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Multi-period network data envelopment analysis model to measure the efficiency of industrial management institute

Arezoo Gazori Nishabori, Kaveh Khalili Damghani, Ashkan Hafezalkotob

Department of Industrial Engineering, South-Tehran Branch, Azad University, Tehran, Iran

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Abstract

Organizations have a complex and dynamic network of processes. However, using network and dynamic models alone is insufficient to evaluate them. Measuring the efficiency of organizations plays a key role in future decisions. This paper proposes a dynamic network data envelopment analysis model to measure the main and sub-processes in a large organization. This model has been designed for the Industrial Management Institute, which is a leading organization that provides management consulting, publishing, and training services. First, the network of processes in the Industrial Management Institute, which includes training, communications, customer affairs, and information technology, is identified. Then, the most important sub-processes and the dynamic relationships among the main processes are determined. Finally, a dynamic network data envelopment analysis model is proposed to measure the efficiency of the main processes over time. The proposed dynamic network data envelopment analysis model is applied to the operational data collected from the Industrial Management Institute through a sixty-month planning horizon. Lastly, using regression analysis, a relationship is established between the efficiency of the whole process and the efficiency of each individual process. Using the proposed model, the managers of the Industrial Management Institute can evaluate the efficiency of the processes and improve the inefficient processes.

Keywords: Multi-period network data envelopment analysis, business performance, industrial management institute, linear programming, measuring 2020 MSC: 90C08

1 Introduction

Holod and Lewis [33] measured the efficiency of the processes as a critical task. The performance of an organization presents for several periods. So, measuring the performance through several periods gives a more insightful vision. The Industrial Management Institute trained thousands of managers. It creates an opportunity for governmental and non-governmental organizations through education, research, and consulting capacities. The IMI is a part of the Iranian Ministry of Industry, Trade, and Mine. It is a professional and independent organization that has played a prominent role in promoting and localizing management models and systems. Industrial Management Institute causes quality improvement in various industries.

*Corresponding author

Email addresses: arezoogazori.66@gmail.com (Arezoo Gazori Nishabori), kaveh.khalili@gmail.com (Kaveh Khalili Damghani), ashkan.hafez@gmail.com (Ashkan Hafezalkotob)

There are complex and dynamic relationships between different units of the Industrial Management Institute. Therefore, the proper evaluation of the performance of this organization, while considering the different types of inputs and outputs and identifying the efficient and inefficient main processes and sub-processes, is of particular importance. Data envelopment analysis is a technique widely used to evaluate the performance of a set of decision-making units with similar inputs and outputs. Since the performance of the Industrial Management Institute is a complex process. Holod and Lewis [33], claimed that the traditional data envelopment analysis models cannot yield a comprehensive evaluation due to the black box perspective. Fare and Grosskopf [25] maintained that the use of network data envelopment analysis models is preferred to examine the operation of the processes and sub-processes. Chen and Dalen, Chen [18, 17], showed that network data envelopment analysis still ignores the important fact that unit processes have a time dimension, and failure to examine this dimension leads to an unrealistic evaluation of performance, which is the case with static models.

Charens et al. [16] presented a data envelopment analysis model known as the Charnes, Cooper and Rhodes model. Banker et al. [11] proposed the Banker, Chames and Cooper model to evaluate units with variable returns to scale. However, Charnes, Cooper and Rhodes and Banker, Chames and Cooper models did not consider the internal process. Golany et al. [32], proposed the network data envelopment analysis models to overcome these limitations.

One of the first and simplest network structures is the two-stage, in which two processes are assumed to work serially in a main decision-making unit. In other words, decision-making units are formed in two serial stages in a way that the outputs of the first stage are used as inputs of the second stage. Castelli et al. [13] proposed a classification framework based on the process type specified in the decision-making unit: joint multistage flow and network models. Multi-stage network structures, which may consist of more than two stages, are connected in series or parallel. Kao [35], maintained that the total performance of a two-stage network whose sub-processes are connected in series is equal to the product of the efficiency of the sub-processes. Kao [35] presented, as a result, a decision-making unit is efficient if its components are efficient too. Chen et al. [20] proposed a model to evaluate two-stage structures. They used the mean weight to show the relationship between total efficiency and partial efficiencies. Despotis et al. [22] introduced Data envelopment analysis models for a variety of sequential multistage structures.

Nemoto and Goto [43, 44] presented a dynamic data envelopment analysis model by dividing the inputs of a unit into two parts: semi-fixed and variable inputs. Emrouznejad and Tanaslis [24] developed a dynamic model that evaluated the efficiency of dynamic production processes by considering intermediate inputs and outputs as an additional variable in the static Data envelopment analysis model. Chen and Dalen [18] showed that in their model, the solution space dimension increases multiplicatively according to the number of inputs and outputs and evaluation periods. Chen et al. [19] proposed a model for evaluating the performance of a dynamic production network consisting of multiple sub-units of production. Kao [35] introduced a data envelopment analysis model for dynamic structures.

Ton and Tsutsui [49] presented a dynamic network model based on slack variables in which intermediate variables could have a positive or negative effect on the output of the next period. Okiran and McChrystal [10] introduced a dynamic network data envelopment analysis model and compared their model with slack-based dynamic network data envelopment analysis models. Amirteimouri [6] presented a dynamic revenue efficiency dynamic data envelopment analysis model to evaluate gas companies in Iran. Chen and Dalen [18] developed a dynamic data envelopment analysis model to evaluate the advertising efficiency of several pharmaceutical and automotive companies in North America. Avkiran, Fukuyama and Weber, Moreno and Lozano, Wu, Ting, Lu and Nourani, Zha, Liang, Wu and Bayan [9, 27, 28, 42, 53, 58], are among other researchers who introduced Data envelopment analysis models to evaluate dynamic network structures.

In order to evaluate the performance of the Industrial Management Institute it is necessary to use data envelopment analysis models that consider both the internal structure and the time, and not only the use of traditional models, but also the use of network models and dynamic models alone are not enough to evaluate the performance of Industrial Management Institute. In this paper, considering the effective process structure of the Industrial Management Institute, dynamic network data envelopment analysis models are developed to measure the total efficiency and the components' efficiency of the Industrial Management Institute in each period. The proposed models are used to evaluate the performance of the Industrial Management Institute over five years.

The innovations of this paper are summarized as follows:

- Developing of network data envelopment analysis model for industrial management organization network structure and measurement of network efficiency and all network components.
- Linearization of the proposed efficiency measurement model and as a result determining the optimal solution for the model.

• Estimating the network efficiency decomposition function of industrial management organization processes by regression analysis.

And the limitations of this paper are summarized as follows:

- Lack of access to data for more than 5 years in Tehran industrial management organization.
- Lack of access to some parameters and use of past data.
- Limitation of research in the field of education, consulting and publication simultaneously with efficiency approaches

The paper is organized as follows. In Section 2, a review of application of data envelopment analysis and network data envelopment analysis in education is presented. In Section 3, the effective process of Industrial Management Institute is introduced. In Section 4, the mathematics of proposed models is developed. In Section 5, the case study and results are presented. Conclusions and future research directions are presented in Section 6.

