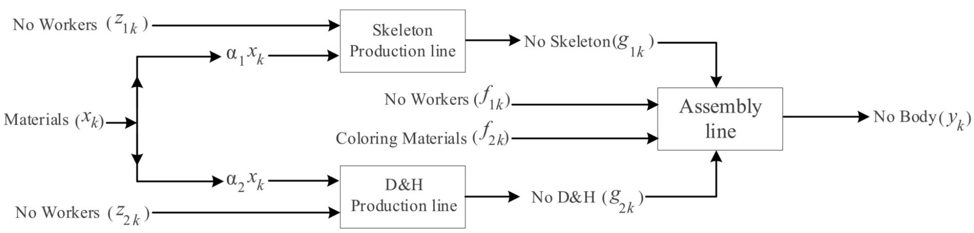

Figure 3: The production process of prefabricated cabin.

207, D_4:RANA, D_5:QUIK, D_6: SINA, D_7:SAHAND) in Iran. The results related to the efficiency of the investigated production system unit are reported in Table 5. From Table 5, there are 2 cars (Peugeot 207 and RANA) that are efficient in both stages in the additive efficiency model.

6 Conclusion

This paper evaluates the efficiency of two-stage production processes using a network DEA-based model. The two-stage production processes consider three processes where two parallel processes with shared input resources to both processes in the first stage are connected serially with the process in the second stage. To assess the efficiency of this two-stage production system, an additive model has been proposed. The additive proposed DEA model in the

Table 4: Input and output data of industrial production for for 7 cars.

DMU_j	$x^{(1)}$	$x^{(2)}$	$z_1^{(1)}$	$z_1^{(2)}$	$g^{(1)}$	$g^{(2)}$	y_1	f_1	f_2
D_1	79875	8875	900	1050	250	1500	250	808	1035
D_2	130320	14480	900	1050	400	2400	400	1292	1035
D_3	147960	16440	900	1050	600	3600	600	1740	1035
D_4	23814	2646	270	360	90	360	90	270	375
D_5	74160	8240	696	810	200	1200	200	590	819
D_6	76860	8540	641	810	200	1200	200	607	819
D_7	83160	9240	747	810	200	1200	200	594	819

Table 5: Results from proposed model (Model (3.2)) on data in Table 4.

DMU_j	E_k	$E_k^{(1)}$	$E_k^{(2)}$	$E_k^{(A)}$	α_1	α_2
D_1	22074.25	525	20862.5	686.75	0.6946	0.3054
D_2	36327	300	35550	477	0.6812	0.3188
D_3	0	0	0	0	0.9000	0.1000
D_4	0	0	0	0	0.9000	0.1000
D_5	28940	396	28060	484	0.5985	0.4015
D_6	31902	341	31060	501	0.5775	0.4225
D_7	38995	447	38060	488	0.5338	0.4662

current paper is based on a modification constructed model in [2]. Also, our proposed model is performed on a data set in [2] to measure the industrial production performance of 17 cases of prefabricated cabin plants and is determined as an optimal split of shared resources uniquely. Finally, as an application, our proposed model is illustrated with a data set for measuring the industrial production performance of 7 cars produced by Iran Khodro Automotive and SIPA Automotive Groups.

References

[1] S. Akbari, J. Heydari, M. Keramatic, and A. Keramati, *Designing a mixed system of network DEA for evaluating the efficiency of branches of commercial banks in Iran*, Adv. Math. Finance Appl. **4** (2019), no. 1, 1–13.

[2] A. Amirteimoori and F. Yang, *A DEA model for two-stage parallel-series production processes*, RAIRO Oper. Res. **48** (2014), no. 1, 123–134.

[3] R.D. Banker, A. Charnes, and W.W. Cooper, *Some models for the estimation of technical and scale inefficiencies in data envelopment analysis*, Manag. Sci. **30** (1984), no. 9, 1078–1092.

[4] S. Chaabouni, *China's regional tourism efficiency: A two-stage double bootstrap data envelopment analysis*, J. Destin. Mark. Manag. **11** (2019), 183–191.

[5] A. Charnes, W.W. Cooper, and E.L. Rhodes, *Measuring the efficiency of decision making units*, Eur. J. Oper. Res. **2** (1978), no. 6, 429–444.

[6] Y. Chen and J. Zhu, *Measuring information technology's indirect impact on firm performance*, Inf. Technol. Management **5** 2004, 9–22.

[7] Y. Chen, W.D. Cook, N. Li, and J. Zhu, *Additive efficiency decomposition in two-stage DEA*, Eur. J. Oper. Res. **196** (2009), no. 3, 1170–1176.