2 Brief Summary of Education Industry applications of Data envelopment analysis and Network Data envelopment analysis

Data envelopment analysis has been widely used in the field of education and research. Some examples are Avkiran, Kyvik, Carlos et al., Casu and Thanassoulis, Glass et al., Leitner et al., Celik and Ecer, Agasisti and PerezEsparrells, Abramo et al., Monfared and Safi [8, 30, 14, 31, 37, 15, 4, 2, 47], which are referred to below.

Parker [46] states: "Performance appraisal is a process which all organizations should perform. This appraisal is in fact protecting the organization against the forthcoming problems." Dagdeviren and Yuksel[57] evaluated the performance of educational centers as a continuous process that requires continuous monitoring in order to maintain the performance of the organization at a high level. Using the data envelopment analysis method, Richman and Summersgater [45] tried to identify the efficient university libraries and introduced the best of them as role models. Journady et al. [34] studied the performance efficiency natural and technical sciences departments at Austria University, and the results showed that the size of a department affects its overall and specialized performance.

Journady et al. [34] studied 209 higher education institutions in eight European countries. Their model evaluates the overall performance of universities. Martin [41] reviewed the performance of the University of Zaragoza. Martin [41] categorized the inputs into three groups: Financial, human, and physical resources and classified the output into two levels of education and research. Golany and Tamir [32], surveyed 45 Turkish public universities by using data envelopment analysis. Celik and Ecer [15] showed that the outputs indicate that the financial departments are the most effective departments and the business management departments are the most ineffective. Agasisti and Perez-Esparrells [4] measured the efficiency of Italian and Spanish universities by using the data envelopment analysis method, and the results showed that Italian universities are relatively more efficient because major technological changes were effective at the time of measurement.

Richmann and Sommersguter [45] evaluated the performance of 58 departments at Brazilian universities. Richmann and Sommersguter [45] classified the outputs into four groups: quantity, quality, research, and service. Lopes and Lanzer [40] considered only the number of full-time professors as the inputs. Lopes and Lanzer [40] evaluated the groups using the fuzzy data envelopment analysis model and obtained the correlation between four dimensions of education, research, quality, and service. Antreas and Estelle [7] used data envelopment analysis to measure the relative efficiency of higher education institutions in the United Kingdom.

Flegg [26] considered the costs and incomes of universities as output. Antreas and Estelle [7] also considered the resources and the students' abilities. Casu and Thanassoulis [14], evaluated the efficiency costs in central administrative United Kingdom university and then data envelopment analysis model set up a framework to identify ways of minimizing cost in the central administrative. Glass et al. [31], used data envelopment analysis to measure the potential effectiveness of the policy and guidance of the UK's higher education budget and the results showed that evaluating the effectiveness of current policy objectives are supported.

Avkiran [8] studied 36 universities in Australia, while the number of teaching and non-teaching staff was considered as the inputs of the model, and the number of graduates and the published papers as outputs. Avkiran [8] used the data envelopment analysis method, and overall efficiency, educational service efficiency and fee-paying enrolments efficiency are measured. Data envelopment analysis is a valuable tool for educational administrators. Carlos et al. [30] proposed a formal evaluation model by CAPES (Brazilian agency for post-graduation programs regulation) using network data envelopment analysis. Flegg et al. [26], evaluated the technical performance of 45 English universities during 1980 to 1992 using the data envelopment analysis method with inputs such as: number of staff, number of undergraduate students, number of graduate students, and total administrative costs, and outputs such as: revenues from research and consulting, number of bachelor's degrees, and number of master's degrees. Beasley [12] measured the teaching and research performance of the physics and chemistry departments of all English universities.

Kuah et al.[36] incorporated weight constraints into the proposed model. Ahn and Charnes [5] used the joint Data envelopment analysis model presented by Kuah et al. [36] to measure educational and research performance simultaneously. Kao [35] proposed a set of performance indicators for university performance analysis and a new structure of network data envelopment analysis to measure efficiency. This approach was tested for evaluating efficiency in Al-Zahra university departments. Abramo et al. [2] improved the weaknesses in previous research on return to size. Abramo et al. [2] conducted for 183 hard sciences at 77 Italian universities over a period of time. Tyagi and Yadav[50] evaluated the overall efficiency, educational efficiency, and research efficiency of 19 educational centers in India using the data envelopment analysis method. Chen et al.[19] proposed a two-stage network approach to measure the operating efficiency of 52 universities in China.

To improve the efficiency of Chinese universities, the funding mechanism needs to be improved, and universities need to have a special program to develop their education and research. Xio et al. [54] provided a two-stage network data envelopment analysis framework that measures the environmental efficiency of 84 resource-based cities in the post-financial crisis period. This framework helps to gain a better understanding of the shortcomings of the internal management of the government and the industrial sector. You-wie[?] proposed a new network data envelopment analysis approach for measuring the efficiency of series multi-stage processes. The equivalence of this approach with the Nash two-stage bargaining approach has been investigated and proven. Corrado [39] used a parallel network data envelopment analysis model to evaluate the Italian water industry's performance.

The efficiencies of individual basic services and the global efficiency of the utility are computed. Hang et al.[56] investigated the dynamic network data envelopment analysis approach to examine the performance of large Chinese and Indian airlines by considering the processes and internal links of airlines in periods. Huijuan et al. [?] showed that the framework of the two-stage network data envelopment analysis leads to a deeper understanding of the public and industrial sectors and increases the cooperation of this sector to improve environmental efficiency. Ya et al.[19] proposed a two-stage dynamic network data envelopment analysis approach to measure 52 Chinese universities. They found that about one-third of universities are efficient, and the others are inefficient.

Xuedong et al. [38] studied a two-stage network data envelopment analysis model to improve the sustainable development of the water resources system in western China. Xu et al. [55] measured efficiency in a multi-period network data envelopment analysis model with feedback. This model is used to evaluate performance by using environmental data from China. Chuanyin et al. [21] proposed a two-stage additive network data envelopment analysis to develop a network model in parametric linear form to determine frontier projection and to calculate efficiency.

Tao et al. [23] evaluated the performance of universities and research institutes is one of the important topics that is addressed. In this paper, a two-stage network data envelopment analysis model is developed to evaluate the performance of Chinese universities. The outputs show approaches to improve the performance of the units and the university. Ya et al. [51] presented a two-stage network data envelopment analysis approach to develop the performance of China's advanced industrial technology. Several suggestions have been made to improve the performance of China's advanced industries. Various studies have evaluated the performance of educational institutions to measure their educational efficiency, which are given in Table 1.

3 Network Data envelopment analysis Model for Effect Process

First, the organizational structure of Industrial Management Institute is presented. Then, the interactions, including the inputs and outputs, are determined. Using the cost analysis, the most important activities are selected. On the other hand, each activity's importance in a process is determined based on its cost divided by the total costs of all activities of the process. Then, the Pareto analysis help to select effective activities using the Pareto law. The resultant network has been confirmed by experts in a brainstorming session. On the basis of this analysis, the main sub-processes are communication and customer affairs, education, information technology. The schematic view of the effective structure for a single period is presented in Fig. 1.

The Industrial Management Institute is working during a multi-period horizon. The multi-period effect structure is presented in Fig. 2.

References	NO.	The name of research	Approach
Ahn et al.	[5]	Some statistical and data envelopment analysis evaluations of relative efficiencies of public and private institutions of higher learning	Data envelopment analysis
Sinuany-Stern et al.	[48]	Academic departments efficiency via DEA	Data envelopment analysis
Beasley	[12]	Determining teaching and research efficiencies	Data envelopment analysis
Gander	[29]	Academic research and teaching productivities: a case study	Data envelopment analysis
Colbert et al.	[21]	Determining the relative efficiency of MBA programs using data envelopment analysis	Data envelopment analysis
Abbott and Doucouliagos	[1]	The efficiency of Australian Universities: A Data Envelopment Analysis	Data envelopment analysis
Tyagi et al.	[50]	Relative performance of academic departments using data envelopment analysis with sensitivity analysis	Data envelopment analysis
Agasisti and Bianco,	[3]	Reforming the university sector: effects on teaching efficiency - evidence from Italy	Data envelopment analysis
Agasisti and Perez-Esparrells	[4]	Comparing efficiency in a cross-country perspective: the case of Italian and Spanish state universities	Data envelopment analysis
Kao	[35]	Dynamic data envelopment analysis: A relational analysis	Dynamic data envelopment analysis

Table 1: Summary of studies about education industry



Figure 1: Network of the Effect Structure for Single Period



Figure 2: Network of the Effect Structure for Multi-Period



Figure 3: Schematic View of a DMU associated with effect Process for single period



Figure 4: Schematic View of a DMU associated with effective Processes for multi-period

4 Proposed Network Data envelopment analysis model for effect Process

In this section, data envelopment analysis models for measuring the effect network's efficiency for single and multiperiod cases are shown in Fig. 3 and Fig. 4, respectively. The mathematical symbols are given in Fig. 3. The sub-processes introduced in Fig. 2 and Fig. 4 are named using sub-process1, sub-process2, and sub-process3. Each DMU is evaluated in multiple scheduling periods. Fig. 3 presents the schematic view of DMU associated with effective processes for a single period.

In Fig. 3 each DMU_j (j = 1, 2, ..., J) consists of three sub-processes. Sub-process1 consumes m inputs X_{ij} (i = 1, 2, ..., m), P_3 intermediate measures $Z_{p3j^3}(p_3 = 1, 2, ..., P_3)$, and P_5 intermediate measures $Z_{p5j^5}(p_5 = 1, 2, ..., P_5)$ to produce s outputs $Y_{rj}(r = 1, 2, ..., s)$, P_1 intermediate measures $Z_{p1j^1}(p_1 = 1, 2, ..., P_1)$, and P_2 Intermediate measures $Z_{p2j^2}(p_2 = 1, 2, ..., P_2)$. Sub-process2 consumes P_2 intermediate measures $Z_{p2j^2}(p_2 = 1, 2, ..., P_2)$, P_6 intermediate measures $Z_{p6j^6}(p_6 = 1, 2, ..., P_6)$, and Q inputs $F_{qj^2}(q = 1, 2, ..., Q)$ to produce D outputs $K_{dj}(d = 1, 2, ..., D)$, P_3 intermediate measures $Z_{p3j^3}(p_3 = 1, 2, ..., P_3)$, and P_4 intermediate measures $Z_{p4j^4}(p_4 = 1, 2, ..., P_4)$. Sub-process3 consumes P_4 intermediate measures $Z_{p4j^4}(p_4 = 1, 2, ..., P_4)$ and A inputs $W_{aj}(a = 1, 2, ..., A)$ to produce D outputs $K_{dj}(d = 1, 2, ..., P_4)$. Sub-process3 $Z_{p5j^5}(p_5 = 1, 2, ..., P_5)$, and P_6 intermediate measures $Z_{p6j^6}(p_6 = 1, 2, ..., P_6)$.

The efficiency of main process, sub-process1, sub-process2, and sub-process3 in period t are parameterized using e_j, e_j^1, e_j^2, e_j^3 , respectively.

Fig. 4 shows the network of effective processes with the inputs, outputs, and intermediate measures for multiperiods. It should be mentioned that the period t is assumed as a DMU. The notations used in the proposed data envelopment analysis models are given in Table 2.

Linear programming Model 4.1 is proposed to measure the efficiency of effective processes for the first period shown in Fig.3.

Table	2:	Model	variables
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	Indices	
J	The number of DMU_S	J = 1,, j
0	DMU which is under assessment	
m	The number of inputs of sub-Process1	i=1,,m
A	The number of inputs of sub-Process2	a = 1,, A
Q	The number of inputs of sub-Process3	q = 1,, Q
P_1	The number of outputs of sub-Process1; the number of inputs of sub-Process2;	$p_1 = 1,, P_1$
P_2	The number of outputs of sub-Process2; the number of inputs of sub-Process1;	$p_2 = 1,, P_2$
P_3	The number of outputs of sub-Process1; the number of inputs of sub-Process3;	$p_3 = 1,, P_3$
P_4	The number of outputs of sub-Process2; the number of inputs of sub-Process3;	$p_4 = 1,, P_4$
P_5	The number of outputs of sub-Process3; the number of inputs of sub-Process1;	$p_5 = 1,, P_5$
P_6	The number of outputs of sub-Process3; the number of inputs of sub-Process2;	$p_6 = 1,, P_6$
r	The number of outputs of sub-Process1	r = 1,, s
b	The number of outputs of sub-Process2	b = 1,, B
d	The number of outputs of sub-Process3	d=1,,D
	Parameters	
$\overline{X_{ij}}$	The ith input of sub-process 1 of DMU_j	
W_{aj}	The ath input of sub-process 2 of DMU_j	
F_{qj}	The qth input of sub-process 3 of DMU_i	
$Z_{n1i}^{\tilde{1}}$	The pth output of sub-process 1 of DMU_j ; the pth input of sub-process 2 of DMU_j	
$Z_{P2i}^{2^{-j}}$	The pth output of sub-process 2 of DMU_i ; the pth input of sub-process 1 of DMU_i	
$Z_{P3i}^{3^{2j}}$	The pth output of sub-process 1 of DMU_j ; the pth input of sub-process 3 of DMU_j	
Z_{P4i}^{4}	The pth output of sub-process 2 of DMU_j ; the pth input of sub-process 3 of DMU_j	
Z_{5}^{5}	The pth output of sub-process 3 of DMU_i : the pth input of sub-process 1 of DMU_i	

The pth output of sub-process 3 of DMU_j ; the pth input of sub-process 1 of DMU_j The pth output of sub-process 3 of DMU_j ; the pth input of sub-process 2 of DMU_j The rth output of sub-process 1 of DMU_j The rth output of sub-process 2 of DMU_j The rth output of sub-process 3 of DMU_j