[8] M.J. Farrell, *The Measurement of Productive Efficiency*, J. Royal Statist. Soc. Ser. A **120** (1957), no. 3, 253–290.

[9] Z. Haibo, Y. Yi, C. Yao and Z. Joe, *Data envelopment analysis application in sustainability: The origins, development and future directions*, Eur. J. Oper. Res. **264** (2018), no. 1, 1–16.

[10] I.C. Henriques, V. Amorim Sobreiro, H. Kimur, and E. Barberio Mariano, *Efficiency in the Brazilian banking system using data envelopment analysis*, Future Bus. J. **4** (2018), no. 2, 157–178.

[11] L.-F. Hsieh and L.-H. Lin, *A performance evaluation model for international tourist hotels in Taiwan-An application of the relational network DEA*, Int. J. Hospital. Manag. **29** (2010), no. 1, 14–24.

[12] D. Kai, A.C. Worthington, and V. Zelenyuk, *Data envelopment analysis, truncated regression and double-bootstrap for panel data with application to Chinese banking*, Eur. J. Oper. Res. **265** (2018), no. 2, 748–764.

[13] Ch. Kao, *Efficiency decomposition in network data envelopment analysis: A relational model*, Eur. J. Oper. Res. **192** (2009), no. 3, 949–962.

[14] Ch. Kao, *Network data envelopment analysis: A review*, Eur. J. Oper. Res. **239** (2014), no. 1, 1–16.

[15] Ch. Kao, *Some properties of Pareto efficiency under the framework of data envelopment analysis*, Int. J. Syst. Sci. **26** (2007), no. 9, 1549–1558.

[16] Ch. Kao and Sh.-N. Hwang, *Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan*, Eur. J. Oper. Res. **185** (2008), no. 1, 418–429.

[17] Ch. Kao and Sh.-N. Hwang, *Efficiency measurement for network systems: IT impact on firm performance*, Decis. Support Syst. **48** (2010), no. 3, 437–446.

[18] H.F. Lewis and Th.R. Sexton, *Network DEA: efficiency analysis of organizations with complex internal structure*, Comput. Oper. Res. **31** (2004), no. 9, 1365–1410.

[19] S. Lim and J. Zhu, *Integrated data envelopment analysis: Global vs. local optimum*, Eur. J. Oper. Res. **229** (2013), no. 1, 276–278.

[20] H. Omrani, Kh. Shafaat, and A. Emrouznejad, *An integrated fuzzy clustering cooperative game data envelopment analysis model with application in hospital efficiency*, Expert Syst. Appl. **114** (2018), 615–628.

[21] A.M. Prieto and J.L. Zofio, *Network DEA efficiency in input-output models: With an application to OECD countries*, Eur. J. Oper. Res. **178** (2007), no. 1, 292–304.

[22] Sangkyu Rho, Jungnam An, *Evaluating the efficiency of a two-stage production process using data envelopment analysis*, Int. Trans. Oper. Rres. **14** (2007), no. 5, 395–410.

[23] L. Shao, X. Yu, and Ch. Feng, *Evaluating the eco-efficiency of China's industrial sectors: A two-stage network data envelopment analysis*, J. Eenvir. Manag. **6247** (2019), 551–560.

[24] S. Soheilirad, K. Govindan, A. Mardani, E. Kazimieras Zavadskas, M. Nilashi, and N. Zakuan, *Application of data envelopment analysis models in supply chain management: A systematic review and meta-analysis*, Ann. Oper. Res. **271** (2017), 915–969.

[25] K. Tone, M. Toloo, and M. Izadikhah, *A modified slacks-based measure of efficiency in data envelopment analysis*, Eur. J. Oper. Res. **287** (2020), no. 2, 560–571.

[26] K. Tone and M. Tsutsui, *Network DEA: A slacks-based measure approach*, Eur. J. Oper. Res. **197** (2009), no. 1, 243–252.

[27] M. Toloo, A. Emrouznejad, and P. Moreno, *A linear relational DEA model to evaluate two-stage processes with shared inputs*, Comput. Appl. Math. **36** (2017), 45–61.

[28] H. Zhao, S. Guo, and H. Zhao, *Provincial energy efficiency of China quantified by three-stage data envelopment analysis*, Energy, **166** (2019), 96–107.

[29] J. Zhu, *DEA under big data: Data enabled analytics and network data envelopment analysis*, Ann. Oper. Res. **309** (2022), no. 2, 761–783.

[30] M.-M. Yu and E.T.J. Lin, *Efficiency and effectiveness in railway performance using a multi-activity network DEA model*, Omega **36** (2008), no. 6, 1005–1017.