Decision variables

a1.	The multiplier of the ith input of sub process1	i-1 m
v_i	The indiciple of the firm input of sub-processi	i = 1,, m
v_a	The multiplier of the ath input of sub-process2	a = 1,, A
v_q	The multiplier of the qth input of sub-process3	$q = 1, \dots, Q$
g_{p1}	The multiplier of the pth output of sub-process1; the multiplier of the lth input of sub-process2	$p_1 = 1,, P_1$
g_{p2}	The multiplier of the pth output of sub-process2; the multiplier of the lth input of sub-process1	$p_2 = 1,, P_2$
g_{p3}	The multiplier of the pth output of sub-process1; the multiplier of the lth input of sub-process3	$p_3 = 1,, P_3$
g_{p4}	The multiplier of the pth output of sub-process2; the multiplier of the lth input of sub-process3	$p_4 = 1,, P_4$
g_{p5}	The multiplier of the pth output of sub-process3; the multiplier of the lth input of sub-process1	$p_5 = 1,, P_5$
g_{p6}	The multiplier of the pth output of sub-process3; the multiplier of the lth input of sub-process2	$p_6 = 1,, P_6$
u_r	The multiplier of the rth output of sub-process1	$r = 1, \dots, s$
u_b	The multiplier of the bth output of sub-process2	b = 1,, B
u_d	The multiplier of the dth output of sub-process3	d=1,,D
e_j	The overall efficiency score of DMU_j	j = 1,, J
0.	The efficiency score of sub process h of DMU .	h = 1, 2, 3;
e_{j^h}	The endency score of sub-process if of DMO_j	j=1,,J

$$\begin{split} Max \quad & e_{o} = \sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}o}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}o}^{5} \qquad s.t. \\ \bullet \quad & \left(\sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}j}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}j}^{5}\right) \\ & - \left(\sum_{i=1}^{m} v'_{i}X_{ij} + \sum_{a=1}^{A} v'_{a}W_{aj} + \sum_{q=1}^{Q} v'_{q}F_{qj}\right) \leq 0, \quad j = 1 \\ \bullet \quad & \left(\sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}^{3}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}j}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}j}^{5}\right) \\ & - \left(\sum_{i=1}^{m} v'_{i}X_{ij} + \sum_{a=1}^{A} v'_{a}W_{aj} + \sum_{q=1}^{Q} v'_{q}F_{qj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}(j-1)}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}(j-1)}^{5}\right) \leq 0, \quad j = 2, \dots, J \\ \bullet \quad & \sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{p_{1}=1}^{P_{1}} g'_{p_{1}}Z_{p_{1}j}^{1} + \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}}Z_{p_{2}j}^{2} - \sum_{i=1}^{m} v'_{i}X_{ij} \leq 0, \quad j = 1 \end{split}$$

•
$$\sum_{r=1}^{s} u'_{r} Y_{rj} + \sum_{p_{1}=1}^{P_{1}} g'_{p_{1}} Z^{1}_{p_{1}j} + \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z^{2}_{p_{2}j} - \sum_{i=1}^{m} v'_{i} X_{ij} - \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}} Z^{3}_{p_{3}(j-1)} - \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z^{5}_{p_{5}(j-1)} \le 0, \quad j = 2, ..., J$$

•
$$\sum_{b=1}^{D} u'_{b} N_{bj} + \sum_{p_{3}=1}^{r_{3}} g'_{p_{3}} Z^{3}_{p_{3}j} + \sum_{p_{4}=1}^{r_{4}} g'_{p_{4}} Z^{4}_{p_{4}j} - \sum_{a=1}^{A} v'_{a} W_{aj} - \sum_{p_{1}=1}^{r_{1}} g'_{p_{1}} Z^{1}_{p_{1}j} - \sum_{p_{6}=1}^{r_{6}} g'_{p_{6}} Z^{6}_{p_{6}j} \le 0, \quad j = 1, 2, ..., J$$

•
$$\sum_{d=1}^{D} u'_{d} K_{dj} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z^{5}_{p_{5}j} + \sum_{p_{6}=1}^{P_{6}} g'_{p_{6}} Z^{6}_{p_{6}j} - \sum_{q=1}^{Q} v'_{q} F_{qj} - \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z^{2}_{p_{2}j} - \sum_{p_{4}=1}^{P_{4}} g'_{p_{4}} Z^{4}_{p_{4}j} \le 0, \quad j = 1, 2, ..., J$$

•
$$\sum_{i=1}^{m} v'_i X_{io} + \sum_{a=1}^{n} v'_a W_{ao} + \sum_{q=1}^{\infty} v'_q F_{qo} = 1$$

•
$$v'_i \ge \varepsilon$$
, $i = 1, ..., m.v'_a \ge \varepsilon$, $a = 1, ..., A.v'_q \ge \varepsilon$, $q = 1, ..., Q$

• $u'_r \ge \varepsilon$, $r = 1, ..., s.u'_b \ge \varepsilon$, $b = 1, ..., B.u'_d \ge \varepsilon$, d = 1, ..., D.

•
$$g'_{p_1} \ge \varepsilon, p_1 = 1, ..., P_1 g'_{p_2} \ge \varepsilon, \quad p_2 = 1, ..., P_2 g'_{p_3} \ge \varepsilon, \quad p_3 = 1, ..., P_3 g'_{p_4} \ge \varepsilon, \quad p_4 = 1, ..., P_4 g'_{p_5} \ge \varepsilon,$$

 $p_5 = 1, ..., P_5 g'_{p_6} \ge \varepsilon, \quad p_6 = 1, ..., P_6$

$$(4.1)$$

Using the global optimum value of the decision variables obtained from Model 4.1, the efficiency of each sub-process can be calculated using Equations 4.2.

$$e_{j}^{(1)} = \left(\sum_{r=1}^{s} u_{r}^{*} y_{rj} + \sum_{p_{1}=1}^{P_{1}} g_{p_{1}}^{*} Z_{p_{1}j}^{1} + \sum_{p_{2}=1}^{P_{2}} g_{p_{2}}^{*} Z_{p_{2}j}^{2}\right) / \left(\sum_{i=1}^{m} v_{i}^{*} x_{ij}\right), \ j = 1$$

$$e_{j}^{(2)} = \left(\sum_{b=1}^{B} u_{b}^{*} N_{bj} + \sum_{p_{3}=1}^{P_{3}} g_{p_{3}}^{*} Z_{P_{3}j}^{3} + \sum_{p_{4}=1}^{P_{4}} g_{p_{4}}^{*} Z_{p_{4}j}^{4}\right) / \left(\sum_{a=1}^{A} v_{a}^{*} W_{aj} + \sum_{p_{1}=1}^{P_{1}} g_{p_{1}}^{*} Z_{p_{1}j}^{1} + \sum_{p_{6}=1}^{P_{6}} g_{p_{6}}^{*} Z_{p_{6}j}^{6}\right), \ j = 1$$

$$e_{j}^{(3)} = \left(\sum_{D=1}^{d} u_{d}^{*} K_{dj} + \sum_{p_{5}=1}^{P_{5}} g_{p_{5}}^{*} Z_{p_{5}j}^{5} + \sum_{p_{6}=1}^{P_{6}} g_{p_{6}}^{*} Z_{p_{6}j}^{6}\right) / \left(\sum_{q=1}^{Q} v_{q}^{*} F_{qj} + \sum_{p_{2}=1}^{P_{2}} g_{p_{2}}^{*} Z_{p_{2}j}^{2} + \sum_{p_{4}=1}^{P_{4}} g_{p_{4}}^{*} Z_{p_{4}j}^{4}\right), \ j = 1$$

$$(4.2)$$

The linear programming Model 4.3 is proposed to achieve the relative efficiency of DMU_j , j = 2, ..., J.

$$\begin{split} Max \quad e_{o} &= \sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}o}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}o}^{5} \\ s.t. \\ & \left(\sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}j}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}j}^{5}\right) - \\ & \left(\sum_{i=1}^{m} v'_{i}X_{ij} + \sum_{a=1}^{A} v'_{a}W_{aj} + \sum_{q=1}^{Q} v'_{q}F_{qj}\right) \leq 0, \quad j = 1 \\ & \left(\sum_{r=1}^{s} u'_{r}Y_{rj} + \sum_{b=1}^{B} u'_{b}N_{bj} + \sum_{d=1}^{D} u'_{d}K_{dj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}j}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}}Z_{p_{5}j}^{5}\right) - \\ & \left(\sum_{i=1}^{m} v'_{i}X_{ij} + \sum_{a=1}^{A} v'_{a}W_{aj} + \sum_{q=1}^{Q} v'_{q}F_{qj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}}Z_{p_{3}(j-1)}^{3} + \sum_{p_{5}=1}^{p_{5}} g'_{p_{5}}Z_{p_{5}(j-1)}^{5}\right) \leq 0, \quad j = 2, \dots, J \end{split}$$

Multi-period network data envelopment analysis model to measure the efficiency of ...

$$\begin{split} \sum_{r=1}^{s} u'_{r} Y_{rj} + \sum_{p_{1}=1}^{P_{1}} g'_{p_{1}} Z_{p_{1}j}^{1} + \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z_{p_{2}j}^{2} - \sum_{i=1}^{m} v'_{i} X_{ij} &\leq 0, \ j = 1 \\ \sum_{r=1}^{s} u'_{r} Y_{rj} + \sum_{p_{1}=1}^{P_{1}} g'_{p_{1}} Z_{p_{1}j}^{1} + \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z_{p_{2}j}^{2} - \sum_{i=1}^{m} v'_{i} X_{ij} - \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}} Z_{p_{3}(j-1)}^{3} - \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z_{p_{5}(j-1)}^{5} &\leq 0, \ j = 2, \dots, J \\ \sum_{b=1}^{B} u'_{b} N_{bj} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}} Z_{p_{3j}j}^{3} + \sum_{p_{4}=1}^{P_{4}} g'_{p_{4}} Z_{p_{4j}j}^{4} - \sum_{a=1}^{A} v'_{a} W_{aj} - \sum_{p_{1}=1}^{P_{1}} g'_{p_{1}} Z_{p_{1}j}^{1} - \sum_{p_{6}=1}^{P_{6}} g'_{p_{6}} Z_{p_{6}j}^{6} &\leq 0, \ j = 1, 2, \dots, J \\ \sum_{d=1}^{D} u'_{d} K_{dj} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z_{p_{5}j}^{5} + \sum_{p_{6}=1}^{P_{6}} g'_{p_{6}} Z_{p_{6j}}^{6} - \sum_{q=1}^{Q} v'_{q} F_{qj} - \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z_{p_{2}j}^{2} - \sum_{p_{4}=1}^{P_{4}} g'_{p_{4}} Z_{p_{4}j}^{4} &\leq 0, \ j = 1, 2, \dots, J \\ \sum_{d=1}^{m} v'_{d} K_{dj} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z_{p_{5}j}^{5} + \sum_{p_{6}=1}^{P_{6}} g'_{p_{6}} Z_{p_{6}j}^{6} - \sum_{q=1}^{Q} v'_{q} F_{qj} - \sum_{p_{2}=1}^{P_{2}} g'_{p_{2}} Z_{p_{2}j}^{2} - \sum_{p_{4}=1}^{P_{4}} g'_{p_{4}} Z_{p_{4}j}^{4} &\leq 0, \ j = 1, 2, \dots, J \\ \sum_{i=1}^{m} v'_{i} X_{io} + \sum_{a=1}^{A} v'_{a} W_{ao} + \sum_{q=1}^{Q} v'_{q} F_{qo} + \sum_{p_{3}=1}^{P_{3}} g'_{p_{3}} Z_{p_{3}o}^{3} + \sum_{p_{5}=1}^{P_{5}} g'_{p_{5}} Z_{p_{5}o}^{5} = 1 \\ v'_{i} \geq \varepsilon, \quad i = 1, \dots, m. \ v'_{a} \geq \varepsilon, \quad a = 1, \dots, A. \ v'_{q} \geq \varepsilon, \quad q = 1, \dots, Q. \\ u'_{r} \geq \varepsilon, \quad r = 1, \dots, r , u'_{b} \geq \varepsilon, \quad b = 1, \dots, P_{3} g'_{p_{3}} \geq \varepsilon, \quad p_{3} = 1, \dots, P_{3} \\ g'_{p_{4}} \geq \varepsilon, \quad p_{4} = 1, \dots, P_{4} g'_{p_{5}} \geq \varepsilon, \quad p_{5} = 1, \dots, P_{5} g'_{p_{6}} \geq \varepsilon, \quad p_{6} = 1, \dots, P_{6} \end{split}$$

The efficiency of each sub-process can be calculated using Equations 4.4.

$$e_{j}^{(1)} = \left(\sum_{r=1}^{s} u_{r_{1}}^{*} Y_{rj} + \sum_{p_{1}=1}^{P_{1}} g_{p_{1}}^{*} Z_{p_{1}j}^{1} + \sum_{p_{2}=1}^{P_{2}} g_{p_{2}}^{*} Z_{p_{2}j}^{2}\right) / \left(\sum_{i=1}^{m} v_{i}^{*} X_{ij} + \sum_{p_{3}=1}^{P_{3}} g_{p_{3}}^{*} Z_{p_{3}(j-1)}^{3} + \sum_{p_{5}=1}^{P_{5}} g_{p_{5}}^{*} Z_{p_{5}(j-1)}^{5}\right), \ j = 2, \dots, J$$

$$e_{j}^{(2)} = \left(\sum_{b=1}^{B} u_{b}^{*} N_{bj} + \sum_{p_{3}=1}^{P_{3}} g_{p_{3}}^{*} Z_{p_{3}j}^{3} + \sum_{p_{4}=1}^{P_{4}} g_{p_{4}}^{*} Z_{p_{4}j}^{4}\right) / \left(\sum_{a=1}^{A} v_{a}^{*} W_{aj} + \sum_{p_{1}=1}^{P_{1}} g_{p_{1}}^{*} Z_{p_{1}j}^{1} + \sum_{p_{6}=1}^{P_{6}} g_{p_{6}}^{*} Z_{p_{6}j}^{6}\right), \ j = 2, \dots, J$$

$$e_{j}^{(3)} = \left(\sum_{d=1}^{D} u_{d}^{*} K_{dj} + \sum_{p_{5}=1}^{P_{5}} g_{p_{5}}^{*} Z_{p_{5}j}^{5} + \sum_{p_{6}=1}^{P_{6}} g_{p_{6}}^{*} Z_{p_{6}j}^{6}\right) / \left(\sum_{q=1}^{Q} v_{q}^{*} F_{qj}^{3} + \sum_{p_{2}=1}^{P_{2}} g_{p_{2}}^{*} Z_{p_{2}j}^{2} + \sum_{p_{4}=1}^{P_{4}} g_{p_{4}}^{*} Z_{p_{4}j}^{4}\right), \ j = 2, \dots, J.$$

$$(4.4)$$

4.1 Theoretical properties of proposed network data envelopment analysis models

The theoretical properties of the proposed models are discussed in this sub-section.

Theorem 4.1. Model 4.1 is always feasible and the value of the objective function is bounded.

Proof. Let's consider $\lambda_j^{(1)}, \lambda_j^{(2)}, \lambda_j^{(3)}$ and θ the associated dual variables of linear Model 4.1. Therefore, the dual form of Model 4.1 can be written as Model 4.5.

$$\begin{split} &Min \quad \theta & s.t. \\ &\sum_{j=1}^{J} \lambda_{j}^{(1)} X_{ij} \leq \theta x_{io}, & i = 1, ..., m \\ &\sum_{j=1}^{J} \lambda_{j}^{(1)} Z_{p_{1}j}^{1} - \sum_{j=1}^{J} \lambda_{j}^{(2)} Z_{p_{1}j}^{1} \geq 0, & p_{1} = 1, ..., P_{1} \\ &\sum_{j=1}^{J} \lambda_{j}^{(1)} Z_{p_{2}j}^{2} - \sum_{j=1}^{J} \lambda_{j}^{(2)} Z_{p_{2}j}^{2} \geq 0, & p_{2} = 1, ..., P_{2} \\ &\sum_{j=1}^{J} \lambda_{j}^{(1)} Y_{rj} \geq Y_{ro}, & r = 1, ..., s \end{split}$$

$$\begin{split} \sum_{j=1}^{J} \lambda_{j}^{(2)} W_{aj} &\leq \theta \; W_{ao}, \qquad a = 1, ..., A \\ \sum_{j=1}^{J} \lambda_{j}^{(2)} g_{p3j} - \sum_{j=1}^{J} \lambda_{j}^{(1)} g_{p3j} \geq 0, \qquad p_{3} = 1, ..., P_{3} \\ \sum_{j=1}^{J} \lambda_{j}^{(2)} N_{bj} \geq N_{bo}, \qquad b = 1, ..., B \\ \sum_{j=1}^{J} \lambda_{j}^{(3)} F_{qj} &\leq \theta F_{qo}, \qquad q = 1, ..., Q \\ \sum_{j=1}^{J} \lambda_{j}^{(3)} K_{dj} \geq K_{do}, \qquad d = 1, ..., D \\ \theta = \text{free} \\ \lambda_{j}^{(1)}, \lambda_{j}^{(2)}, \lambda_{j}^{(3)} \geq 0, \qquad j = 1, ..., J \end{split}$$
(4.5)

Consider arbitrary solution 4.6 for Model 4.5.

$$\begin{cases} \lambda_{j}^{(1)} = \lambda_{j}^{(2)} = \lambda_{j}^{(3)} = 0, \quad j = 1, ..., J, \quad j \neq 0 \\\\ \theta = 1 \\\\ \lambda_{0}^{(1)} = \lambda_{0}^{(2)} = \lambda_{0}^{(3)} = 1 \end{cases}$$

$$(4.6)$$

It can be concluded that the solution 4.6 is always a feasible solution for Model 4.5. Thus, Model 4.1 is feasible. According to the dual theorem in linear programming, the value of the objective function of the primary and the dual models is equal, which completes the proof.

Theorem 4.2. Model 4.3 is always feasible and the value of the objective function is bounded.

Proof . According to Theorem 1, the proof is straightforward. \Box

5 Results of network data envelopment analysis Model for Effect Process

Sixty periods on the basis of the monthly reports from 2011 to 2015 are selected to analyze. The descriptive statistics of the data are given in the appendix. The efficiency of main process and sub-processes depicted in Fig. 4 have been calculated using the proposed models. The proposed models are coded using LINGO software. The results are given in Table 3.

According to the results in Table 3, DMU_9 and DMU_{11} are efficient in the first and second stages. These DMU_s are inefficient in the third stage. Also DMU_{16} and DMU_{25} are efficient in the second and third stages, and these DMU_s are inefficient in the first stage. DMU_4 and DMU_{29} are efficient in the first and second stages. However, they are not efficient in all stages. In other words, the efficiency of these two stages is effective enough to reach the efficiency of the total efficiency effect processes. The average efficiency scores of a network of effect processes in Industrial Management Institute in the sub-processes of communication and customer affairs, education and IT are 97.01%, 98.63% and 88.81% respectively. The network of the effect processes during the planning period in each sub-process is 0.08%, 28.33% and 0.06% efficient in the stages respectively. The efficiency scores of the third sub-process is less than the first and second sub-processes. In addition, it can be said that the sub-process of education is one of the effect processes such as $(DMU_4, DMU_9, DMU_{10}, DMU_{11}, DMU_{33}, DMU_{43}, DMU_{55}$ etc.), the efficiency score of the second stage is complete and the efficiency score of the first stage is higher than the third stage is higher than the third stage and this shows the importance of the sub-processes of communication and customer affairs is higher than the third stage is higher than the importance of the sub-processes of communication and customer affairs is higher than the third stage is higher than the importance of the sub-processes of communication and customer affairs is higher than the third stage is higher than the importance of the sub-processes of communication and customer affairs.

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DMU	Overall	Sub-process 1	Sub-process 2	Sub-process 3	DMU	Overall	Sub-process 1	Sub-process 2	Sub-process 3
1	0.9658	0.9441	0.9733	0.8129	31	0.9935	0.978	0.98	0.8342
2	0.9675	0.9645	0.9433	0.8623	32	0.9875	0.9901	1	0.8387
3	0.9485	0.9731	0.9641	0.8787	33	1	0.9781	1	0.8922
4	0.9735	1	1	0.8389	34	0.9761	0.9799	0.9762	0.9121
5	1	0.9761	0.9745	0.8994	35	0.9982	0.9732	0.9652	0.9432
6	0.9132	0.9631	0.9981	0.8923	36	0.9456	0.9631	0.9799	0.8612
7	1	1	0.9321	0.8317	37	0.9714	0.9673	0.9915	0.8221
8	0.9221	0.9761	0.9918	0.9003	38	0.9519	0.8718	0.9961	0.8412
9	1	1	1	0.8661	39	0.9751	0.9932	0.9739	0.8419
10	1	0.9561	1	0.8661	40	0.9751	0.9932	0.9739	0.8419
11	1	1	1	0.8944	41	0.9326	0.9989	0.9989	0.8127
12	0.9931	0.9423	1	0.9421	42	0.9753	0.9721	0.9976	0.9232
13	0.9851	0.9341	0.9927	0.8538	43	0.9403	0.9431	1	0.8356
14	0.9943	0.9652	0.9984	0.8739	44	0.9731	0.9976	0.9421	0.8015
15	0.998	0.9678	1	0.7941	45	0.9512	0.9712	0.9567	0.8532
16	0.9421	0.9973	1	1	46	0.9751	0.9652	0.9998	0.8361
17	0.9218	0.8914	1	0.8717	47	0.9817	0.9514	0.9918	0.8912
18	0.9515	0.9713	1	0.8119	48	0.9514	0.9914	0.9614	0.9653
19	0.9714	0.9317	1	0.9516	49	0.9615	0.9615	0.9918	0.8318
20	0.9962	0.9615	0.9912	0.9612	50	0.9809	0.9926	0.9413	0.9276
21	0.9515	0.9815	0.9995	1	51	0.9514	0.9481	0.9509	0.7876
22	0.9817	0.9915	1	1	52	0.9912	0.9985	0.9617	0.9816
23	0.9924	0.9816	0.9918	0.9817	53	0.9615	0.9786	0.9912	0.8915
24	0.9765	0.9317	0.9918	0.9565	54	0.9876	0.9568	0.9807	0.8318
25	0.9987	0.9615	1	1	55	0.9816	0.9216	1	0.8976
26	0.9518	0.9615	0.9919	0.9716	56	0.9632	0.9763	0.9992	0.8782
27	0.9983	0.9761	0.9989	0.9415	57	0.9987	0.9817	0.9986	0.8912
28	0.9973	0.9981	0.9993	0.8342	58	0.9716	0.9712	0.9987	0.8312
29	0.9965	1	1	0.9917	59	0.9722	0.9904	0.9761	0.8517
30	0.9776	0.9912	0.9817	0.9814	60	0.9731	0.9614	0.9918	0.8773

Table 3: Efficiency scores of the effect processes in 60 periods

in a network of effect processes in the Industrial Management Institute. According to the obtained results the highest efficiency is observed in the first and second sub-processes (communication and customer affairs), and about 13.33% of the planning period are weak in term of performance that has been reported in the sub-processes of communication and customer affairs, education. The average efficiency scores are calculated for five years. The results are given in Table 4. Fig. 5 shows the average efficiency scores of the three sub-processes in five years.

Tab	ole 4:	Average	efficiency	scores	for	five	years
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Year	Overall	Sub-process 1	Sub-process 2	Sub-process 3
1	0.974	0.975	0.981	0.874
2	0.972	0.959	0.997	0.921
3	0.985	0.979	0.989	0.917
4	0.963	0.968	0.982	0.855
5	0.975	0.970	0.982	0.873

One of the main issues in network data envelopment analysis models is the methods in which the total efficiency score of the main DMU is decomposed to the efficiency score of the sub-processes. Wang [52], proposed various classical decompositions methods in this regard such as: bargaining game among stages, mean of efficiency of stages, harmonic mean of efficiency of stages, weighted mean of efficiency of stages, multiplication of efficiency of stages and geometric mean of efficiency of stages. In this paper, the efficiency scores are analyzed statistically. On the other hand, a curve fitting method, including regression analysis, is used to develop a relationship between the independent variables (efficiency score of the sub-processes) and the dependent variables (efficiency score of the total effect process). Linear, logarithmic, quadratic, cubic and growth curves are tested to determine the best fitness. In curve fitting, R^2 , is a measure of how well the function is fitted with the real observations, F, is a significant test value of the model and sig, shows the statistical significance of the regression analysis model. The results of the five regression models



Figure 5: The average efficiency scores of the effect process in five years

are given in Table 5.

Equation	\mathbb{R}^2	F	sig
Quadratic	0.181	29.012	0.000
Linear	0.690	38.740	0.000
Cubic	0.336	28.712	0.000
Growth	0.290	37.120	0.000
Logarithmic	0.363	37.520	0.000

Table 5: Parameter estimated in regression model

In Table 5, the significance level of the F-test of all regression models is smaller than 0.05. Therefore, all models are significant. The linear regression with F=38.740 is the best regression model because its F value is higher. The estimated linear regression function is $e_j = 0.211e_j^1 + 0.593e_j^2 + 0.731e_j^3 + 0.110$. The notations e_j^1 , e_j^2 , and e_j^3 are the efficiency of communication and customers, education, and information technology, respectively. R^2 is the impact of dependent variables, including the efficiency of the total effect network on the independent variables. The significance level is smaller than 0.05, which confirms the regression analysis result and the estimated parameters.

6 Conclusions

The classic data envelopment analysis models ignore the internal processes of a main structure. Therefore, they cannot decompose the total efficiency score into the efficiency score of the sub-processes. Thus, network models should be used to evaluate the efficiency of the processes with a complicated network. This paper proposed a dynamic network data envelopment analysis model to measure the main and sub-processes in a large organization. This model was designed for the Industrial Management Institute, which is a leading organization that provides management consulting, publishing, and training services. According to the monthly data, the efficiency of the effective processes and its sub-processes are calculated using the proposed network data envelopment analysis model in 5 years from 2011 to 2015. This network includes three sub-processes of communication and customers, education, and information technology. The proposed models are employed for 60 months in the Industrial Management Institute, and the results showed that the education sub-process has the most significant impact on the total efficiency of the effect processes network. The minimum impact is obtained for information technology. A regression analysis was used to develop a relationship between the independent variables (efficiency score of the sub-processes) and the dependent variables (efficiency score of the total effect process). The uncertainty and qualitative nature of parameters can be modeled. Variable return to scale assumptions, extra inputs and outputs and resource allocation and target setting in the proposed network structure can be interesting ideas for future research.

Appendix A Descriptive statistics of input, output and intermediate measure variables for sub-processes

Sub-Process Name	Indices	Туре	Measurement Unit	Average	Standard Deviation	Max	Min
Communication and customer affairs	Performance of annual plans The clients' opinions on the consulting unit The clients' opinions on publication unit The sent ideas and suggestions The account of the attracted man force that have been trained and developed The detracted man force that have been trained and developed The detracted man force that have been trained and developed The designing requests from all units Designers network Calendar events and occasions Institutional events and occasions Institutional events and occasions Institutional events and occasions Internet researches Internet researches Internet researches Internet researches Products and services of the institute Information banks Products and services of the institute Information banks Products and services of the institute Information banks Outsourced plans The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The analysis of the opinions and requests of the clients The comprehensive agreements in the field of all services Marketing plans The comprehensive agreements in the field of all services Marketing plans Involvement of the institute in the events The reports provided by this unit Identification of the clients ineds The reports provided by this unit Identification of the clients ineds The comprehensive agreements with the international institutes The comprehensive agreements with the international institutes The reports of marketing researches The reports of marketing researches The reports of marketing researches The reports of marketing researches The analysis of the opo	Input Output Intermediate measure Intermediate measure Intermediate measure Intermediate measure	amount number	$\begin{array}{c} 3.4\\ 8\\ 28\\ 5\\ 5\\ 15\\ 7\\ 8\\ 34\\ 225\\ 3\\ 2\\ 25\\ 6\\ 6\\ 3\\ 3\\ 2\\ 2215\\ 16\\ 4\\ 4\\ 2\\ 222\\ 7\\ 50\\ 11\\ 17\\ 4\\ 713\\ 2\\ 8\\ 5\\ 2\\ 2\\ 4\\ 2\\ 1\\ 3.48\\ 2\\ 49\\ 6\\ 27\\ 3\\ 3.48\\ 2\\ 13\\ 3\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\$	$\begin{array}{c} 1.37\\ 4.19\\ 15.41\\ 2.72\\ 6.02\\ 2.72\\ 6.02\\ 1.55\\ 13.88\\ 3.1\\ 3.51\\ 0.9\\ 3.11\\ 3.31\\ 2.87\\ 1.34\\ 1.88\\ 1.34\\ 1.89\\ 1.287\\ 1.34\\ 1.89\\ 1.89\\ 1.89\\ 1.89\\ 1.89\\ 1.79\\ 6.22\\ 2.943\\ 6.6\\ 2.08\\ 412.3\\ 1.7\\ 2.943\\ 6.62\\ 2.08\\ 412.3\\ 1.48\\ 4.89\\ 2.25\\ 1.7\\ 1.44\\ 2.95\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.95\\ 1.7\\ 2.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 2.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.7\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.85\\ 1.4\\ 0.97\\ 1\\ 1.42\\ 2.412\\ 2.412\\ 1.42\\ 1.42\\ 2.412\\ 1.42\\ 2.412\\ 1.42\\ 1.42\\ 2.412\\ 1.42\\ 1$	$\begin{array}{c} 5\\ 15\\ 54\\ 10\\ 25\\ 15\\ 15\\ 55\\ 30\\ 10\\ 10\\ 10\\ 5\\ 7\\ 410\\ 21\\ 7\\ 5\\ 34\\ 15\\ 98\\ 22\\ 30\\ 7\\ 1487\\ 4\\ 15\\ 8\\ 5\\ 4\\ 9\\ 5\\ 8\\ 4\\ 3\\ 5\\ 4\\ 99\\ 11\\ 54\\ 10\\ 5\\ 69\end{array}$	$\begin{array}{c} 2\\ 1\\ 2\\ 1\\ 2\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 1\\ 1\\ 1\\ 1\\ 0\\ 1\\ 1\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\$
Education	The sent information fields used to launch CRM system Ability and potential of the institute to meet the needs of the clients The expectations and requests of the clients The legal services provided Performance of annual plans The goods sent from the warehouse and procurement section The trained learners Visiting professors Educational experts Specific educational contracts New courses Abolished courses The educational contracts New sub-disciplines The analyzed opinions and requests of the education clients The analyzed opinions and requests of the education clients The request opinions and requests of the education clients The analyzed opinions and requests of the educations The request dopinions and instructions The request dopinions and instructions The request set on the library The sent request to the library The sent requests to formulate the instructions and processes The growed gent and force, change the personnel status and training the publication unit The sent requests to formulate the instructions and processes The growed set of the instructions and processes The sent request to receive financial resources The sent request to receive financial resources The sent requests to receive financial resources The sent request to formulate the instructions and processes The request to formulate the instructions and processes The sent request to receive financial resources The sent request	Intermediate measure Input Output Output Output Output Output Output Intermediate measure Intermediate measu	number amount amount number	$\begin{array}{c} 13\\ 3.2\\ 4.8\\ 2\\ 3.2\\ 2066\\ 2966\\ 3.3\\ 2\\ 1\\ 4.6\\ 2.9\\ 1\\ 5\\ 10\\ 5\\ 3\\ 4\\ 198\\ 10\\ 4\\ 5\\ 3\\ 3.4\\ 6\\ 4\\ 9\\ 2\\ 236\\ 6\\ 73\\ 2210\\ 70\\ 65\\ 5\\ 1\\ 1\\ 7\\ 3\\ 283\\ 26\\ 4\end{array}$	$\begin{array}{r} 24.12\\ 1.18\\ 0.4\\ 1.86\\ 0.85\\ 251.46\\ 300.12\\ 14.27\\ 0.49\\ 1.70\\ 0.74\\ 0.49\\ 1.04\\ 0.72\\ 3.27\\ 6.21\\ 3.09\\ 2.21\\ 1.327\\ 3.19\\ 2.21\\ 1.325\\ 1.83\\ 0.49\\ 2.21\\ 1.35\\ 3.19\\ 2.21\\ 1.83\\ 0.49\\ 3.68\\ 2.46\\ 5.17\\ 1.47\\ 225.52\\ 4.48\\ 1.59.13\\ 8.7\\ 10.81\\ 6.26\\ 0.85\\ 3.28\\ 318.12\\ 14.3\\ 3.48\\ \end{array}$	$\begin{array}{c} 69\\ 5\\ 5\\ 7\\ 4\\ 8\\ 50\\ 910\\ 264\\ 49\\ 8\\ 7\\ 3\\ 5\\ 31\\ 4\\ 11\\ 22\\ 10\\ 6\\ 7\\ 412\\ 15\\ 7\\ 11\\ 7\\ 4\\ 16\\ 9\\ 917\\ 82\\ 823\\ 79\\ 9\\ 22\\ 18\\ 823\\ 79\\ 922\\ 3\\ 2\\ 18\\ 15\\ 9924\\ 56\\ 10\\ \end{array}$	$\begin{array}{c} 0\\ 0\\ 2\\ 4\\ 0\\ 2\\ 18\\ 42\\ 218\\ 25\\ 0\\ 0\\ 4\\ 28\\ 0\\ 0\\ 4\\ 28\\ 0\\ 0\\ 0\\ 1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\$
ITC	The attracted man force that have been trained and developed for IT unit The sent ideas and suggestions The goods sent from warehouse and procurement section The request sent by the hardware and software services to install and launch the communication and customer affairs units The request sent by the hardware and software services to install and initiate from the education unit Performance of annual plans Receiving the required financial resources The request sent by the hardware and software services from all units The sent request to formulate instructions and processes The sent information fields used to launch CRM The information fields sent for good performance Similar breakdowns The processes and instructions formulated for all units The processes and instructions formulated by the communication and customer affairs unit	Input Input Input Input Input Input Input Input Input Output Output Output Intermediate measure	number number number number amount amount amount number number number number	$5 \\ 5 \\ 5 \\ 3 \\ 10 \\ 3.71 \\ 8 \\ 7 \\ 4 \\ 13 \\ 1 \\ 4 \\ 5 \\ 2$	$\begin{array}{c} 3\\ 3.18\\ 3.36\\ 1.62\\ 6.36\\ 1.11\\ 4.56\\ 4.68\\ 2.71\\ 24.21\\ 1.04\\ 2.27\\ 3.27\\ 1.21\\ \end{array}$	$\begin{array}{c} 10 \\ 10 \\ 5 \\ 20 \\ 5 \\ 15 \\ 15 \\ 9 \\ 69 \\ 3 \\ 7 \\ 10 \\ 4 \end{array}$	$egin{array}{cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $

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